# An Urban Mass Transit Optimization Model for the Federal Capital Territory in Nigeria 

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#### Abstract

In accomplishing its mission, the Abuja Urban Mass Transport Company (AUMTCO)ofthe Federal Capital Territory in Nigeria randomly multiplied urban mass transit buses along city routes.This strategy did not seem to solve the magnitude of the problems as it increased the risk of accidents, congestion and stampede thereby leading to further delay to destinations and loss of properties via theft. This is beside the challenges of long waiting time, long queue of passengers, lateness to work which continued to characterize the system over time due to this non-scientific approach. An immediate solution to this problem is therefore to develop an efficient bus Scheduling Shift System for minimizing the total number of buses in operation across routes.It is on this backdrop that this study is focused on formulating an Urban Mass Transit Optimization Model via two Shift Systems namely; the Capacity-Mix and the Non-Capacity-Mix Shift System using the Integer Linear Programming methodology for optimizing the Abuja Mass Transit System on seven routes.The Capacity-Mix Shift System for Gwagwalada route on Tuesdays gave an optimal number of 10 buses that can sufficiently convey the passengers with shift 1 assigned 2 Marcapolo and 2 Ashok buses; shift 2 assigned 1 Yutong bus and shift 3 assigned 1 Marcapolo and 4 Ashok buses. While the Non-Capacity-Mix Shift System on this same route on Mondays, stipulates that 13 Marcapolo buses should be assigned to this route and distributed as 5, 2 and 6 buses across shifts 1, 2, and 3 respectively. The study successfully determined that either Shift System can be applied to the Zuba and Nyanya routes for each weekday since they yield approximately the same result.


Keywords: Urban Mass Transit, Shift Model, Integer Progranming

## I. Introduction

Abuja is the capital city of Nigeria. It is a planned city and was built mainly in the 1980s. It officially became Nigeria's capital on 12th December 1991, replacing Lagos as the headquarter. As at 2006 census, the city of Abuja had a population of 776,298 , making it one of the ten most populous cities in Nigeria. According to the United Nations, Abuja grew at the rate of $139.7 \%$ between 2000 and 2010, making it the fastest growing city in the world. As at 2015, the city experienced an annual growth of at least $35 \%$, still retaining its position as the fastest growing city on the African continent and one of the fastest in the world. Abuja has witnessed a huge influx of people into the city; the growth has led to the emergence of satellite towns such as Karu Urban Area, Suleja, Gwagwalada, Lugbe, Kuje and smaller settlements to which the planned city is sprawling.

Abuja Main Attractions are:Zuma Rock, Aso Rock, and The Minister's Hill. Other Attractions in Abujainclude;The National Arboretum, Abuja Stadium, Guerara Falls, The National Church of Nigeria and The National Mosque of Nigeria. Indeed, the population growth of Abuja city cannot be overemphasized, hence the need for an organized transportation system.

With the rate of population growth in Nigeria, especially in Abuja, the intra-urban transport is becoming worrisome because the demand for movement of goods and passengers increases per day. It was on this backdrop that the Federal Capital Territory Administration (FCTA) established an Urban Mass Transport Company (AUMTCO) on 13th November, 1989, registered under the companies and allied matters Act 1990. The company's mission statement reads "Our mission is to provide the best value for more and safest, most reliable schedule and bus hire services in Nigeria".

This mission if accomplished is believed will alleviate the transport challenges of citizens dwelling in and around Abuja. In order to achieve this, over 1.5 billion naira was spent to procure 192 buses to ply different satellite town within the city at affordable price to solve the mobility challenges. Among those challenges are long waiting time, long queue of passengers' at various bus stops, stampeding due to rush, loss of valuable items as a result of struggle for buses, lateness to work, among others.This situation has reached an alarming
proportion which has attracted the attention of the FCTA by procuring additional 200 high capacity buses for transportation within FCT in order to improve its availability, reliability, safety, efficiency, comfortability, accessibility and convenience. However, this strategy does not seem to solve the magnitude of problems with the present developments in Abuja because, excessive addition of buses for public transport increases the risk of accidents, congestion and stampede thereby leading to further delay to destinations and loss of properties via theft.

An immediate solution to this problem is therefore not the multiplication of Buses, but an efficient Bus Scheduling Shift System for minimizing the total number of busesin operationacross routes, considering the passenger demands for Buses, the peak days and peak period of the day in such a way that will benefit both the passengers and the management of AUMTCO. It is on this backdrop that the researchers focused at formulating two shift system models namely; the Capacity-Mix and the Non Capacity-Mix Shift System Integral Linear Programming Models for optimizing the Abuja Mass Transit system.

## II. Literature Review

There are vast applications of Linear Programming to Bus Scheduling Problems. The following applications are few among so many; Bixuan (2015) developed a framework that optimizes Bus assignments to routes with the objective of minimizing both operating costs and the environmental impacts of emissions. The optimization model is applied in a case study of Metro Transit in Minneapolis. The results of this study showed a set of tradeoff relationships between operating costs and emissions. The optimized vehicle assignments generated by the model significantly reduce both the operating costs and emissions of the current fleet.

Also on the list is the work of Eshetieet al. (2014) on the "modeling and analysis of bus scheduling systems of urban public bus transportation" using the Anbessa City Bus Service Enterprise (ACBSE) as case study. The finding of the study showed that the new model formulated reveals better performances in the operating cost, bus utilization, trips and distance covered compared with the existing scheduling system. The company's bus utilization improved by the new system and cut costs on the one hand and improved the service quality to passengers on the other hand.

Continuing on this list is the work of Ceder (2011) where he author formulated an optimization framework to address the vehicle scheduling problem while considering the characteristics of different trips (urban, inter-city, etc.) and the vehicle type required for the particular trip. The objective is to minimize total operating plus deadheading costs by changing the numbers of different types of buses.

Further on this listis Li and Head (2009) who carried out a research on bus scheduling problem with the aim of minimizing operating costs and vehicle emissions, under the constraints of a limited budget to purchase new buses and a timetable of bus trips. The authors develop a time-space network to optimize the vehicle movements needed to cover all routes on a timetable, and include emission constraints and penalties in its formulations. The outcome of their study significantly improved the operation cost of the company under study.

Further in this vein, Van (2008) carried out a research on "Integrating timetabling and vehicle scheduling in public Bus transportation". The finding of his research showed that a significant reduction of the operational cost can be achieved by optimization of the type and number of vehicles performing a service trip and by splitting service trips.

Considering the vast and successful application of the Liner Programming in solving Bus Scheduling problems, the bus assignment and scheduling problems faced by AUMTCO was modeled and analyzed using Linear Programming (LP) with integer constraints. Implementation was by the Branch and Bound (B\&B) algorithm with the sole aim of determining the optimum bus assignments that improved the existing bus schedule and assignment system. Hence, this research focused on developing demand oriented Integral Linear Programming (ILP) model for the bus assignment problem of AUMTCO. The rest of the paper is sectioned as follows; Methodology, Result, Discussion, Conclusion and Recommendation.

## III. Methodology

This section focuses on the source and nature of data used for the research, an over-view of the linear programming problem, integer programming (Branch and Bound Algorithm) and sensitivity analysis.

## 1. Sources and Nature of Data

The data for this research work was collected from the head office of Abuja Urban Mass Transport Company (AUMTCO) andfrom the field. Certainly, information gathered will be both primary and secondary. In addition to this, informal interviews as well as expert opinionwould be sourced where needed.Empirical data on ridership for each time shifts known as demands, number of trips per day and bus capacities will be sourced as both primary and secondary information while the number of routes, time of operation, mode of operations, bus distribution to route would be sourced as secondary data from the head office.

