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Differential Inequalities in Nonlocal Boundary Value Problems

[1]*Goteti V.Sarma, [2]Habtayghebrewold
Department Of Mathematics, Eritrea Institute Of Technology, Mainefhi, Eritrea.
Corresponding Author: Goteti V.Sarma

Abstract: *This Article Deals With A Nonlocal Five-Point Boundary Value Problem Associated With Third Order Differential Equation. Differential Inequalities Arise In This Problem Is Studied And Using These Inequalities And Solution Matching Technique, The Existence And Uniqueness Of Solution Is Obtained.*
Keywords: *Differential Inequalities, Nonlocal Boundary Value Problem, Solution Matching Technique.*
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I. Introduction

This Article Deals With The Differential Inequalities Arises In Nonlocal Real Valuedfive Point Boundary Value Problem Associated With 3rdoreder Differential Equation
$y'''(x) = f(x, y, y', y'')$ (1.1)
Satisfying $y(a) - y(x_1) = y_1, y(b) = y_2, y(c) - y(x_2) = y_3$ (1.2)
Where $a < x_1 < b < x_2 < c$ And $f \in C\big[[a,c]XR^3, R\big]$. Using These Inequalities And The Solution Matching Technique, Its Solution Is Constructed. It Is Assumed The Existence And Uniqueness Of Initial Value Problems Associated With (1.1). A Monotonicity Restriction On f Ensures That The Following Three Point Boundary Value Problems:
$y'''(x) = f(x, y, y', y'')$ (1.3)
Satisfying One Of The Boundary Conditions
$y(a) - y(x_1) = y_1, y(b) = y_2, y'(b) = m$ (1.4)
$y(a) - y(x_1) = y_1, y(b) = y_2, y''(b) = m$ (1.5)
$y(b) = y_2, y'(b) = m, y(c) - y(x_2) = y_3$ (1.6)
$y(b) = y_2, y''(b) = m, y(c) - y(x_2) = y_3$ (1.7)

Have Atmost One Solution For Any Real Constant m And With The Added Hypothesis That The Solution Exists To (1.1) Satisfying One Of (1.4)-(1.7), A Unique Solution To The Five Point Boundary Value Problem Is Constructed With The Help Of Solution Matching Technique.Non Local Boundary Value Problems Raises Much Attention Because Of Its Ability To Accommodate More Boundary Points Than Their Corresponding Order Of Differential Equations [5], [8]. Considerable Studies Were Made By Bai And Fag [2], Gupta [4] And Web [9]. Solution Matching Technique Which Is Used To Obtain The Existence And Uniqueness Of Solutions To Two Point Boundary Value Problems Is First Initiated By Baily [1] Et Al Is Further Extended By Moorty And Garner To Study Larger Class Of Boundary Value Problem. Later Lakshmikantham And Murthy [6] Developed The Theory Of Differential Inequalities For Two Point Boundary Value Problems To Obtain Existence And Uniqueness Of Solutions To Three Point Boundary Value Problems. Later Murthy And Sarma[7] Generalized These Inequalities To Study The Existence And Uniqueness Of Solutions To Three Point Bvps Associated With Nth Order Non Linear System Of Differential Equations.This Article Is Organized As Follows: Section 2 Deals With The Differential Inequalities In Three Point Boundary Value Problems Associated With Third Order Differential Equations. Later Section 3 Establishes The Existence And Uniqueness Of Solution To A Five Point Boundary Value Problem Associated With Third Order Differential Equation.

1.2 Differential Inequalities: Before Deriving Differential Inequalities Let Us Define The Following Sets And Classes Of Functions:
Suppose $y \in C^3\big[[a,c], R^3\big]$
$\Psi_1(y) = \{ x\colon\ If\ x \in [a,b)\ then\ y''(x) = 0\ \ and\ \ y(x)y'(x) < 0 \}$
$\Psi_2(y) = \{ x\colon\ If\ x \in (b,c]\ \ then\ y''(x) = 0\ \ and\ \ y(x)y'(x) > 0 \}$

$$G_1 = \begin{Bmatrix} G\colon G \in C\big[[a,b]\ X\ R^3, R\big]\ and \\ for\ x \in [a,b), \quad G(x, t_1, t_2, t_3) > 0\ whenever\ t_1 t_2 < 0 \end{Bmatrix}$$

$$G_2 = \begin{cases} G: G \in C\left[[a,b]\ X\ R^3, R\right]\ and \\ for\ x \in (b,c], \qquad G(x,t_1,t_2,t_3) > 0\ whenever\ t_1 t_2 > 0 \end{cases}$$

Now We Will Derive The Differential Inequalities Arises In Nonlocal Boundary Value Problems In The Following Lammas.

