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Electron Capture Cross-Sections In $D^{+}+H(1s)\rightarrow D(1s)+H^{+}$ Collisions

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Abstract: *Electron capture differential and integral cross-sections for collisions of deuterons with hydrogen atom in the ground state have been calculated in the high energy range from 20keV to 40MeV using the Coulomb-Born approximation. The differential cross-sections have been calculated in the energy range from 0-4mrad.The isotopic effect of mass on the integral cross-sections have been compared with that of proton-hydrogen collisions.*
Key Words: *Differential cross sections, deuterons, Coulomb Born approximation, Isotopic effect*

I. Introduction:

The study of the simplest system $H^{+}+H(1s)\rightarrow H(1s)+H^{+}$ in which a proton collides with hydrogen atom in the ground state have been investigated in several previous theoretical works extensively by different authors as Dalgarno and Yadav[1],McDowell [2],Dalgarno[3], Bares and Williams[4],Parcell and May[5],Smith[6],Chen and Watson[7], Bates and Tween[8],Bates and Sprevak[9] and many other pioneer texts by McDowell and Coleman[10],Bransden and McDowell[11],Bransden[12],Joachain[13] and Eichler[14].Some review articles as Basu et al[15],Bransden[15],Fritsch and Lin[16] have given good account of the simplest charge transfer for proton-hydrogen collisions. For the reaction $D^{+}+H(1s)\rightarrow D(1s)+H^{+}$ in which a deuteron collides with the hydrogen atom in the ground states have been studied by Hunter and Kuriyan[17] and Igarashi and Lin[18]. Hunter and Kuriyan have calculated charge transfer cross-section for the reaction $\mathrm{D}^{+}-H(1s)$ in the very low energy range from 0.001 to 7.5 eV by using the Born-Oppenheimer method while Igarashi and Lin have studied the same reaction in hyperspherical coordinates in the low energy range and claim the results better than that of Hunter and Kuriyan. We have studied first time the reaction $D^{+}-H(1s)$ in the frame work of Coulomb-Born approximation (CBA) first proposed by Fujiwara [19] in high energy range from 20keV to 40MeV. We have compared our calculated integral cross-sections with that of the $H^{+}-H(1s)$ results as to see the isotopic effect on the integral cross-sections.

We used the atomic units throughout this paper ($e=m=\hbar=4\pi\varepsilon_0=1$) except for cross-sections which are expressed in πa_0^2.

II. Theory

We have considered the charge transfer reaction

$$D^{+}+H(1s)\rightarrow D(1s)+H^{+} \tag{1}$$

The differential cross-sections for the process (1) can be written as

$$\frac{d\sigma}{d\Omega}=\frac{\mu_i\mu_f}{4\pi^2}\frac{k_f}{k_i}\left|T_{if}\right|^2 \tag{2}$$

Where,

$$\mu_i=\frac{m_P(m_T+1)}{m_P+m_T+1} \text{ and } \mu_f=\frac{m_T(m_P+1)}{m_P+m_T+1}$$

Where again, $\mathrm{m}_P=$ mass of projectile ion and $\mathrm{m}_T=$ mass of target atom

The transition matrix T_{if} from an initial state i to a final state f in the CB approximation for the process (1) are given by

$$T_{if}=\int\Phi_f^{*}V_f^{*}\Phi_i d\vec{r}d\vec{R}_f \tag{3}$$

Where Φ_i and Φ_f are the wave functions for the process (1) in the initial and final channels respectively and are given by

$$\Phi_i=\phi_i\left(\vec{r}\,'\right)\psi_i\left(\vec{R}_i\right) \tag{4}$$

and

$$\Phi_f=\phi_f\left(\vec{r}\right)\psi_f\left(\vec{R}\right) \tag{5}$$

Where again,

$\phi_i\left(\vec{r}\,'\right)=\dfrac{1}{\sqrt{\pi}}e^{-Sr_i}$ = wave function of the H -atom in its ground state in the initial channel $\left(S=1\right)$

$\Phi_f\left(\vec{r}\right)=\dfrac{1}{\sqrt{\pi}}e^{-\beta r}$ = wave function of the deuterium atom in $1s$ state of the final channel $\left(\beta=1\right)$

$\varphi_i\left(\vec{R}_i\right)=e^{i\vec{k}_i.\vec{R}_i}$ is a plane wave in the initial channel.

