IOSR Journal of Applied Physics (IOSR-JAP)
e-ISSN: 2278-4861.Volume 9, Issue 6 Ver. IV (Nov. - Dec. 2017), PP 31-38
www.iosrjournals.org

The Effect of Damping on Thermal Vibrations of Isotropic Elastic Square Plate

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Abstract:- *A mathematical model is presented with an aim to assist the design engineers for the making of different structure used in space technology and others relative fields, many structural devices have to operate under elevated temperatures. This paper describes thermal vibrations of square plates of uniform thickness. Frobenius method has been employed to obtain the frequency of vibrations for first four modes of vibration using the functions based plates. The frequency parameter of the plate with elastically restrained edge conditions are presented for various values of damping and temperature parameters. A comparison of the results with those available in the literature obtained by power series solution method and coordinate functions shows an excellent agreement.*

Keywords:- *Damping, Frequency Parameter ,Square, Thermal gradient, Young modulus,*

Date of Submission: 20-12-2017 Date of acceptance: 31-12-2017

I. Introduction

In engineering, we cannot move without considering the vibration effect because almost all mechanical structures experiences vibrations. Vibrations almost affect the structure design. For healthy performance of the structure, the design engineers and scientist, it has always been a necessity to optimize or control of vibrations .The different material plates play a significant role in design the structural device. The reasons for it is because of engineering applications are working under high temperatures. Square plates have many engineering applications i.e. Vehicles, spacecrafts, missiles, ships, aircrafts, bridges etc. The vibrating plate type structures made up of composite materials have a significant role in various industrial mechanical structures, aerospace industries and other engineering applications The Present work is a full-fleshed endeavor to assist the designers.

Tomotika has observed the axisymmetric vibrations of a square plate clamped at all the four edges [1]. Tomar has studied the flexural vibrations of an isotropic, thin, square plate with different boundary conditions [2-3]. Roy has analyzed the large deflection of a square plate of non-homogeneous material subjected to normal pressure and temperature [4]. Kumar and Ramaieh have determined the vibrations of clamped square plates by Raleigh Ritz method with asymptotic solution [5]. Sabir and Davies have observed a lowest natural frequency of square plates containing circular holes is investigated using the finite element method for simply supported or clamped plates [6]. Sakiyama has analyzing the free vibration problem of orthotropic square plate with a square hole with non-uniform thickness by Green function method [7]. Metin and Taner have observed the vibration frequencies of antisymmetric angle ply laminated thin square composite plates having deferent boundary conditions by Ritz method [8]. Shuaand others have apply the two-dimensional least-square-based finite difference (LSFD) method for solving free vibration problems with simply supported and clamped edges [9]. Nesterov has analyzing square homogeneous plate clamped along its contour by resonance method [10]. Khanna and Sharma have studied the effect of linear thickness variation on vibration of visco-elastic square plate having clamped boundary condition on all the four edges with 2D temp. Fields by using Rayleigh Ritz technique [11-12]. Khanna and others have analyzing linear thermal effects on frequency of free vibrations of a visco-elastic square plate of parabolically varying thickness by Rayleigh Ritz method [13]. Sharma and others have studied the vibration of visco-elastic isotropic square plate with thermal effect on two direction varying thickness parabolically by using Rayleigh-Ritz technique with a twoterm deflection function. [14]. Khanna and sharma have studied the effect of thermal gradient on vibration of square plate of bi-parabolic varying thickness by Rayleigh Ritz technique. [15]. Khanna and others have analyzing study the 2D thermal effect on the vibration of non-homogeneous square plate of exponentially (in x direction) varying thickness having clamped boundary by Rayleigh Ritz technique[16] . Sharma and others have studied the effect of thermal gradient on vibration of non-homogenous square plate with varying thickness in x and y directions by Rayleigh-Ritz technique. Meera and others have studied the large amplitude free vibration behavior of uniform moderately thick square plates with axially immovable edges [18]

In the recent years, thermal effect on solid bodies has highly increased because of rapid developments in space technology and high speed flights. In these problems, the thermal dependence of frequency of plates of different shapes is of great importance and designing many scientific devices. The equation of motion is solved by Frobenius method and frequency parameters for first four modes of vibration at clamped and simply-supported edges have been computed. The numerical results are shown in graphical form. The present investigations are helpful in designing many scientific devices where uniform structure are exposed to high intensity heat fluxes due to which the material properties undergo significant change in vibrations.