## 2. Model Formulation

The general linear programming problem (LPP) is stated in this section and the model is applied to reflect the bus scheduling problem at hand. Details of the method of solution are also presented.

### 2.1. General Linear Programming problem (LPP)

The general linear programming problem is given below;
Optimize: $\mathrm{Z}=C^{T} X$
Subject to;
$A X \leq b$
With $\begin{gathered}\mathrm{X} \geq 0 \\ \mathrm{~A}=\left[\begin{array}{ccc}a_{11} & a_{12} \ldots & a_{1 n} \\ a_{21} & a_{22} \ldots & a_{2 n} \\ \vdots & \vdots & \vdots \\ a_{m 1} & a_{m 2} \ldots & a_{m n}\end{array}\right], \mathrm{b}=\left[\begin{array}{c}b_{1} \\ b_{2} \\ \vdots \\ b_{m}\end{array}\right], \quad \mathrm{X}=\left[\begin{array}{c}x_{1} \\ x_{2} \\ \vdots \\ x_{m}\end{array}\right], \mathrm{c}=\left[\begin{array}{c}c_{1} \\ c_{2} \\ \vdots \\ c_{m}\end{array}\right]\end{gathered},=$...
Where X is a $m \times 1$ column vector of unknowns including all slacks, surplus and artificial variables. $\mathrm{C}^{\mathrm{T}}$ is a $1 \times n$ row vector of corresponding costs (Objective function coefficients). A is a $m x n$ coefficient matrix of the constraints equations and $b$ is column vector of the right hand sides of the constraint equations.

### 2.2.Model Formulation for Abuja Urban Mass Transport Problem

The model to be formulated is a time shift system with the sole aim of solving the Abuja Mass Transport Problem. We begin with a diagram for the time shift given below.

## Time Shift Chart



Figure 1:Time Shift for each Route Covered by AUMTCO

## 1. Shift System I: Non Capacity-Mix

This shift system uses the same bus type (same capacity) across shifts. It is modeled as;
Minimize $Z=\left(\begin{array}{llllll}1 & 1 & 1 & 1 & \ldots & 1\end{array}\right)$

$$
\left(\begin{array}{c}
x_{1} \\
x_{2} \\
x_{3} \\
\vdots \\
x_{p}
\end{array}\right)
$$

subjectto;
$\left(\begin{array}{ccccc}2 c & 0 & 0 & \ldots & 0 \\ 0 & 2 c & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 2 c\end{array}\right)\left(\begin{array}{c}x_{1} \\ x_{2} \\ x_{3} \\ \vdots \\ x_{p}\end{array}\right) \geq\left(\begin{array}{c}D_{S_{1}} \\ D_{S_{2}} \\ D_{S_{3}} \\ \vdots \\ D_{S_{p}}\end{array}\right)$
$x_{1}, x_{2}, x_{3}, \ldots, x_{p} \geq 0$ andintegral

## 2.Shift System II: Capacity-Mix

This shift system uses different bus types (Mix Capacities) across shifts. It is modeled as;

$$
\begin{aligned}
\text { MinimizeZ }= & x_{11}+x_{12}+x_{13}+\cdots+x_{1 k}+x_{21}+x_{22}+x_{23}+\cdots+x_{2 k}+x_{31}+x_{32}+x_{33}+\cdots+x_{3 k} \\
& +\cdots+x_{x_{1} 1}+x_{p 2}+x_{p 3}+\cdots+x_{p k}
\end{aligned}
$$

```
\(2 C_{1} x_{11}+2 C_{2} x_{12}+2 C_{3} x_{13}+\cdots+2 C_{k} x_{1 k} \geq D_{s_{1}}\)
\(2 C_{1} x_{21}+2 C_{2} x_{22}+2 C_{3} x_{23}+\cdots+2 C_{k} x_{2 k} \geq D_{S_{2}}\)
\(2 C_{1} x_{31}+2 C_{2} x_{32}+2 C_{3} x_{33}+\cdots+2 C_{k} x_{3 k} \geq D_{s_{3}}\)
:!:!
\(2 C_{1} x_{p 1}+2 C_{2} x_{p 2}+2 C_{3} x_{p 3}+\cdots+2 C_{k} x_{p k} \geq D_{s_{p}}\)
    \(x_{11}, x_{12}, x_{13}, \ldots, x_{1 k}, x_{21}, x_{22}, x_{23}, \ldots, x_{2 k}, x_{31}, x_{32}, x_{33}, \ldots, x_{3 k}, \ldots, x_{p 1}, x_{p 2}, x_{p 3}, \ldots, x_{p k}\)
```

$\geq 0$ and integral.
Where
$2 C_{1}, 2 C_{2}, 2 C_{3}, \ldots, 2 C_{k}$ are the capacities of the buses per trip.
and
$x_{1}=x_{11}+x_{12}+x_{13}+\cdots+x_{1 k}$
$x_{2}=x_{21}+x_{22}+x_{23}+\cdots+x_{2 k}$
$x_{3}=x_{31}+x_{32}+x_{33}+\cdots+x_{3 k}$
::
$x_{p}=x_{p 1}+x_{p 2}+x_{p 3}+\cdots+x_{p k}$
where $x_{1}, x_{2}, x_{3}, \cdots, x_{p}$ are the number of buses required in each shift respectively.
The above model can be represented in matrix form below;
Minimize $\mathrm{Z}=\left(\begin{array}{llllllllll}x_{11} & x_{12} & \ldots x_{1 k}, & x_{21} & x_{22} \ldots & x_{2 k}, & \ldots x_{p 1} & x_{p 2} & \ldots & x_{p k}\end{array}\right)$
subjectto;
$\left(\begin{array}{ccccccccccccccccc}\boldsymbol{x}_{\mathbf{1 1}} & \boldsymbol{x}_{\mathbf{1 2}} & \ldots & \boldsymbol{x}_{\mathbf{1 k}} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \boldsymbol{x}_{\mathbf{2 1}} & \boldsymbol{x}_{22} & \ldots & \boldsymbol{x}_{\mathbf{2 k}} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \boldsymbol{x}_{\mathbf{3 1}} & \boldsymbol{x}_{\mathbf{3 2}} & \ldots & \boldsymbol{x}_{\mathbf{3 k}} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \boldsymbol{x}_{\boldsymbol{p} 1} & \boldsymbol{x}_{\boldsymbol{p} 2} & \ldots & \boldsymbol{x}_{\boldsymbol{p k}}\end{array}\right)\left(\begin{array}{c}2 c_{1} \\ 2 c_{2} \\ \vdots \\ 2 c_{5} \\ 2 c_{6} \\ \vdots \\ 2 c_{k}\end{array}\right) \geq\left(\begin{array}{c}D_{S_{1}} \\ D_{S_{2}} \\ D_{S 3} \\ \vdots \\ D_{S_{p}}\end{array}\right)$
$x_{11}, x_{12}, x_{13}, \ldots, x_{1 k}, x_{21}, x_{22}, x_{23}, \ldots, x_{2 k}, x_{31}, x_{32}, x_{33}, \ldots, x_{3 k}, \ldots, x_{p 1}, x_{p 2}, x_{p 3}, \ldots, x_{p k} \geq 0$ and integral.
The time shift model formulated is actually anInteger Linear Programming Problem (ILPP) as indicated by the constraints;
$x_{p} \geq 0$ and integral (Shift system 1)
and the constraint;
$x_{p k} \geq 0$ and integral (Shift system 2)
The integral condition is due to the fact that the number of buses cannot be fractional but integral. This necessitated a refinement of the fractional number of buses obtained by an integer programming methodology known as the Branch and Bound Algorithm (See section3.4)

### 2.3. The Simplex Method of Solving Linear Programming Problem (LPP)

This method is captured in the following algorithm;
Step one: Converting the linear programming problem to standard form so as to determine the starting basic feasible solution by setting n-m appropriate (non-basic) variables at Zero level.
Step two:Selectan entering variable from among the current non basic variables which when increased above zero, can improve the value of the objective function. If none exists, stop; the current basic feasible solution is optimal. Proceed to step 3

Step three: Select the leaving variable from among the basic variable that must be set to zero (become none basic) while entering variable becomes basic. Note that in minimization problems, theoptimality condition calls for selecting the entering variable as the nonbasic variable withthe most positive objective coefficient in the objective equation, the exact opposite ruleof the maximization case. This follows because max $z$ is equivalent to $\min (-z)$. As forthefeasibility condition for selecting the leaving variable, the rule remains unchanged
Step four: Determining the new basic solution by making the entering variable basic and the leaving variable non basic, until optimal solution is achieved.

