IOSR Journal of Business and Management (IOSR-JBM)
e-ISSN:2278-487X, p-ISSN: 2319-7668. Volume 28, Issue 7. Ser. 1 (July. 2026), PP 07-13
www.iosrjournals.org

A Linear Approach To Solve Fuzzy Assignment Problem With The Help Of Yager's Ranking And Membership Function

Author

Abstract
This paper presents a specialized linear programming approach to solve fuzzy assignment problem with the help of yager's ranking and membership function. As we know traditional models assume precise cost or time parameters, real world scenarios often involved inherent ambiguity and imprecise data. To address this, we employ membership function to transform fuzzy parameters into crisp numerical values effectively reducing uncertainty in the assignment matrix. The resulting deterministic model solved using LP techniques. Comparative analysis demonstrates that the proposed method not only streamlines the computational process but also provided more robust results than traditional assignment problem. A numerical example is given to validate the accuracy and efficiency of fuzzy based approach in optimizing resource allocation.
Keywords: *Assignment problem, Linear programming, Hungarian method.*

Date of Submission: 23-06-2026 Date of Acceptance: 03-07-2026

I. Introduction:

In the context of business operations, the optimal distribution of limited resources is a highly significant managerial endeavour [1-4]. It demands a holistic and foundational approach to facilitate accurate decision-making. As stated in the Operational Researcher Quarterly (1962) [5-8], an assignment model is characteristically linked to a transportation challenge where the primary objective is to minimize the costs of assigning various jobs to multiple individuals or facilities, ensuring that each individual or facility is tasked with only one job. The conventional solution to the Assignment Problem (AP) is the Hungarian Algorithmic Approach (HAA), which was pioneered by Kuhn in 1955 [9-11]. Broadly speaking, it is also an integral part of the linear programming model that underscores the challenges of assigning each facility to a single job to optimize efficiency [12-14]. Its configuration aligns with the transportation problem model, where both supply at sources and demand at the destination are limited to a singular entity.

The aim of this classical linear methodological model is to guarantee that management operations are conducted effectively, utilizing a scientific quantitative management approach. This approach is anticipated to facilitate the determination of the minimum cost associated with assigning workers to specific jobs or allocating certain machines to workstations by adhering to optimal operational scenarios. Optimality is attained when the total cost is minimized and overall profit is maximized [4], [15-17]. In practice, within various industrial operational activities, managers encounter critical decision points that compel them to pursue an optimal assignment of 'n' facilities to 'j' jobs [6], [18-20]. Primarily, during the scheduling of operational activities, it is essential to assign optimal resources to achieve the organization's predefined ultimate goal.

The existing case is also unable to carry out and refine the assignment operation, which compels it to incur an extra 10% in quarterly operational expenses beyond its pre-planned budget. This is a result of the failure to optimally assign the 'i' operator to the 'M/c' machine through the application of quantitative management techniques. In most previous research, the assignment model has been employed, emphasizing mathematical calculations for both balanced and unbalanced models [14], [19]. For example, a number of scholars have proposed various methods to solve generalized Assignment Problems (APs) in real-world contexts [7], [19-20]. An unbalanced matrix is adjusted to a balanced form or square matrix by adding sufficient numbers of dummy (imaginary worker or machine) elements, highlighting the challenges that problem analysts must confront, with all costs for these newly added columns or rows being zero, thereby not modifying the pre-defined objective function.