II. Analysis

It is assumed that the rectangular plate material is subjected to a linear temperature distribution along the length i.e. in X-direction.

$$T = T_0(1-X) \tag{1}$$

where T is the temperature excess above the reference temperature at any point X and T_0 is the temperature excess above the reference temperature at the end X=0. The temperature dependence of modulus of elasticity generally for large number of materials [19, 20, and 21] is given by

$$\overline{E}(T) = \overline{E}_0(1-\xi T) \tag{2}$$

Where $\overline{E}_0$ is the modulus of elasticity of the material at the reference temperature and ξ is a constant. Now considering the temperature at the end of the plate i.e. at X=1, as the reference temperature, the modulus variation becomes.

$$\overline{E}(X) = \overline{E}_0\{1-\eta(1-X)\} \tag{3}$$

Where, $\eta = \xi\, T_0$, $(0 \le \eta < 1)$ is called the thermal gradient and ξ is an arbitrary constant. The governing equation of motion in non-dimensional variables is given by

$$\left[\overline{E}\right]\frac{\partial^4\overline{W}}{\partial X^4}+\left[2\frac{\partial\overline{E}}{\partial X}\right]\frac{\partial^3\overline{W}}{\partial X^3}+\left[\frac{\partial^2\overline{E}}{\partial X^2}-2\alpha^2\overline{E}\right]\frac{\partial^2\overline{W}}{\partial X^2}-\left[2\alpha^2\frac{\partial\overline{E}}{\partial X}\right]\frac{\partial\overline{W}}{\partial X}$$
$$+\left[\left\{\alpha^4\overline{E}\right\}-\left\{v\alpha^2\frac{\partial^2\overline{E}}{\partial X^2}\right\}-\left\{\frac{3(1-v^2)K^2}{\overline{\rho}a^2H^4}\right\}-\left\{\frac{12(1-v^2)\overline{\rho}a^2p^2}{H^2}\right\}\right]\overline{W}(X)=0 \tag{4}$$

Where, $\overline{W}=\frac{W}{a}, X=\frac{x}{a}, \overline{E}=\frac{E}{a}, H=\frac{h}{a}$ *and* $\overline{\rho}=\frac{\rho}{a}$ and $\alpha^2 = (m\pi)^2$

Using equation (3), equation (4) reduces to

$$\left[\{1-\eta+\eta X\}\right]\frac{\partial^4\overline{W}}{\partial X^4}+\left[2\eta X\right]\frac{\partial^3\overline{W}}{\partial X^3}-\left[2\alpha^2\{1-\eta+\eta X\}\right]\frac{\partial^2\overline{W}}{\partial X^2}-\left[2\alpha^2\eta\right]\frac{\partial\overline{W}}{\partial X}$$
$$+\left[\left\{\alpha^4(1-\eta+\eta X)\right\}-\left\{\left(D_k^2 I^{*2}\right)+\left(\Omega^2 I^*\right)\right\}\right]\overline{W}(X)=0 \tag{5}$$

Respectively, Where,

$$D_k^2=\left\{\frac{3k^2(1-v^2)}{a^2\overline{\rho}\overline{E}_0}\right\}, I^*=\left\{\frac{1}{H^2}\right\} \text{ and } \Omega^2=\left\{\frac{12(1-v^2)\overline{\rho}\,a^2\,p^2}{\overline{E}_0}\right\}$$

Hence, D_k is damping parameter; P is circular frequency and Ω is frequency parameter. A is the width of the plate and a prime denotes the derivative with respect to X.

III. Solution And Its Convergence

Let the solution for $\overline{W}$ is assured in the series from as:

$$\overline{W}(X)=\sum_{\lambda=0}^{\infty}a_\lambda X^{C+\lambda} \text{ with } a_0 \neq 0 \tag{6}$$

Where C is the exponent of singularity,

Using equation (6) & (5) one obtains

$$\sum_{\lambda=0}^{\infty} a_\lambda \left[b_\lambda^{(3)} T_1^{(1)}\right] X^{C+\lambda-4} + \sum_{\lambda=0}^{\infty} a_\lambda \left[b_\lambda^{(3)} T_2^{(1)} + b_\lambda^{(2)} T_2^{(2)}\right] X^{C+\lambda-3}$$
$$+ \sum_{\lambda=0}^{\infty} a_\lambda \left[b_\lambda^{(1)} T_3^{(1)}\right] X^{C+\lambda-2} + \sum_{\lambda=0}^{\infty} a_\lambda \left[b_\lambda^{(1)} T_4^{(1)} + b_\lambda T_4^{(2)}\right] X^{C+\lambda-1} + \sum_{\lambda=0}^{\infty} a_\lambda \left[T_5^{(1)}\right] X^{C+\lambda}$$
$$+ \sum_{\lambda=0}^{\infty} a_\lambda \left[T_6^{(1)}\right] X^{C+\lambda+1} = 0 \tag{7}$$