$\varphi_f\left(\vec{R}_f\right)=e^{-\pi\alpha/2}\Gamma\left(1-i\alpha\right)e^{i\vec{k}_f.\vec{R}_f}\,{}_1F_1\left(i\alpha;-ik_fR_f-i\vec{k}_f.\vec{R}_f\right)$ is Coulomb wave

Where $\vec{k}_i$ and $\vec{k}_f$ are the momentum vectors in the initial and final channels respectively.

$\alpha=\dfrac{\mu_f}{\vec{k}_f}$ is the repulsive Coulomb parameter and ${}_1F_1\left(i\alpha;-ik_fR_f-i\vec{k}_f.\vec{R}_f\right)$ is the confluent hyper-geometric function.

Now the transition matrix element given in equation (3) will be

$$T_{if}=\int d\vec{r}d\vec{R}_fe^{i\left(\vec{k}_i.\vec{R}_i-\vec{k}_f.\vec{R}_f\right)}\frac{e^{-Sr_i}}{\sqrt{\pi}}e^{-\pi\alpha/2}\Gamma\left(1+i\alpha\right){}_1F_1\left(i\alpha;-ik_fR_f-i\vec{k}_f.\vec{R}_f\right)$$
$$\times\left(-\frac{1}{r_i}\right)\frac{1}{\sqrt{\pi}}e^{-\beta r}$$

$$T_{if}=\frac{1}{\pi}e^{-\pi\alpha/2}\Gamma\left(1+i\alpha\right)\int d\vec{r}d\vec{R}_fe^{i\left(\vec{k}_i.\vec{R}_i-\vec{k}_f.\vec{R}_f\right)}{}_1F_1\left(i\alpha;-ik_fR_f-i\vec{k}_f.\vec{R}_f\right)e^{-\beta r}\times\left(-\frac{e^{-Sr_i}}{r_i}\right) \tag{6}$$

$$T_{if}==e^{-\pi\alpha/2}\Gamma\left(1+i\alpha\right)I \tag{7}$$

Where

$$I=\int d\vec{r}d\vec{R}_fe^{i\left(\vec{k}_i.\vec{R}_i-\vec{k}_f.\vec{R}_f\right)}{}_1F_1\left(i\alpha;-ik_fR_f-i\vec{k}_f.\vec{R}_f\right)e^{-\beta r}\times\left(-\frac{e^{-Sr_i}}{r_i}\right) \tag{8}$$

Integration of equation (8) yields

$$I=-\frac{4\pi}{\mu_a^6}\int_0^1 xdx\frac{1}{g^3}\frac{1}{\lambda}\frac{d}{d\lambda}\frac{1}{\lambda}\frac{d}{d\lambda}\varphi\left(\lambda\right) \tag{9}$$

Where

$$\varphi\left(\lambda\right)=\left(\frac{q^2+\lambda^2}{2}\right)^{-i\alpha-1}\left(\frac{q^2+\lambda^2}{2}+\vec{k}_f.\vec{q}-i\lambda k_f\right)^{i\alpha} \tag{10}$$

$$g=x+\frac{1-x}{\mu_a^2}\ ,\quad \mu_a=\frac{m_P}{m_P+1}$$

Now the one dimensional integral from 0 to 1 in equation are done by using Gauss –Legendre quadrature by insuring divergence by increasing number of Gaussian points.

The total cross-sections are obtained by integrating the expressions in equation (2) as

$$\sigma = 2\pi\int_{0}^{\pi}\frac{d\sigma}{d\Omega}\sin\theta d\theta \qquad (11)$$

III. Results and discussions:

The integral cross-sections in the energy range from 20keV to 40MeV are given in the table. In the second column and third column, we have given the cross-sections for the reactions $H^{+}+H(1s)\rightarrow H(1s)+H^{+}$ and $D^{+}+H(1s)\rightarrow D(1s)+H^{+}$ respectively in the units of πa_0^2.

It is found that the cross-section for the $D^{+}-H$ system is higher than the system $H^{+}-H$. Here it is to be noted that these differences are due to the isotopic mass.