### 2.4.Optimality Condition

Optimality condition states that in the case of maximization (minimization) if all the non-basic variables have non-negative (non positive) coefficient in the Z-equation of the current tableau (the current solution is optimal). Otherwise, the non-basic variable with the most negative (most positive) coefficient is selected as the entering variable.

### 2.5Feasibility Condition

This selects the leaving from among the current basic variables. This will be the variable, which has the minimum ratio of the RHS of the problem to the associated positive coefficient of the entering variable.

## 3. Branch and Bound method of solving Integer Linear Programming Problem (ILPP)

The branch and bound technique is explained by Bronson and Naadimuthur (1997) is captured under the following sub-headings;

### 3.1. First Approximation

An integer program is a linear program with the added requirement that all variables be integral. Therefore, a first approximation to the solution of any integer program may be obtained by ignoring the integer requirement and solving the resulting linear program. If the optimal solution to the linear program happens to be integral, then this solution is also the optimal solution to the original integer program. Otherwise, one may round the components of the first approximation to nearest feasible integers and obtain a second approximation.

### 3.2. Branching

If the first approximation contains a variable that is not integral, say $x_{j}{ }^{*}$, then $i_{1}<x_{j}{ }^{*}<i_{2}$ where $i_{1}$ and $i_{2}$ are consecutive, non negative integers.
Two new integer programs are then created by augmenting the original integer program with either the constraint $x_{j} \leq i$, or the constraint $x_{j} \geq i_{2}$.
This process, called branching, has the effect of shrinking the feasible region in a way that eliminates from further consideration of the current nonintegral solution for $x_{j}$ but still preserves all possible integral solutions to the original problem. This is called branching.

### 3.2. Bounding

Assume that the objective function is to be maximized. Branching continues until an integral first approximation (which is thus an integral solution) is obtained. The value of the objective for this first integral solution becomes a lower bound for the problem, and all programs whose first approximations, integral or not, yield values of the objective function smaller than the lower bound are discarded. This process is termed bounding.

## 4. Sensitivity Analysis

Sensitivity of the model (Shift System 1) to changes introduced by bus capacity mix would be deduced from the run of shift system 2 (i.e, model 2 ) across routes.

This is to enable the management to determine the optimal number of bus capacity mix that will minimize the total number of buses for each time shift across the routes.

### 4.1. Comparing the Existing Bus Schedule System with the Proposed System

The number of buses allocated to some selected routes in the existing system was compared with the number allocated to each time shift in the proposed shift system. The argument is that, the allotted number of buses per route in the existing system may not all be utilized in a shift.

### 4.2. Use of Software

Because of the large dimension of the matrix involved in this work, and for computational ease and accuracy, the Tora software package was be employed in implementing the integer programming algorithm.

## IV. Results

This chapter presentsthevarious bus types (Capacities, Fuel Consumption Rate per Kilometer, Percentage Breakdowns and Passengers' Preference Percentage) ofpassengers demand matrix, the optimal bus distribution plan for Abuja Urban Mass Transport Company, shift matrix and the results of the sensitivity analysis.

Table1: Bus Type, Capacity, Fuel Consumption Rate, Percentage Breakdowns and Passengers' preference

| Bus Type | Capacity | Fuel consumption per <br> kilometer | Percentage of <br> Breakdowns | Passengers' <br> preference (\%) |
| :--- | :--- | :--- | :--- | :--- |
| Marcapolo | 49 | 0.30 | 10 | 20 |
| Tata | 47 | 0.34 | 60 | 8 |
| Yutong | 44 | 0.45 | 10 | 68 |
| Ashok | 42 | 0.28 | 75 | 4 |

Source: AUMTCO, 2017.

Table 2:Passengers' Demand Matrix

| Waiting time (minutes) | Frequency | Percent (\%) |
| :--- | :--- | :--- |
| $1-10$ | 136 | 34 |
| $11-20$ | 28 | 7 |
| $21-30$ | 64 | 16 |
| $31-40$ | 140 | 35 |
| $41-50$ | 8 | 2 |
| $51-60$ | 4 | 1 |
| $60 \&$ above | 8 | 2 |
| No response | 12 | 12 |
|  | 400 | 100 |

Source: Nwankwoet al. (2016).
Table 3: Distribution of Optimal Number of Buses across Shifts for Gwagwalada Route

| Day | Bus type | Capacity per trip | Optimal Number of buses | Bus distribution across Shifts |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Shift 1 | Shift 2 | Shift 3 |
| Monday | Marcapolo | 98 | 13 | 5 | 2 | 6 |
|  | Tata | 94 | 13 | 5 | 2 | 6 |
|  | Yutong | 88 | 13 | 5 | 2 | 6 |
|  | Ashok | 84 | 15 | 6 | 2 | 7 |
| Tuesday | Marcapolo | 98 | 10 | 4 | 1 | 5 |
|  | Tata | 94 | 10 | 4 | 1 | 5 |
|  | Yutong | 88 | 10 | 4 | 1 | 5 |
|  | Ashok | 84 | 13 | 5 | 2 | 6 |
| Wednesday | Marcapolo | 98 | 9 | 4 | 1 | 4 |
|  | Tata | 94 | 10 | 4 | 1 | 5 |
|  | Yutong | 88 | 10 | 4 | 1 | 5 |
|  | Ashok | 84 | 10 | 4 | 1 | 5 |
| Thursday | Marcapolo | 98 | 10 | 4 | 1 | 5 |
|  | Tata | 94 | 10 | 4 | 1 | 5 |
|  | Yutong | 88 | 10 | 4 | 1 | 5 |
| Friday | Ashok | 84 | 11 | 5 | 1 | 5 |
|  | Marcapolo | 98 | 10 | 4 | 1 | 5 |
|  | Tata | 94 | 10 | 4 | 1 | 5 |
|  | Yutong | 88 | 10 | 4 | 1 | 5 |
|  | Ashok | 84 | 11 | 5 | 1 | 5 |



Fig. 2:Bus distribution across shifton weekdays for Gwagwalada route.