Mathematical Model
The mathematical model of the assignment problem can be stated as linear programming problem:

$$\text{Min} \quad Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{for all i (recourse availability)}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad \text{for all j (activity requirement)}$$

and $\qquad x_{ij} = 0 \; or \; 1 \quad$ for all i and j

Where c_{ij} represents the cost of assignment of recourse i to activity j. Then consider a problem of assignment of n resource to n activities so as to minimize the overall cost or time in such a way that each source can be associate with one and only one job. The effectiveness matrix is given as under:

Table 1

	Activity					Available
Resource		A_1	A_2	⋯	A_n	
	R_1	C_{11}	C_{12}	⋯	C_{1n}	1
	R_2	C_{21}	C_{22}	⋯	C_{2n}	1
	⋮	⋮	⋮	⋮	⋮	⋮
	R_n	C_{n1}	C_{n2}	⋯	C_{nn}	1
Required		1	1	⋯	1	

Let x_{ij} denote the assignment of facility i to j such that

$$x_{ij} = \begin{cases} 1 & if\ facility\ i\ is\ assigned\ to\ job\ j \\ 0 & otherwise \end{cases}$$

Some definitions:

1. Yager's ranking function: yager's ranking function is a defuzzification technique used to convert fuzzy numbers into crisp values for comparison and optimization. It is based on the concept of averaging the mid points of all possible alpha cuts of a fuzzy number. Formally for a fuzzy number $\tilde{A}$ with membership function $\mu_A(x)$, the ranking indexed is defines as

$$R(\tilde{A}) = \int_0^1 \frac{1}{2}(A_L(\alpha) + A_U(\alpha))d\alpha$$

Where:

- $A_L(\alpha)$= lower bound of the α-cut of $\tilde{A}$
- $A_U(\alpha)$= upper bound of the α-cut of $\tilde{A}$
- $R(\tilde{A})$= crisp ranking value of the fuzzy number

In this paper we use triangular fuzzy number (a,b,c) for which yagers ranking function simplifies to

$$R(\tilde{A}) = \frac{a+b+c}{3}$$

2. Alpha cut: Let $\tilde{A}$ be a fuzzy set defined on a universe of discourse X with membership function $\mu_A(x)$.For any $\alpha \in [0,1]$, the **α-cut** (or α-level set) of $\tilde{A}$ is the crisp set of all elements in X whose membership degree is greater than or equal to α.

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$$

3. Membership function: In this paper, we will fuzzy our penalties (objectives) such as (transportation cost, delivery time, etc), to convert them from crisp region to fuzzy region (solution space) to minimize a set of (p) objectives, the membership function used for that is defined as follows:

$$M_r(x_{ij}^r) = \begin{cases} 1, x_{ij}^r \leq L_r \\ \frac{U_r - x_{ij}^r}{U_r - L_r}, L_r \leq x_{ij}^r \leq U_r \\ 0, x_{ij}^r \geq U_r \end{cases} \qquad (1)$$

Where L_r *is* the lowest crisp value of x_{ij}^r and U_r is the highest crisp value of x_{ij}^r

Simplex method:

Step 1. Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximizing it by using the result

$$Minimum\ z = -Maximum\ (-z)\ldots\ldots\ldots (2).$$

Step 2. Here, we check all values of b_i (*i = 1, 2, ... m*) are non-negative. If the value of any b_i is negative then we multiply with -1 respective inequation of the constraints, so we get all b_i (*i = 1, 2, ... m*) are non-negative.

Step 3. Introducing slack/surplus variables to change all the inequations of the constraints into equations. Here we have the value of variables equal to zero.

Step 4. To find an initial basic feasible solution to LP problem in the form $x_B = B^{-1}b$ and put it in the first column of the simple table.

Step 5. Compute the net evaluations $z_j - c_j$ (*j = 1, 2, ... n*) by using the relation $z_j - c_j = c_B y_j - c_j$ where $y_j = B^{-1}a_j$.

Inspect the sign $z_j - c_j$as follows

(i) If all $z_j - c_j \geq 0$ then the initial basic feasible solution x_B is an optimum basic feasible solution.

(ii)If at least one $z_j - c_j < 0$, proceed on to the next step.