Respectively, where

$$b_\lambda^{(3)} = (C+\lambda)(C+\lambda-1)(C+\lambda-2)(C+\lambda-3), b_\lambda^{(2)} = (C+\lambda)(C+\lambda-1)(C+\lambda-2),$$
$$b_\lambda^{(1)} = (C+\lambda)(C+\lambda-1), b_\lambda = (C+\lambda), T_1^{(1)} = (1-\eta), T_2^{(1)} = \eta,\ T_2^{(2)} = 2\eta,$$
$$T_3^{(1)} = -2\alpha^2(1-\eta),\ T_4^{(1)} = T_4^{(2)} = -2\alpha^2(\eta),\ T_5^{(1)} = \left[\alpha^4(1-\eta) - \left\{D_k^2 I^{*2} + \Omega^2 I^*\right\}\right], T_6^{(1)} = \alpha^4 \eta,$$

For the series expression (6) to be solution, the coefficients of the powers of X in the equation (7) must be identically zero. Hence equating to zero the coefficient of the lowest power of X, the indicial roots C=0, 1, 2, 3 are obtained. For higher power of X, the constants a_1, a_2, a_3 are indeterminate for C=0 and these can be taken as along with a_0, similarly the constants $a_\lambda(\lambda=4)$ are obtained in terms of a_0, a_1, a_2 and a_3. The remaining unknown constant $a_\lambda(\lambda=5, 6, 7, 8 \ldots)$ are determined from the recurrence relation:

$$\left[(C+\lambda+5)(C+\lambda+4)(C+\lambda+3)(C+\lambda+2)T_1^{(1)}\right] a_{\lambda+5}$$
$$+(C+\lambda+4)(C+\lambda+3)(C+\lambda+2)\left[(C+\lambda+1).T_2^{(1)} + T_2^{(2)}\right] a_{\lambda+4}$$
$$+(C+\lambda+3)(C+\lambda+2)\left[T_3^{(1)}\right] a_{\lambda+3} + (C+\lambda+2)\left[\left\{(C+\lambda+1)T_4^{(1)} + T_4^{(2)}\right\}\right] a_{\lambda+2}$$
$$+\left[T_5^{(1)}\right] a_{\lambda+1} + \left[T_6^{(1)}\right] a_\lambda = 0 \tag{8}$$

If the notations:

$$a_\lambda = f_\lambda^{(3)} a_3 + f_\lambda^{(2)} a_2 + f_\lambda^{(1)} a_1 + f_\lambda^{(0)} a_0 \quad with\ \ \lambda = 0,1,2,3\ldots\ldots$$

Are introduced one finds that

$$f_m^{(n)} = \left.\begin{matrix} 0, m \neq n \\ 1, m = n \end{matrix}\right] \left.\begin{matrix} m = 0,1,2,3 \\ n = 0,1,2,3 \end{matrix}\right|$$

And $f_\lambda^{(n)}(\lambda = 4,5,6\ldots\ldots and\ n = 0,1,2,3)$ *are function of* η, D_k, I^* *and* Ω

The solution for $\overline{W}$, corresponding to C = 0, is

$$\overline{W} = a_0 F_0(X,\Omega) + a_1 F_1(X,\Omega) + a_2 F_2(X,\Omega) + a_3 F_3(X,\Omega) \tag{9}$$

Where,

$$\left.\begin{aligned} F_0(X,\Omega) &= 1 + \sum_{\lambda=4}^{\infty} f_\lambda^{(0)} X^\lambda \\ F_1(X,\Omega) &= X + \sum_{\lambda=4}^{\infty} f_\lambda^{(1)} X^\lambda \\ F_2(X,\Omega) &= X^2 + \sum_{\lambda=4}^{\infty} f_\lambda^{(2)} X^\lambda \\ F_3(X,\Omega) &= X^3 + \sum_{\lambda=4}^{\infty} f_\lambda^{(3)} X^\lambda \end{aligned}\right\} \tag{10}$$