Fig.3: Bus distribution across shifts on weekdays on Lugbe route
Table 4:Distribution of optimal number of buses across shifts for Nyanya route

| Day | Bus type | Capacity per trip | Optimal Number <br> of buses | Bus distribution across Shifts |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | Shift 1 | Shift 2 | Shift 3 |  |
| Monday | Marcapolo | 98 | 62 | 25 | 7 | 30 |
|  | Tata | 94 | 64 | 26 | 7 | 31 |
|  | Yutong | 88 | 69 | 28 | 7 | 34 |
| Tuesday | Ashok | 84 | 72 | 29 | 8 | 35 |
|  | Marcapolo | 98 | 54 | 22 | 6 | 26 |
|  | Tata | 94 | 56 | 23 | 6 | 27 |
|  | Yutong | 88 | 59 | 24 | 6 | 29 |
|  | Ashok | 84 | 63 | 26 | 7 | 30 |
|  | Marcapolo | 98 | 56 | 23 | 6 | 27 |
|  | Tata | 94 | 58 | 24 | 6 | 28 |
|  | Yutong | 88 | 62 | 25 | 7 | 30 |
|  | Ashok | 84 | 65 | 26 | 7 | 32 |


| Thursday | Tata | 94 | 47 | 19 | 5 | 23 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Yutong | 88 | 51 | 21 | 5 | 25 |
|  | Ashok | 84 | 54 | 22 | 6 | 26 |
|  | Marcapolo | 98 | 48 | 20 | 5 | 23 |
|  | Friday | 94 | 49 | 20 | 5 | 24 |
|  | Yata | 88 | 54 | 21 | 6 | 26 |
|  | Antong | 84 | 56 | 23 | 6 | 27 |



Fig. 4:Bus Distribution across shifts on weekdays on Kubwaroute
Table 5:Distribution of optimal number of buses across shifts for B wari route

| Day | Bus type | Capacity per trip | OptimalNumber of buses | Bus distribution across Shifts |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Shift 1 | Shift 2 | Shift 3 |
| Monday | Marcapolo | 98 | 17 | 7 | 2 | 8 |
|  | Tata | 94 | 17 | 7 | 2 | 8 |
|  | Yutong | 88 | 18 | 7 | 2 | 9 |
|  | Ashok | 84 | 19 | 8 | 2 | 9 |
| Tuesday | Marcapolo | 98 | 16 | 6 | 2 | 8 |
|  | Tata | 94 | 17 | 7 | 2 | 8 |
|  | Yutong | 88 | 17 | 7 | 2 | 8 |
|  | Ashok | 84 | 18 | 7 | 2 | 9 |
| Wednesday | Marcapolo | 98 | 15 | 6 | 2 | 7 |
|  | Tata | 94 | 15 | 6 | 2 | 7 |
|  | Yutong | 88 | 17 | 7 | 2 | 8 |
|  | Ashok | 84 | 17 | 7 | 2 | 8 |
| Thursday | Marcapolo | 98 | 15 | 6 | 2 | 7 |
|  | Tata | 94 | 15 | 6 | 2 | 7 |
|  | Yutong | 88 | 15 | 6 | 2 | 7 |
| Friday | Ashok | 84 | 16 | 6 | 2 | 8 |
|  | Marcapolo | 98 | 13 | 5 | 2 | 6 |
|  | Tata | 94 | 13 | 5 | 2 | 6 |
|  | Yutong | 88 | 15 | 6 | 2 | 7 |
|  | Ashok | 84 | 15 | 6 | 2 | 7 |



Fig. 5:Bus Distribution across shifts on weekdays on Mpape route
Table 6:Distribution of optimal number of buses across shifts for Zuba route

| Day | Bus type | Capacity per trip | Optimal Number of buses | Bus distribution across Shifts |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Shift 1 | Shift 2 | Shift 3 |
| Monday | Marcapolo | 98 | 45 | 18 | 5 | 22 |
|  | Tata | 94 | 47 | 19 | 5 | 23 |
|  | Yutong | 88 | 50 | 21 | 5 | 24 |
|  | Ashok | 84 | 53 | 21 | 6 | 26 |
| Tuesday | Marcapolo | 98 | 39 | 16 | 4 | 19 |
|  | Tata | 94 | 40 | 16 | 4 | 20 |
|  | Yutong | 88 | 44 | 18 | 5 | 21 |
|  | Ashok | 84 | 45 | 18 | 5 | 22 |
| Wednesday | Marcapolo | 98 | 36 | 15 | 4 | 17 |
|  | Tata | 94 | 37 | 15 | 4 | 18 |
|  | Yutong | 88 | 39 | 16 | 4 | 19 |
|  | Ashok | 84 | 41 | 17 | 4 | 20 |
| Thursday | Marcapolo | 98 | 37 | 15 | 4 | 18 |
|  | Tata | 94 | 39 | 16 | 4 | 19 |
|  | Yutong | 88 | 42 | 17 | 5 | 20 |
| Friday | Ashok | 84 | 44 | 18 | 5 | 21 |
|  | Marcapolo | 98 | 30 | 12 | 3 | 15 |
|  | Tata | 94 | 32 | 13 | 4 | 15 |
|  | Yutong | 88 | 34 | 14 | 4 | 16 |
|  | Ashok | 84 | 35 | 14 | 4 | 17 |

Table 7: Distribution of Optimal number of Buses across shifts for Gwagwalada, Lugbe and KubwaRoute (Capacity-Mix)

| Route/Day | Optimal number of Buses | Shifts |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{X}_{1}$ | $\mathbf{X}_{2}$ | $\mathbf{X}_{3}$ |
| Gwagwalada |  |  |  |  |
| Monday | 11 | $\mathrm{M}=5$ | $\mathrm{M}=1$ | $\mathrm{M}=5$ |
| Tuesday | 10 | $\mathrm{M}=2$ | $\mathrm{Y}=1$ | $\mathrm{M}=1$ |
|  |  | $\mathrm{A}=2$ |  | $\mathrm{A}=4$ |
| Wednesday | 9 | $\mathrm{A}=4$ | $\mathrm{A}=1$ | $\mathrm{M}=3$ |
|  |  |  |  | $\mathrm{Y}=1$ |
| Thursday | 10 | $\mathrm{M}=1$ | $\mathrm{A}=1$ | $\mathrm{A}=5$ |
|  |  | $\mathrm{A}=3$ |  |  |
| Friday | 10 | $\mathrm{M}=1$ | $\mathrm{A}=1$ | $\mathrm{A}=5$ |
|  |  | $\mathrm{A}=3$ |  |  |
| Lugbe Route |  |  |  |  |
| Mondays | 12 | $\mathrm{M}=5$ | $\mathrm{M}=1$ | $\mathrm{M}=6$ |
| Tuesdays | 10 | $\mathrm{M}=2$ | $\mathrm{T}=1$ | $\mathrm{M}=2$ |
|  |  | $\mathrm{A}=2$ |  | $\mathrm{A}=3$ |
| Wednesdays | 10 | $\mathrm{M}=1$ | $\mathrm{Y}=1$ | $\mathrm{A}=5$ |
|  |  | $\mathrm{A}=3$ |  |  |
| Thursdays | 9 | $\mathrm{A}=4$ | $\mathrm{A}=1$ | $\mathrm{M}=3$ |
|  |  |  |  | $\mathrm{A}=1$ |

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| Fridays | 8 | $\mathrm{M}=3$ | $\mathrm{~A}=1$ | $\mathrm{M}=2$ <br> $\mathrm{~A}=2$ |
| :--- | :--- | :--- | :--- | :--- |
| Kubwa Route | 8 | $\mathrm{M}=2$ | $\mathrm{~A}=1$ | $\mathrm{~A}=4$ |
| Mondays |  | $\mathrm{A}=1$ |  |  |
| Tuesdays | 7 | $\mathrm{~A}=3$ | $\mathrm{~A}=1$ | $\mathrm{M}=1$ |
|  |  | $\mathrm{~A}=3$ | $\mathrm{~A}=1$ | $\mathrm{~A}=2$ |
| Wednesdays | 7 | $\mathrm{~A}=3$ | $\mathrm{~A}=1$ | $\mathrm{~A}=3$ |
| Thursdays | 7 | $\mathrm{~A}=3$ | $\mathrm{~A}=1$ | $\mathrm{~A}=3$ |
| Fridays | 7 |  | $\mathrm{~A}=3$ |  |