Step 6. If there are more than one negative $z_j - c_j$, then choose the most negative of them. Let it be $z_r - c_r$, for some j = r.
i. If all $z_j - c_j \geq 0$ then the initial basic feasible solution x_B, is an optimum basic feasible solution.
ii.If at least one $z_j - c_j < 0$ proceed on to next step,

Step 7. Calculate the ratios $\{\frac{xBi}{}, y > 0, i = 1,2,\cdots,m\}$ and pick the minimum of them. Let the minimum y_{ir} ir of these ratios be $\frac{xBk}{}$ Then the vector y will level the basis y. The common element y which is in the y_{kr} k Bkr k^th^ row and the r^th^ column is recognized as the leading element (or pivotal element) of the table.

Step 8. Change the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zeroes by making use of the relations:

$$\hat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir} \qquad i = 1,2,\cdots,m+1; i \neq k$$

and

$$\hat{y}_{kj} = \frac{y_{kj}}{y_{kr}} \qquad i = 0,1,2,\cdots,n$$

Step 9. Go to Step 5 and replication the solving procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

Numerical problem: here we consider triangular balanced fuzzy assignment problem.
Problem: A logistic company has four delivery agents and four delivery zones. The cost of assigning each agent to each zone is given below:

Table 2: Balanced Assignment Problem

Agents/Zones	Z1	Z2	Z3	Z4
A1	(7,8,9)	(5,6,7)	(6,7,8)	(4,5,6)
A2	(5,6,7)	(6,7,8)	(7,8,9)	(5,6,7)
A3	(6,7,8)	(7,8,9)	(5,6,7)	(6,7,8)
A4	(4,5,6)	(5,6,7)	(6,7,8)	(7,8,9)

Now applying yagers ranking function to change fuzzy number into crisp value, the new matrix is given as below,

Table 3

Agents/Zones	Z1	Z2	Z3	Z4
A1	8	6	7	5
A2	6	7	8	6
A3	7	8	6	7
A4	5	6	7	8

Now assignment problem changes into linear programming problem as follows
$Min\ Z = 8x_{11} + 6x_{12} + 7x_{13} + 5x_{14} + 6x_{21} + 7x_{22} + 8x_{23} + 6x_{24} + 7x_{31} + 8x_{32} + 6x_{33} + 7x_{34} + 5x_{41} + 6x_{42} + 7x_{43} + 8x_{44}$

Subject to constraint
$x_{11} + x_{12} + x_{13} + x_{14} = 1$
$x_{21} + x_{22} + x_{23} + x_{24} = 1$
$x_{31} + x_{32} + x_{33} + x_{34} = 1$
$x_{41} + x_{42} + x_{43} + x_{44} = 1$
$x_{11} + x_{21} + x_{31} + x_{41} = 1$
$x_{12} + x_{22} + x_{32} + x_{42} = 1$
$x_{13} + x_{23} + x_{33} + x_{43} = 1$
$x_{14} + x_{24} + x_{34} + x_{44} = 1$
$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34}, x_{41}, x_{42}, x_{43}, x_{44} \geq 0$

To reduce numerical value, apply membership function in the above table with help of equation (1) then obtained following linear programming problem as follows
$Min\ Z = 0.00x_{11} + 0.80x_{12} + 0.40x_{13} + 0.60x_{14} + 0.80x_{21} + 0.60x_{22} + 0.20x_{23} + 0.80x_{24} + 0.60x_{31} + 0.40x_{32} + 0.80x_{33} + 0.60x_{34} + 1.00x_{41} + 0.80x_{42} + 0.20x_{43} + 0.40x_{44}$

Subject to constraint
$x_{11} + x_{12} + x_{13} + x_{14} = 1$
$x_{21} + x_{22} + x_{23} + x_{24} = 1$
$x_{31} + x_{32} + x_{33} + x_{34} = 1$
$x_{41} + x_{42} + x_{43} + x_{44} = 1$
$x_{11} + x_{21} + x_{31} + x_{41} = 1$
$x_{12} + x_{22} + x_{32} + x_{42} = 1$
$x_{13} + x_{23} + x_{33} + x_{43} = 1$
$x_{14} + x_{24} + x_{34} + x_{44} = 1$
$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34}, x_{41}, x_{42}, x_{43}, x_{44} \geq 0$