It is evident that no new solution will arise corresponding to C=1, 2, 3. The solution corresponding to these values of C are already contained in the solution (9). The technique used by Lamb [23] has been applied for convergence of solution (9). If $\lambda \to \infty$ one finds that equation (9) is uniformly convergent for $0 \le X \le 1 \, when \, |\mu| < 1, Where, \mu = \lim it_{\lambda \to \infty} \left(\frac{a_{\lambda+1}}{a_\lambda} \right)$

IV. Boundary Conditions And Frequency Equation

The following combinations of boundary conditions at the edge X = 0 and X = 1 have been considered while the other two edges Y =0 and Y=1 are simply supported in all the cases.
(1) Clamped at both the edges X=0 and X=1 (C-SS-C-SS) (2). Clamped at X=0 and simply supported at X=1 (C-SS-SS-SS) and (3) simply supported at both the edges X=0 and X=1 (SS-SS-SS-SS). The boundary conditions for different edge conditions are:

For clamped edge: $w = 0 \; and \; \frac{\partial w}{\partial x} = 0$ (11)

For simply supported edge: w=0 and M_8 =0 (12)

4.1 [C-SS-C-SS] – Plates :

Using equation (9) , (10) and (11) , then obtain governing frequency equation is below as:

$$\begin{vmatrix} F_2(1,\Omega) & F_3(1,\Omega) \\ F_2^1(1,\Omega) & F_3^1(1,\Omega) \end{vmatrix} = 0 \quad (13)$$

where a prime denotes the derivative with respect to x.

4.2 |C-SS-SS-SS| - Plates:

Again using equation (9), (12), (10) and (13), the frequency equation for this plate is given below as

$$\begin{vmatrix} F_2(1,\Omega) & F_3(1,\Omega) \\ F_2^n(1,\Omega) & F_3^n(1,\Omega) \end{vmatrix} = 0 \quad (14)$$

4.3 |SS-SS-SS-SS| - Plates:

Similarly using equation (9) on (12) , (10) and (13), the frequency equation for this plate after eliminating a_0, a_1, a_2 and a_3 at this edges, is

$$\begin{vmatrix} F_1(1,\Omega) & F_3(1,\Omega) \\ F_1^n(1,\Omega) & F_3^n(1,\Omega) \end{vmatrix} = 0 \quad (15)$$

V. Results And Discussion

Numerical results for an isotropic, elastic, non-homogeneous, square plate of uniform thickness have been computed from the equations **(13), (14) and (15)** when the temperature fields varies as linearly, using latest computer technology for various combinations of thermal gradients η, damping parameter D_k and length to breadth ratio (a/b=1). In all the cases considered, the Poisson's ratio (0.3) and thickness (0.1) of the plate has been assumed to remain constant. Terms of the series upto an accuracy of 10^{-8} in their absolute values have been retained. The variation of frequency parameters Ω corresponding to the first four modes of vibration have been computed for (C-SS-C-SS), (C-SS-SS-SS) and (SS-SS-SS-SS) plates for different values of η and D_k. However the results of first two modes have been plotted and shown in the figures (1 to 4). Also the results for frequency parameters Ω with η=0=D_k have been compared with Soni [22]. The results, plotted in figures from 1 to 4 depicts the effect of damping parameter D_k on the frequency parameter Ω of square plates of uniform thickness corresponding to the first four modes of vibration for various combinations of thermal gradient η with (C-SS-C-SS), (C-SS-SS-SS) and (SS-SS-SS-SS) boundary condition. For all the boundary conditions considered here, it is observed that the frequency parameters Ω decreases with the increasing value of damping parameters D_k for heated aswell as unheated (η=0) plates. One can also noted that the for higher values of D_k the fall in frequency parameters Ω is very sharp specially for (SS-SS-SS-SS) boundary conditions when the η is higher. The frequencies for (C-SS-C-SS) plate is higher than the corresponding to (C-SS-SS-SS) and (SS-SS-SS-SS) plates for all the four modes. The variation of Ω with thermal gradient η for different values of damping parameter D_k is plotted in figures from 5 to 8 for all the three edge conditions for first four modes of vibrations. It is noted that the Ω decreases with the increasing value of thermal gradient η for all the three edge conditions for first four modes of vibrations considered here. Hence it is clearly noted that the for first four modes of non homogeneous square plate, as the value of'η and D_k increases the frequency parameter Ω also decrease for all the three cases of boundary conditions considered here.