Bus type :M= Marcapolo, $\mathrm{T}=$ Tata, $\mathrm{Y}=$ Yutong, $\mathrm{A}=$ Ashok, $\mathrm{X}_{1}=$ shift $1, \mathrm{X}_{2}=$ shift 2 and $\mathrm{X}_{3}=$ shift 3.
Table 8: Capacity-Mix and Distribution of Optimal Number of Buses across Shifts for Bwari, Mpape, Zuba and Nyanya Route

| Route/Days | Optimal Number of buses | Shifts |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathbf{X}_{3}$ |
| Bwari |  |  |  |  |
| Mondays | 15 | $\mathrm{M}=6$ | $\mathrm{M}=2$ | $\mathrm{M}=7$ |
| Tuesdays | 15 | $\mathrm{M}=6$ | $\mathrm{M}=2$ | $\mathrm{M}=7$ |
| Wednesdays | 14 | $\mathrm{M}=6$ | $\mathrm{M}=1$ | $\mathrm{M}=7$ |
| Thursdays | 12 | $\mathrm{M}=5$ | $\mathrm{M}=1$ | $\mathrm{M}=6$ |
| Fridays | 13 | $\mathrm{M}=3$ | $\mathrm{A}=2$ | M=4 |
|  |  | $\mathrm{A}=2$ |  | $\mathrm{A}=2$ |
| Mpape |  |  |  |  |
| Mondays | 8 | $\mathrm{M}=2$ | $\mathrm{A}=1$ | $\mathrm{A}=4$ |
|  |  | $\mathrm{A}=1$ |  |  |
| Tuesdays | 7 | $\mathrm{A}=3$ | $\mathrm{A}=1$ | M=2 |
|  |  |  |  | $\mathrm{T}=1$ |
| Wednesdays | 7 | $\mathrm{A}=3$ | $\mathrm{A}=1$ | $\mathrm{M}=2$ |
|  |  |  |  | $\mathrm{Y}=1$ |
| Thursdays | 7 | $\mathrm{A}=3$ | $\mathrm{A}=1$ | $\mathrm{M}=2$ |
|  |  |  |  | A=1 |
| Fridays | 7 | $A=3$ | $\mathrm{A}=1$ | $\mathrm{A}=3$ |
| Zuba |  |  |  |  |
| Mondays | $\mathrm{M}=44$ | 18 | 4 | 22 |
| Tuesdays | $\mathrm{M}=37$ | 15 | 4 | 18 |
| Wednesdays | $\mathrm{M}=34$ | 14 | 3 | 17 |
| Thursdays | M $=37$ | 15 | 4 | 18 |
| Fridays | $\mathrm{M}=29$ | 12 | 3 | 14 |
| Nyanya |  |  |  |  |
| Mondays | $\mathrm{M}=61$ | 25 | 6 | 30 |
| Tuesdays | $\mathrm{M}=53$ | 22 | 5 | 26 |
| Wednesdays | $\mathrm{M}=54$ | 22 | 5 | 27 |
| Thursdays | $\mathrm{M}=44$ | 28 | 4 | 22 |
| Fridays | $\mathrm{M}=47$ | 19 | 5 | 23 |

Key:M= Marcapolo, T=Tata, $Y=$ Yutong, $A=$ Ashok, $X_{1}=$ shift $1, X_{2}=$ shift 2 and $X_{3}=$ shift 3.


Fig. 6: Bus Capacity-Mix Distribution across shifts on weekdays on Lugbe route


Fig. 7: Bus Capacity-Mix Distribution across shifts on weekdays on Kubwa route


Fig. 8: Bus-Capacity Mix Distribution across shifts on weekdays on Mpape route
Table 9:Distribution of Buses across the selected sevenroutesin the Existing System

| Routes | Number of Buses |
| :--- | :---: |
| Gwagwalada | 12 |
| Lugbe | 7 |
| Kubwa | 5 |
| Bwari | 9 |
| Mpape | 5 |
| Zuba | 18 |
| Nyanya | 30 |

AUMTCO: Fleet department, (2017)

## V. Discussion

The result displayed in Table 1 shows that AUMTCO uses four brand of buses; Marcapolo, Tata, Yutong and Ashok with capacity 49, 47, 44 and 42 respectively. Yutongbrand of buses has the highest fuel consumptionrate of 0.45 litres per kilometer, The Tata brand of buseshas 0.34 fuel consumption ratewhile the Ashok brand of buses hasfuel consumption rate of 0.28 litres per kilometer. TheMarcapolo brand of buses has0.30fuel consumption rate.The Yutongand Marcapolo brands of Buses have the least breakdown rate of 10\% per week, while Ashok and Tata have the highest breakdown rate of $75 \%$ and $60 \%$ per week.It is needful to point out that the Yutong brand of Busesis the passengers'preferred fleet. This is probably due to the comfort and the Air Conditioner facility they provide.

The management of the company may like to serve the mass as earlier stated in their mission statement in this work by using Yutong buses only (bearing in mind the consumption rate), then they will have to spend more on fuel.Meanwhile if the management decides to reduce the expenditure on fuel by using Ashok then two problem should be considered as incurred challenge; for one, Ashok has the highest breakdown rate and secondly Ashok is the least of passengers preference.The Marcapolo and Tata brand of buses used by the company are highly depreciated; passengers hardly patronize them.It therefore depends on the management to makeinformed decision on the use of these buses considering the accruals ofmaintenancecosts due to vehicle depreciation.It was difficult to build in the effect of fuel consumption and breakdown rates as well as customers' preference for Buses into the models. This is due to the difficulty in assessing other relevant information such as daily amount of Diesel fuel made available by the company and the number of each Bus type made available per each weekday. Nevertheless, the aforementioned information (on fuel consumption rate, breakdown rate and customers preference) can be used by management in complementing the result of the study for profitable decision making.

The result displayed in table 2 above shows passengers' demand matrix. The table reveals that majority of passengers do wait between thirty one to forty minutes for AUMTCO buses though a very good number claim to wait for AUMTCO buses between one to ten minutes, this result clearly shows that the passengers are not satisfied due to scarcity of buses experienced at different time of the day which must have amounted to majority waiting for a very long time (31-40mins) to access the service. The study by Nwankwo et al. (2016) shows that the majority of Abuja residents are civil servants closely followed by business men (private
employees). It therefore implies that the occupation of the people in the area account for the peak period noticed between the hours of 7.00 am to 10.30 am and 2.30 pm to 7.30 pm . It can be construedthat the first and second peak periods are the rush hours to offices or businesses and return from offices or business respectively. These peak periods informed the time shift system formulated in this study.