Table:
Integral cross-sections for the process $H^{+}+H(1s)\rightarrow H(1s)+H^{+}$ and $D^{+}+H(1s)\rightarrow D(1s)+H^{+}$ in units of πa_0^2 are given.

Energy in(keV)	$H^{+}+H(1s)\rightarrow H(1s)+H^{+}$	$D^{+}+H(1s)\rightarrow D(1s)+H^{+}$
20	4.873(0)	1.066(1)
25	3.842(0)	9.216(0)
30	3.012(0)	7.797(0)
40	1.870(0)	5.479(0)
50	1.192(0)	3.861(0)
60	7.818(-1)	2.759(0)
70	5.266(-1)	2.005(0)
80	3.635(-1)	1.482(0)
90	2.564(-1)	1.112(0)
100	1.844(-1)	8.4709(-1)
120	1.004(-1)	5.104(-1)
150	1.405(-1)	2.589(-1)
200	1.429(-2)	9.821(-1)
250	5.498(-3)	4.320(-2)
300	2.417(-3)	2.159(-2)
400	6.179(-4)	6.372(-3)
500	2.041(-4)	2.371(-3)
600	8.029(-5)	1.021(-3)
700	3.586(-5)	4.90(-4)
800	1.764(-5)	2.554(-4)
900	9.360(-6)	1.422(-4)
1000	5.280(-6)	8.349(-5)
2000	1.098(-7)	2.142(-6)
3000	1.075(-8)	2.275(-7)
4000	2.030(-9)	4.495(-8)
5000	5.519(-10)	1.261(-8)
6000	1.891(-10)	4.435(-9)
7000	7.614(-11)	1.825(-9)
8000	3.453(-11)	8.431(-100
9000	1.717(-11)	4.257(-100
10000	9.194(-12)	2.306(-11)
15000	8.459(-13)	2.146(-11)
20000	1.609(-13)	3.944(-12)
25000	4.514(-14)	1.063(-12)
30000	1.603(-14)	3.666(-13)
35000	6.657(-15)	1.500(-13)
40000	3.090(-15)	6.958(-14)

In the table, integral cross-sections in units of πa_0^2.The numbers in brackets denote powers of the ten by which each entry should be multiplied.

In the figures from 1 to 3, the differential cross-sections are shown. In fig.1, at 90 keV energy a pronounced dip has been observed at angle about 3.4 mrad.Othe curves are smooth. In fig.2 at energy 150keV two peaks have been observed one at about .2 mrad and the other at 2mrad.In the curve for 500keV a pronounced dip has been found at about 1 mrad. In fig.3 a dip and a peak have been observed at 1 mrad and 1.8 mrad respectively.

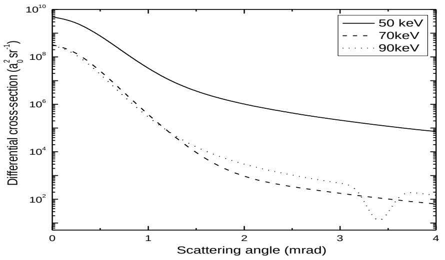

Fig. 1

Solid line(__) for energy 50keV, dashed line(----) for 70 keV and dotted line(…..) for 90keV.

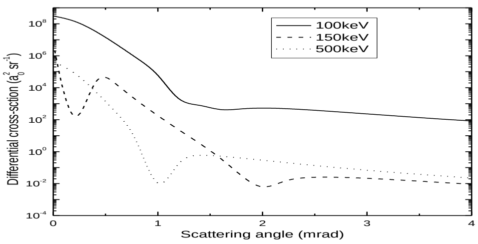

Fig. 2

Solid line(__) for energy 100keV, dashed line(----) for 150 keV and dotted line(…..) for 500keV.

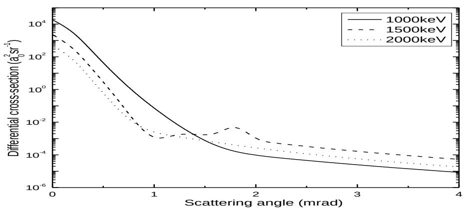

Fig. 3

Solid line (__) for energy 1000keV, dashed line (----) for 1500 keV and dotted line (…..) for 2000keV. Further investigations are needed for the isotopic mass effect in the cross-sections in these charge transfer reactions.

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