VI. Conclusion

Mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

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Figure 1: Frequencies of a Square Plate with D_K For First Mode of Vibration Under the Thermal Gradient 'η' [Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]

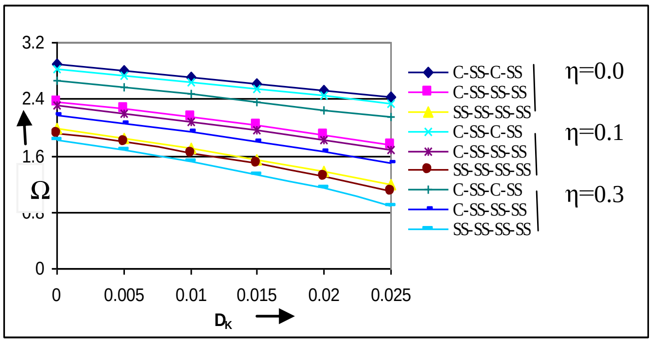


Figure 2: Frequencies of a Square Plate with D_K For Second

Mode of Vibration Under the Thermal Gradient 'η'
[Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]

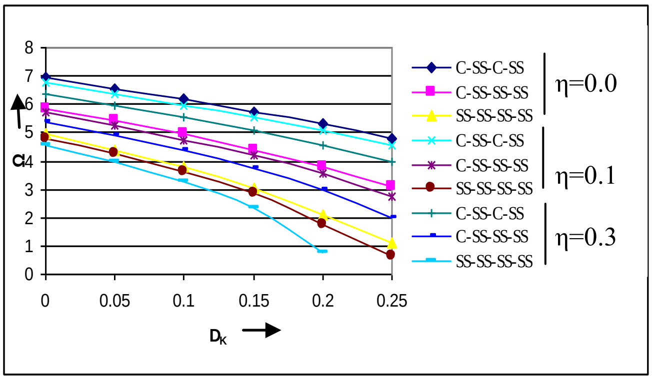


Figure 3: Frequencies of a Square Plate with D_K For Third
Mode of Vibration Under the Thermal Gradient 'η'
[Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]

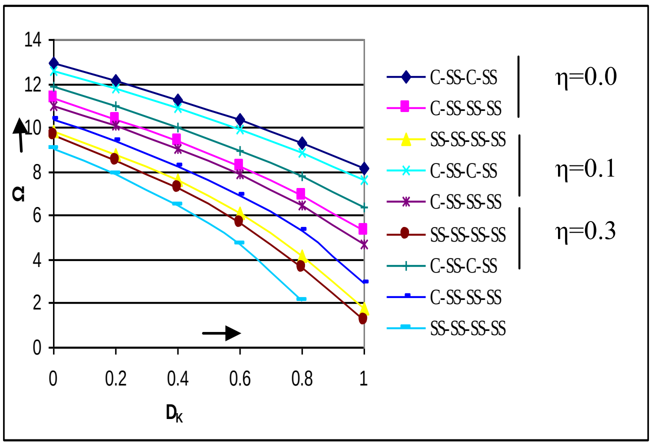


Figure 4: Frequencies of a Square Plate with D_K For Fourth
Mode of Vibration Under the Thermal Gradient 'η'
[Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]

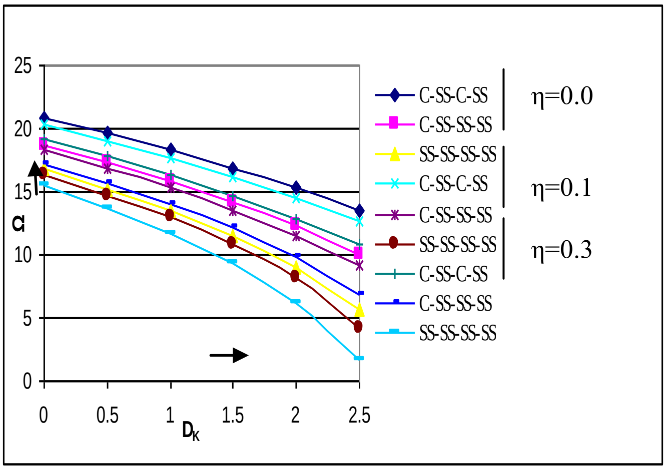


Figure 5: Frequencies of a Square Plate with η For First
Mode of Vibration Under the Damping Parameter D_K
[Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]