### 5.1Non Capacity-Mix (Shift System I)

The distribution of the optimal number of buses across shifts for Gwagwalada route is shown in Table 3 above. Gwagwalada route covers a total distance of 111 km from area 1 to Gwagwada town. The result of the non capacity mix model reveals that the maximum number of buses expected to serve the commuters on the route should not exceed 13 which should be distributed as follows; on Mondays, shift 1 should be assigned 5 Marcapolo buses, shift 2 should be assigned 2 Marcapolo buses and shift 3 should be assigned 6 Marcapolo buses. Considering the use of Tata buses or Yutong the result is same with the distribution for Marcapolo on Mondays but differed for Ashok buses on Mondays with 15 buses, of which 6 buses should be assigned to shift 1,2 buses assigned to shift 2 and 7 buses should be assigned to shift 3 . On Tuesdays, the result showed that the demand can be met with 10 buses (Marcapolo, Tata or Yutong buses) at most with 4,1 and 5 buses assigned to shift1, shift2 and shift3 respectively. Using Ashok buses will amount to a total demand of 13 buses with 5, 2 and 6 buses assigned to shift 1, shift2 and shift3 respectively. On Wednesdays, a total of 9 Marcapolo buses will sufficiently meet the demand with 4,1 and 4 buses assigned to the three shifts (shift1, shift2 and shift3 respectively). But the result for Tata, Yutong and Ashok remain the same with a maximum of 10 buses and a distribution of 4,1 and 5 buses across the shifts (shift 1, shift 2 and shift 3 respectively). For Thursdays, the demand can be sufficiently met with a total of 10 buses (i.e the respective distribution for Marcapolo, Tata and Yutong buses are the same) with 4,1 and 5 buses distributed across the shifts (shift 1 , shift 2 and shift 3 respectively) while a maximum of 11 Ashok buses are expected with 5,1 and 5 bus distributed across the shifts (shift 1, shift2 and shift3 respectively). Then on Fridays, a maximum of 10 Marcapolo buses will be sufficient for the demand with 4,1 and 5 buses distributed across the shifts (shift1, shift2 and shift3 respectively). The result for Marcapolo on Fridays is the same for Tata and Yutong buses with same distribution across the shifts. If Ashok is to be used on Gwagwalada route on Fridays then the maximum number of buses sufficient to meet the demand should be 11 buses with a distribution of 5,1 and 5 buses distributed across the shifts (shift1, shift2 and shift3 respectively). See figure 2 above for the distribution of buses on Gwagwalada route for the days of the week.

Table 5 shows the distribution of optimal number of buses across shifts for Nyanya route having a distance of 34 km from Nyanya to Berger Junction. The result reveals that the maximum number of Marcapolo buses needed to sufficiently carter for the demand along Nyanya route on Mondays is 62 with 25, 7 and 30 buses distributed across the shifts (shift 1, shift 2 and shift 3 respectively). The demand for Tata buses on Mondays should be 64 buses which should be distributed thus; 26 buses for shift 1, 7 buses for shift two and 31 buses for shift three. Meanwhile, the demand for Yutong on Mondays further increase the maximum number of buses to 69 buses with 28,7 and 34 buses distributed across the shifts (shift 1 , shift 2 and shift 3 respectively). 72 buses (Ashok) will be expected with 29,8 and 35 buses distributed across the shifts (shift 1, shift 2 and shift 3 respectively) for Mondays. At most, 54 buses (Marcapolos) will be sufficient to handle the demand on Tuesdays with 22, 6 and 26 buses distributed across the shifts (shift 1, shift 2 and shift 3 respectively). 56 Tata buses will be optimal for Tuesdays with 23, 6 and 27 distributed across the shifts (shift1, shift2 and shift3 respectively). This trend continued with the demand for Yutong as the optimal number of buses increased to 59 buses and a distribution of 24,6 and 29 buses distributed across the three shifts (shift 1, shift2 and shift3 respectively). Meanwhile, the highest demand for buses is on Ashok brand with optimal number of 63 buses of which 26,7 and 30 buses should be distributed across the shifts (shift1, shift2 and shift3 respectively). Wednesdays, a total of 56 Marcapolo buses will sufficiently meet the demand of which 23, 6 and 27 buses should be distributed across the three shifts for shift1, shift2 and shift 3 respectively. The result for Tata buses shows that a total of 58 buses will be optimal with 24,6 and 28 of the buses distributed to the three shifts (shift1, shift2 and shfit3 respectively). The result also showed that a total of 62 Yutong buses will be optimal to serve the demand for Wednesdays on Nyanya route with 25, 7 and 30 buses distributed across the shifts (shift1, shift2 and shfit3 respectively) and the optimal number for Ashok buses is 65 with 26, 7 and 32 bus distribution across the shifts (shift1, shift2 and shfit3 respectively). For Thursdays, the demand can be sufficiently met with a total of 46 Marcapolo buses with 19, 5 and 22 buses distributed to shift1, shift 2 and shfit 3 respectively. The optimal number of Tata buses needed to carter for the passengers demand on Thursdays increased to 47 buses (Tata) of which 19, 5 and 23 buses distributed to the shifts (shift1, shift2 and shfit3 respectively). The demand on Yutong buses is 51 with 21, 5 and 25 of the buses assigned to the shifts (shift1, shift2 and shfit3 respectively) while a maximum of 54 Ashok buses should be expected with 22, 6 and 26 buses distributed to shift 1, shift 2 and shift 3 respectively. Then on Fridays, a maximum of 48 Marcapolo buses should be sufficient for the demand with 20,

5 and 23 buses distributed across to shift1, shift2 and shift 3 respectively. Considering Tata buses, a total of 49 buses should be expected with 20,5 and 24 buses distributed across the shifts (shift1, shift2 and shfit3 respectively) with regard to Friday demands. Meanwhile, 54 Yutong buses should be optimal number of buses sufficient to handle the demand on Nyanya route on Fridays with a distribution of 21, 6 and 26 buses across the shifts (shift1, shift2 and shfit3 respectively). Then, the total number of required buses (Ashok) should not exceed 56 buses with 23, 6 and 27 buses distributed across the shifts (shift1, shift2 and shfit 3 respectively) (see figure 4).

Table 7 shows the distribution of optimal number of buses across shifts for Bwari route which covers a distance of 95 km to Area 1. The optimal number of expected buses for Mondays should be 17 buses (either Marcapolo or Tata buses) with 7, 2 and 8 buses assigned to shift 1 , shift 2 and shift 3 respectively however, the optimal number of busses increased to 18 buses (Yutong) with a distribution of 7, 2 and 9 buses to shift 1, shift 2 and shift 3 respectively and a total of 19 Ashok buses with a distribution of 8,2 and 9 Ashok buses for the shifts (shift1, shift2 and shfit3 respectively). The result also reveals that the optimal number of buses needed to convey passengers on Tuesdays should be 16 with 6,2 and 8 buses distributed across the three shifts (shift 1 , shift 2 and shift 3) respectively for Marcapolo brand of buses. A total of 17 buses (either Tata or Yutong buses) with 7,2 and 8 buses distributed across the shifts (shift1, shift2 and shfit3 respectively), meanwhile, considering the use of Ashok buses, a total of 18 buses with a 7, 2 and 9 buses distributed across the three shifts (shift 1, shift 2 and shift 3) respectively on Tuesdays. Wednesdays' demand for Marcapolo and Tata remain the same with an optimal number of 15 buses and 6,2 and 7 buses distributed across the three shifts (shift 1 , shift 2 and shift 3) respectively but differed by demand for Yutong and Ashok having an optimal number of 17 buses (either Yutong or Ashok buses) with 7, 2 and 8 buses distributed across the three shifts (shift 1, shift 2 and shift 3 respectively). The demand on Thursdays reduced to 15 buses (either Marcapolo, Tata or Yutong buses) with 6, 2 and 7 buses distributed across the three shifts (shift 1 , shift 2 and shift 3 respectively). Taking into account, the demand on Ashok buses, the optimal number of buses on Thursdays increased to 16 buses with 6,2 and 8 buses distributed across the three shifts (shift 1, shift 2 and shift 3 ) respectively. Fridays being the last working day of the week has optimal demand of 13 buses (either Marcapolo or Tata buses) and a distribution of 5, 2 and 6 buses across the shifts (shift 1 , shift 2 and shift 3 respectively), while the demand of 15 buses (Yutong and Ashok) was achieved with 6,2 and 7 buses distributed across the three shifts (shift 1 , shift 2 and shift 3 ) respectively for Bwari route (see figure 6).