To apply simplex method firstly introducing artificial variable, we found that
$Min\ Z = 0.00x_{11} + 0.80x_{12} + 0.40x_{13} + 0.60x_{14} + 0.80x_{21} + 0.60x_{22} + 0.20x_{23} + 0.80x_{24}$
$+ 0.60x_{31} + 0.40x_{32} + 0.80x_{33} + 0.60x_{34} + 1.00x_{41} + 0.80x_{42} + 0.20x_{43} + 0.40x_{44}$
$+ MA_1 + MA_2 + MA_3 + MA_4 + MA_5 + MA_6 + MA_7 + MA_8$

Subject to constraint
$x_{11} + x_{12} + x_{13} + x_{14} + MA_1 = 1$
$x_{21} + x_{22} + x_{23} + x_{24} + MA_2 = 1$
$x_{31} + x_{32} + x_{33} + x_{34} + MA_3 = 1$
$x_{41} + x_{42} + x_{43} + x_{44} + MA_4 = 1$
$x_{11} + x_{21} + x_{31} + x_{41} + MA_5 = 1$
$x_{12} + x_{22} + x_{32} + x_{42} + MA_6 = 1$
$x_{13} + x_{23} + x_{33} + x_{43} + MA_7 = 1$
$x_{14} + x_{24} + x_{34} + x_{44} + MA_8 = 1$
$x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{34}, x_{41}, x_{42}, x_{43}, x_{44}, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8 \geq 0$

Now apply all the necessary steps of simplex method and obtained the optimal solution as follows

Table 4: Optimal Solution Balanced Assignment Problem with Simplex Method

	C_j	0.4	0	0.6	0.2	0.8	0.2	1	0.4	0.6	0.4	0.8	0.6	0	0	0	0	
C_B	Basic	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	A 1	A_2	A_3	A_4	RHS
0	x_{12}	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1
0.4	x_{24}	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	1
0.6	x_{31}	1	0	1	0	1	0	1	0	1	1	1	0	0	-1	0	-1	1
0	A_1	0	0	-1	0	0	0	-1	0	0	0	-1	0	1	1	0	1	0
0.2	x_{22}	0	0	0	-1	1	1	1	0	0	0	0	-1	0	0	0	-1	0
0	A_3	0	0	1	0	0	0	1	0	0	0	-1	0	0	0	1	0	1
0.4	x_{32}	-1	0	-1	0	-1	0	-1	0	0	0	0	1	0	1	0	1	0
Z_j	Z = 1	0.2	0	0.2	0.2	0.4	0.2	0.4	0.4	0.4	0.6	0.6	0.6	0	-0.2	0	0	
$Z_j - C_j$		-0.2	0	-0.4	0	-0.4	0	-0.6	0	0	0	-0.2	0	0	-0.2	0	0	

Hence the optimal solution is arrived with the following values as follows
$x_{11} = 1,$ $x_{23} = 1,$ $x_{32} = 1,$ $x_{44} = 1$
Put the above value in equation (2) and get optimal solution is as follows
$Min\ Z = 8 * 1 + 8 * 1 + 8 * 1 + 8 * 1$
$Min\ Z = 8 + 8 + 8 + 8$
$Min\ Z = 32\ Rs$

Table 4: Comparative Table of Optimal Solution of Balanced Problem

Name of Method	Allocation of Work	Optimal Solution
Hungarian Method	$x_{11} = 1, x_{23} = 1, \ x_{32} = 1, \ x_{44} = 1$	32 Rs
Simplex Method	$x_{11} = 1, x_{23} = 1, \ x_{32} = 1, \ x_{44} = 1$	32 Rs

II. Result And Discussion:

The obtained results demonstrate the effectiveness of the proposed approach in solving the balanced fuzzy assignment problem. After applying the membership function the numerical complexity of the data was significantly reduced, enabling linear transformation into a linear programming model. The numerical example shows that both the Hungarian method and the simplex method yielded identical optimal allocations with a minimum cost of 32 Rs, indicating consistency and reliability of both techniques.