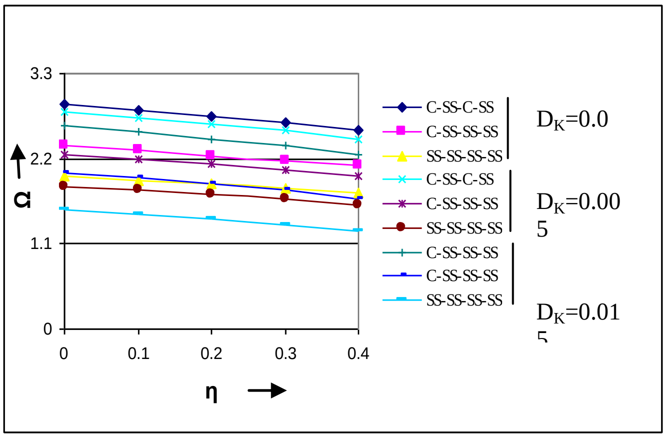


Figure 6: Frequencies of a Square Plate with η For Second Mode of Vibration Under the Damping Parameter D_K
[Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]

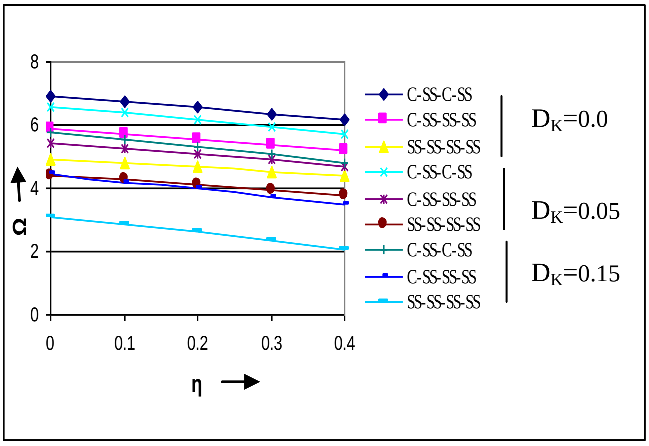


Figure 7: Frequencies of a Square Plate with η For Third Mode of Vibration Under the Damping Parameter D_K
[Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]

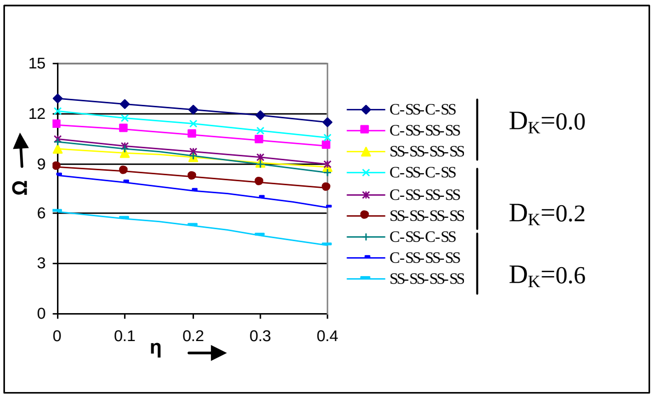
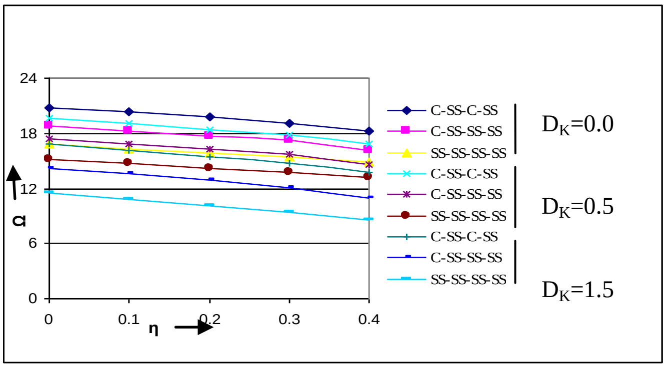


Figure 8: Frequencies of a Square Plate with η For Fourth Mode of Vibration Under the Damping Parameter D_K
[Legend : H=0.1, ν=0.3, m=1.0,C = Clamped and SS = Simply Supported]



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IOSR Journal of Applied Physics (IOSR-JAP) (IOSR-JAP) is UGC approved Journal with Sl. No. 5010, Journal no. 49054.

Narender Kumar Sarswat " The Effect of Damping on Thermal Vibrations of Isotropic Elastic Square Plate.” IOSR Journal of Applied Physics (IOSR-JAP) , vol. 09, no. 06, 2017, pp. 31-38.