Table 9 shows the distribution of optimal number of buses across shifts for Zuba route which covers a distance of 84 km from Berger Junction. The result reveals that the maximum number of buses needed to sufficiently carter for the demand along Zuba route on Mondays is 45 Marcapolo buses with 18, 5 and 22 of the buses distributed across the shifts (shift 1, shift 2 and shift 3 respectively). The demand for Tata buses on Mondays is 47 which will need to be distributed thus; 19 buses for shift 1,5 buses for shift 2 and 23 buses for shift 3. The demand for Yutong on Mondays further increased the maximum number of buses to 50 buses with 21,5 and 24 of the buses distributed across the shifts (shift 1, shift 2 and shift 3 respectively). 53 buses (Ashok) will be expected with 21, 6 and 26 buses distributed across the shifts (shift 1, shift 2 and shift 3 respectively) for Mondays. At most, 39 buses (Marcapolo) will be sufficient to handle the demand for Tuesdays with 16, 4 and 19 bus distribution across the shifts (shift 1, shift 2 and shift 3 respectively). 40 Tata buses will be optimal for Tuesdays with 16, 4 and 20 buses distributed across the shifts (shift1, shift2 and shift3 respectively). This trend continued with the demand for Yutong on Tuesdays, as the number of buses increased to 44 buses and a distribution of 18,5 and 21 buses distributed across the three shifts (shifts1, shift2 and shift3 respectively). Meanwhile, the highest demand for buses is on Ashok brand of buses with a maximum expectation of 45 buses and 18, 5 and 22 buses distributed across the shifts (shift1, shift2 and shift3 respectively). On Wednesdays, a total of 36 Marcapolo buses will sufficiently meet the demand with 15,4 and 17 of the buses distributed across the three shifts for shift1, shift2 and shift3 respectively. The result for Tata shows that a total of 37 buses with 15,4 and 18 of the buses distributed to the three shifts (shift1, shift2 and shfit3 respectively). A total of 39 Yutong buses will be expected to serve the demand on Wednesdays for Zuba route with 16, 4 and 19 of the buses across the shifts (shift1, shift2 and shfit3 respectively) and the optimal number of buses (Ashok bus) is 41 with 17,4 and 20 buses distributed across the shifts (shift1, shift2 and shfit3 respectively) as displayed in figure 34. For Thursdays, the demand can be sufficiently met with a total of 37 buses with 15,4 and 18 buses distributed across the shifts (shift1, shift2 and shfit3 respectively) of Marcapolo buses only, which is quite similar to the demand on Tata with maximum of 39 buses with 16,4 and 19 buses assigned to the shifts (shift1, shift2 and shfit3 respectively). The demand on Yutong buses is 42 with 17,5 and 20 buses assigned to the shifts (shift1, shift2 and shfit3 respectively) while a maximum of 44 Ashok buses is expected with 18, 5 and 21 buses distributed to shift 1, shift2 and shift3 respectively. Then on Fridays, a maximum of 30 Marcapolo buses will be sufficient for the demand with 12, 3 and 15 buses distributed to shift1, shift2 and shift3 respectively. Considering Tata buses, a total of 32 buses will be expected with 13,4 and 15 buses distributed across the shifts (shift1, shift2 and shfit3 respectively) with regard to Friday demands. Meanwhile, 34 Yutong buses should be
sufficient to handle the demand on Zuba route on Fridays with a distribution of 14, 4 and 16 buses across the shifts (shift1, shift2 and shfit3 respectively). Then, the total number of required Ashok buses should be 35 with 14,4 and 17 buses distributed across the shifts (shift1, shift2 and shfit 3 respectively) as shown in figure 8.

The shifts are linear in nature, as such the three shifts does not run concurrently hence buses that run in shift 1 can be opportune to run in shift 2 or shift 3 but to avoid over utilization (for easy maintenance purposes) and congestion likelihood, the buses should ply within the scheduled time. More so, other routes like Mararaba, Central Area, Karmo route among other routes that are yet to be covered by AUMTCO buses can be covered by the reserved buses awaiting shifts. It should be noted that the shift 3 (for all the days irrespective of the route) contains the maximum needed buses sufficient for operation at all the time for shift 1 , shift 2 and shift 3 as the case may be. Hence, the optimal number of buses shown in shift 3 for all the routes can sufficiently handle the demands across the shifts' demand.

### 5.2 Capacity-Mix (Shift System I)

The model mixes the capacity of different brand of buses used by AUMTCO to determine the optimal buses needed to convey the passengers from source to destinations. Table 7 shows the optimal number of buses across shifts for Gwagwalada, Lugbe and Kubwa route using the Capacity Mix system. For Gwagwalada route, on Mondays; 11 buses are optimal with 5Marcapolo in shift 1, 1 marcapolo in shift 2 and 5Marcapolo in shift 3 . A total of 10 buses can sufficiently convey the passengers on Tuesdays of which shift 1 should have a total of 4 buses ( 2 Marcapolo and 2 Ashok buses), shift 2 should be assigned 1 Yutong bus and shift 3 should also be assigned 5 buses ( 1 Marcapolo and 4 Ashok buses). Due to the decline of demand on Wednesdays, the optimal number of required buses reduced to 9 buses with 4 Ashok buses assigned to shift1, 1 Ashok in shift 2 and 4 buses ( 3 Marcapolo and 1 Yutong buses) assigned to the third shift. The demand on Thursdays and Fridays are very close which required 10 buses to convey the passengers on each of the days; shift 1 should be assigned 4 buses ( 1 Marcapolo and 3 Ashok), shift 2 should be assigned 1 Ashok and Shift 3 should be assigned 5 Ashok buses for Thursday and Friday demand.

The demand along Lugbe's route on Mondays can sufficiently be met with 12 Marcapolo buses which should be distributed as follows; shift 1 should be assigned 5 buses, shift 2 should be assigned 1 bus and shift 3 should be assigned 6 buses. The Tuesday demand on Lugbe route requires 10 buses of which 4 buses ( 2 Marcapolo and 2 Ashok) should be made available for shift 1, 1 Tata bus should be made available for shift 2 and 5 buses ( 2 Marcapolo and 3 Ashok) should be available for shift 3 . Wednesday demand is very close to the demand on Tuesday which requires a total of 10 buses (for each of the days) which should be distributed thus; 4 buses (1 Marcapolo and 3 Ashok) should be made available for shift 1, 1 Yutong should be made available for shift 2 and 5 Ashok bus for shift 3. The optimal number of buses declined to 9 buses on Thursdays along Lugbe route of which shift 1 should be assigned 4 Ashok buses, shift 2 should be assigned 1 Ashok and shift 3 should be assigned 4 buses ( 3 Marcapolo and 1 Ashok). The trend continued for Friday demand as the optimal number of buses required to convey passengers declined to 8 buses which should be distributed as follow; shift 1 should be assigned 3 Marcapolo, shift 2 should be assigned 1 Ashok bus and shift 3 assigned 4 buses ( 2 Marcapolo and 2 Ashok buses). See figure 6 for details.

Considering the demand on Kubwa route, 8 buses will sufficiently meet the demand on Mondays which should be distributed thus; shift 1 should be assigned 3 buses ( 2 Marcapolo and 1 Ashok), shift 2 should be assigned 1 Ashok bus and shift 3 should be assigned 4 Ashok buses. On Tuesday, 7 buses will be sufficient to meet the demand along the route with 3 Ashok buses assigned to the first shift, 1 Ashok bus for shift 2 and 3 buses (1 Marcapolo and 2 Ashok buses) assigned to shift 3. The demand for Wednesday, Thursday and Friday is very close such that equal number of buses will be sufficient for the days with the same distribution. Hence 7 Ashok buses should be made available with 3 buses assigned to shift 1,1 bus assigned to shift 2 and 3 buses assigned to shift 3 from Wednesday to Friday. See figure 7 for details.