III. Conclusions:

This study concludes that the integration of yagers and membership function with optimization techniques provides an efficient framework for solving fuzzy assignment problems. The transformation of problem into linear programming models simplifies computation and enhances solution accuracy. The comparative analysis reveals that while both the simplex and Hungarian method perform equally well for balanced problem. Overall, the proposed methodology improves decision making by reducing computational difficulty and ensuring optimal allocation. Future work may focus on extending this approach to fuzzy Multi objective assignment problems and real-world large-scale applications.

Reference:

[1]. Khanna, S., Dholakia, T., Prakash, S., Gupta, S., & Jain, U. (2021). Application Of The Hungarian Assignment Method In The Aviation Industry. International Journal Of Innovative, Science,6(10),300-305. Https://Ijisrt.Com/Assets/Upload/Files/IJISRT21OCT220.Pdf

[2]. Akpan, N. P., & Abraham, U. P. (2016). A Critique Of The Hungarian Method Of Solving Assignment Problem To The Alternate Method Of Assignment Problem By Mansi. International Journal Of Sciences: Basic And Applied Research, 29(1), 43–56. Https://Www.Researchgate.Net

[3]. Schäfer, F., Walther, M., Hübner, A., & Kuhn, H. (2019). Operational Patient-Bed Assignment Problem In Large Hospital Settings Including Overflow And Uncertainty Management. Flexible Services And Manufacturing Journal, 31, 1012-1041. Https://Doi.Org/10.1007/S10696-018-9331-0

[4]. Limbong, T., Simarmata, J., Fauzi, A., Siagian, P., Tarigan, P., Katolik, U., & Medan, D. (2018). Optimization Of Employee Assignment In Content Management System Making With Hungarian Method. Proceedings Of The Joint Workshop Ko2pi And The 1st International Conference On Advance And Scientific Innovation (Pp. 84-91). EAI. Https://Doi.Org/10.4108/Eai.23-4-2018.2277594

[5]. Rai, N., & Khan, A. J. (2019). A Brief Review On Classic Assignment Problem And Its Applications,09(9),73–81. Https://Iosrjen.Org/Papers/Vol9_Issue9/Series1/M0909017381.Pdf

[6]. Jain, L. (2022). Finding An Optimal Solution Of Assignment Problem. YMER Digital, 21(08), 259–272. Https://Ymerdigital.Com/Uploads/YMER210822.Pdf

[7]. Gong, X., Li, H., & Yang, X. (2022, April). A Task Planning Simulation Method Based On Hungarian Algorithm. Journal Of Physics: Conference Series (Vol. 2219, No. 1, P. 012041). IOP Publishing. Https://Doi.Org/10.1088/1742- 6596/2219/1/012041

[8]. Jain, L., & Singh, C. (2022). Finding An Optimal Solution Of Assignment Problem. YMER Digital, 21(8), 259–272. Https://Doi.Org/10.37896/Ymer21.08/23

[9]. Ammar, A., Pierreval, H., & Elkosentini, S. (2013). Workers Assignment Problems In Manufacturing Systems: A Literature Analysis. Proceedings Of 2013 International Conference On Industrial Engineering And Systems Management (IESM) (Pp. 1-7). IEEE. Https://Ieeexplore.Ieee.Org/Abstract/Document/6761486

[10]. Cui, H., Zhang, J., Cui, C., & Chen, Q. (2016). Solving Large-Scale Assignment Problems By Kuhn-Munkres Algorithm Hong Cui1 Jingjing Zhang2 B Chunfeng Cui3 C Qinyu Chen4 D [Presentation]. 2nd International Conference On Advances In Mechanical Engineering And Industrial Informatics (AMEII 2016) (Pp. 822-827). Https://Www.Atlantis- Press.Com/Article/25854748.Pdf