The result displayed in table 8 shows that the demand on Bwari route on Mondays and Tuesdays are very close, as such same number of buses can serve for the days, thus 15 Marcapolo buses is optimal with 6 buses assigned to shift 1, 2 buses assigned to shift 2 and 7 buses assigned to shift 3 for Mondays and Tuesdays. 14 Marcapolo buses is optimal for Wednesday demand with 6 buses assigned to shift1, 1 bus assigned to shift 2 and 7 buses assigned to shift 3 . Similarly, 12 Marcapolo buses is optimal for Thursday demand of which 5 buses should be assigned to shift 1,1 bus assigned to shift 2 and 4 buses assigned to shift 3 . Friday's demand optimized the bus demanded to 13 buses with 5 buses ( 3 Marcapolo and 2 Ashok) assigned to shift 1, 2 Ashok buses assigned to shift 2 and 6 buses ( 4 Marcapolo and 2 Ashok buses) assigned to shift 3 .

On this same table, the demand is relatively low onMpape route, as such, 8 buses is optimal for Mondays' demand with 3 buses ( 2 Marcapolo and 1 Ashok) assigned to shift 1, 1 Ashok bus assigned to shift 2 and 4 Ashok buses assigned to shift 3 . On Tuesdays, 7 buses are optimal with 3 Ashok buses assigned to shift 1, 1 Ashok for shift 2 and 3 buses ( 2 Marcapolo and 1 Tata) assigned to shift 3. Similarly, the demand for Wednesday, Thursday and Friday are very close such that 7 buses are optimal with 3 Ashok buses assigned to
shift 1, 1 Ashok bus assigned to shift 2 for the three days. On Wednesday shift 3 has 3 buses ( 2 Marcapolo and 1 Yutong) assigned to it, 3 buses ( 2 Marcapolo and 1 Ashok) should also be assigned to shift 3 on Thursday and 3 Ashok buses should be assigned to shift 3 on Friday. See figure 8 for details.

The table also reveals that Zuba route is one of the busiest routes among the routes managed by AUMTCO. The result shown in table 11 showed that Marcapolo brand was the only bus type used in the optimization. This probably occurred because of the high capacity of Marcapolo buses. Monday demand can be met with 44 Marcapolo buses with 18 buses assigned to shift 1,4 buses assigned to shift 2 and 22 buses assigned to shift 3 . Tuesday and Thursday's demand are very close with 37 buses required to convey passengers and 15,4 and 18 buses distributed across the shift (shift1, shift 2 and shift 3 respectively). 34 buses are optimal for the demand on Wednesday with 14, 3 and 17 buses assigned to shift 1, shift 2 and shift 3 respectively. 29 buses are sufficient for the demand on Fridays with 12 buses assigned to shift 1, 3 buses assigned to shift 2 and 14 buses assigned to shift 3 .

Finally, on this table, Nyanya route is seen to be the busiest routes among the routes managed by AUMTCO. Similar toZuba route, Marcapolo brand of bus is the only bus type used in the optimization. The demand on Mondays can be met with 61 Marcapolo buses; with 25 buses assigned to shift 1, 6 buses assigned to shift 2 and 30 buses assigned to shift 3 . Tuesday's demand requires 53 buses in conveying passengers with; 22, 5 and 26 buses distributed across shift1, shift 2 and shift 3 respectively. 54 buses are optimal for the demand on Wednesday with 22,5 and 27 buses assigned to shift 1 , shift 2 and shift 3 respectively. Thursday's demand can be met with 44 Marcapolo buses with 18 buses assigned to shift 1,4 buses assigned to shift 2 and 22 buses assigned to shift 3 , and 47 buses are sufficient for the demand on Fridays with 19 buses assigned to shift 1,5 buses assigned to shift 2 and 23 buses to shift 3 .

### 5.3Comparing the Existing System with theNon Capacity-Mix and the Capacity-Mix Shift Systems

Considering the breakdown rate and fuel consumption rate (tables 1 and 2) so as to effectively determine the cost of maintaining each of the brand of the buses, it is therefore necessary to establish a situation where each bus type is singly utilized in order to determine the associated costs. This is one of the significance of the Non Capacity Mix Shift System. This was not captured in the existing system where all the bus types were assigned to ply a certain route at the same time (see table 9).

The Non Capacity-Mix and the Capacity-Mix Shift Systems took into consideration the peak times and off peak times of the day so as to determine the optimal number of buses needed to serve the people efficiently at that very time of the day but the existing system uses constant number of buses to serve the demands all through the day leading to additional expenses on the side of the management. The reason been that during the off-peak hours of the day, there is the possibility that most Buses would ply empty or if the decide not to, passengers will queue for a very long time. This will lead to lost in customers.

Unlike the existing system, the two shift systems also take into account the peak days (of demand) of the week in determining the optimal number buses for each weekday across routes. See tables 3 to 9 for details. One can infer that using these shift systems, buses and their drivers as well as conductors would not be over utilized. Staff can take rest during the periods they are not on shift or arrange for convenient shifts. While buses that are not plying any route at a particular period can be assigned to other routes or taken for maintenance.

### 5.4Comparing the Non Capacity-Mix and the Capacity-Mix Shift Systems

There are more similarities between the two Shift Systems than disparities. The Shift System 1 (Non Capacity Mix) as earlier mentioned, utilizes buses of same capacity (same type) to optimize the total buses used by the company to serve the populace while the shift system 2 (Capacity-Mix) utilizes buses of mix capacity (mixed Bus types) for the same purpose. Both systems optimize for Buses of higher capacities as this will help minimize the total number of Buses required across routes. In this vein, the Marcapolo Bus was selected by Shift System I (Non Capacity-Mix) to provide the least optimal number of Buses for each weekday across routes.

Interestingly, the Marcapolo Bus was selected by Shift System II (Capacity-Mix) on each weekday for Zuba and Nyanya routes. Comparison of the optimal number of theMarcapolo Buses in both systems shows that it differs only by one Bus in each respective weekday.This shows that either shift system can be employed on these routes without a significant difference in the result. See tables 5,9 and 11 and figures $4,8,14$ and 15 for details. Beside these routes, the Bus types were optimally mixed across shifts for Shift System II.
The management of AUMTCO can decide which shift system to use considering the one that yield the least optimal number of buses alongside the frequency of breakdown, fuel consumption rate and customers preference. Although the frequency of breakdown, fuel consumption rate and customers preferencewere not incorporated in the models, but the historic records on them as shown in table 1 can be used as a guide.

## VI. Conclusion and Recommendation

This section presents the conclusion and recommendations of this study.

### 6.1. Conclusion

The study concludes that;
(i)The current or existing system of operation of AUMTCO can be improved by developing an Optimization Time Shift System Models (Capacity-Mix and Non-Capacity-Mix models) across the routes.
(ii)Optimal number of buses across the selected routes has been determined using theaforementioned models. (iii)The study successfully determined that either Shift System can be applied to the Zuba and Nyanya routes for each weekday since they yield approximately the same result.

### 6.2. Recommendation

This study recommendsthat;
(i)The Capacity-Mix and Non Capacity-MixTime Shift System models formulated should be used for assignment of fleets across the routes managed by the Abuja Urban Mass Transport Company.
(ii)The variability in the passenger demands for Buses across routes, the vehicle fuel consumption and breakdown rates as well as the customers' preference for Bus type should be built into a Stochastic Linear Programming model in a further research.

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