[11]. Alam, S. T., Sagor, E., Ahmed, T., Haque, T., Mahmud, M. S., Ibrahim, S., ... & Rebate, M. (2022). Assessment Of Assignment Problem Using Hungarian Method. Proceedings Of The First Australian International Conference On Industrial Engineering And Operations Management (Pp. 1-11). IEOM Society International. Https://Ieomsociety.Org/Proceedings/2022australia/498.Pdf

[12]. Kabiru, S., Saidu, B. M., Abdul, A. Z., & Ali, U. A. (2017). An Optimal Assignment Schedule Of Staff-Subject Allocation. Journal Of Mathematical Finance, 7(04), 805. Https://Www.Scirp.Org/Html/1-1490461_79506.Htm

[13]. Iriania, Y., & Andrian, T. (2019). Optimization Of Employee Assignment Problem Base On Time Work Settlement By Using Hungarian Methods. International Journal Of Innovation, Creativity And Change, 6(5), 197-207. Https://Www.Ijicc.Net/Images/Vol6Iss5/6517_Iriani_2019_E_R.Pdf

[14]. Lavanya, M., & Santhi, B. (2023). Retraction Note To Hungarian Optimization Technique Based Efficient Resource Allocation Using Clustering Unbalanced Estimated Cost Matrix. Journal Of Ambient Intelligence And Humanized Computing, 14(S1), 521. Https://Doi.Org/10.1007/S12652-022-04228-7

[15]. Keyser, R. S., & Khalifa, H. A. E. W. (2024). A Fuzzy Goal Programming Approach For Solving Multi-Objective Minimum Cost Flow Problems With Possibilistic Coefficients. Uncertaintydiscourseandapplications,1(1),2940, Https://Uda.Reapress.Com/Journal/Article/View/20

[16]. Gaspars-Wieloch, H. (2021). The Assignment Problem In Human Resource Project Management Under Uncertainty. Risks, 9(1), 25. Https://Doi.Org/10.3390/Risks9010025 Niknafs, A., Denzinger, J., & Ruhe, G. (2013). A Systematic Literature Review Of The Personnel Assignment Problem [Presentation]. Proceedings Of The International Multiconference Of Engineers And Computer Scientists, Hongkong. Https://Www.Iaeng.Org/Publication/IMECS2013/IMECS2013_Pp1121-1128.Pdf

[17]. Niknafs, A., Denzinger, J., & Ruhe, G. (2013). A Systematic Literature Review Of The Personnel Assignment Problem

[Presentation]. Proceedings Of The International Multiconference Of Engineers And Computer Scientists, Hongkong. Https://Www.Iaeng.Org/Publication/IMECS2013/IMECS2013_Pp1121-1128.Pdf
[18]. Prajapati, P. R., Haque, A., Jain, J., & Singh, P. S. (2021). A Study On Solving Assignment Problem. International Journal For Research And Innovation, 1(4), 21–26. Https://Www.Viva-Technology.Org/New/IJRI/2021/107.Pdf
[19]. Kadhim, H. J., Shiker, M. A., & Al-Dallal, H. A. (2021, July). A New Technique For Finding The Optimal Solution To Assignment Problems With Maximization Objective Function. Journal Of Physics: Conference Series (Vol. 1963, No. Pap1, P. 012104). IOP Publishing. Https://Iopscience.Iop.Org/Article/10.1088/1742-6596/1963/1/012104/Meta
[20]. Hidri, L., Gazdar, A., & M. Mabkhot, M. (2020). Optimized Procedure To Schedule Physicians In An Intensive Care Unit: A Case Study. Mathematics, 8(11), 1976. Https://Doi.Org/10.3390/Math8111976