Lemma 2.1: There Exists No $y \in C^3\left[[a,b], R^3\right]$ Satisfying

i. $y(a) - y(x_1) = 0,\ y(b) = 0$ And Either $y^{'}(b) = 0, y^{''}(b) < 0$ Or $y^{'}(b) > 0, y^{''}(b) = 0$
ii. $y^{'''}(x) > G\left(x, y(x), y^{'}(x), y^{''}(x)\right)$ For Some $x \in \Psi_1$, For Some $G \in G_1$

Proof: Suppose There Exists $y \in C^3\left[[a,b], R^3\right]$ Satisfying The Hypothesis (I) And (Ii). Then Clearly $y(x) \neq 0$ On $[a,b]$.

Let Us First Consider The Case

$y(a) - y(x_1) = 0, y(b) = 0$ And $y^{'}(b) = 0, y^{''}(b) < 0$

Then There Exists $r \in (a, x_1)$ Such That $y^{'}(r) = 0$ And Hence There Exists $p \in (r,b)$ Such That $y^{'}{'}(p) = 0$ And Since $y^{''}(b) < 0$ We Can Assume Without Loss Of Generality That $y^{''}(x) < 0\ on\ x \in (p,b]$.

This Gives $\int_x^b y^{''}(x)dx = -y^{'}(x) < 0$ Which Shows That $y^{'}(x) > 0 on\ x \in (p,b]$.Again With The Same Logic And Using The Fact That $y(b) = 0$ We Can Show That $y(x) < 0\ on\ x \in (p,b]$. This Means $y(x) and\ y^{'}(x)$Are Decreasing And Increasing On$(p,b]$ And Using The Fact That $y(b) = 0\ and\ y^{'}(b) = 0$ We Can Conclude That $y(p)y^{'}(p) < 0$. This Shows That $p \in \Psi_1$.

Hence By Our Assumption $y^{'''}(p) > 0$

However$y^{'''}(p) = \text{Lim}_{x \to p^+} \frac{y^{''}(x) - y^{''}(p)}{x - p} \leq 0$ Which Is A Contradiction.

Let Us Consider The Second Case

$y(a) - y(x_1) = 0, y(b) = 0$ And $y^{'}(b) > 0, y^{''}(b) = 0$

Now $y(a) - y(x_1) = 0$ Gives An $r \in (a, x_1)$ Such That $y^{'}(r) = 0$ And

Since $y^{'}(b) > 0$ We Can Assume Without Loss Of Generality That $y^{'}(x) > 0$ On $(r,b]$

With This We Can Get A $p \in (r,b)$ Such That $y''(p) > 0$. For Otherwise If $y^{'}{'}(x) \leq 0$ On (r,b) Then $\int_r^x y^{''}(x)dx = y^{'}(x) \leq 0$ On (r,b) Which Is A Contradiction.

Since $y''(p) > 0$ And $y^{''}(b) = 0$ Gives That There Exists A $q \in (p,b]$ Such That

$y^{''}(q) = 0$ And $y^{''}(x) > 0$ On $[p,q)$.

Clearly Since $q \in (p,b], y^{'}(q) > 0$

Againintegrating The Fact That $y^{'(x)} > 0$ On $(r,b]$ And Using The Condition That $y(b) = 0$ Gives $y(x) < 0$ On (r,b) And As $q \in (p,b] \in (r,b)$ Hence $y(q) < 0$

Therefore $y(q)y^{'}(q) < 0$ Hence $q \in \Psi_1$.

Consequently Because Of Our Assumption (Ii), $y^{'''}(q) > G\left(q, y(q), y^{'}(q), y^{''}(q)\right) > 0$

But $y^{'''}(q) = \text{Lim}_{x \to q^-} \frac{y^{''}(x) - y^{''}(q)}{x - q} \leq 0$ Which Is A Contradiction.

Hence There Exists No $y \in C^3\left[[a,b], R^3\right]$ Satisfying The Hypothesis (I) And (Ii).

This Proves Our Lemma.

Lemma 2.2: There Exists No $y \in C^3\left[[b,c], R^3\right]$ Satisfying

i. $y(b) = 0,\ y(c) - y(x_2) = 0$ And Either $y^{'}(b) = 0, y^{''}(b) > 0$ Or $y^{'}(b) > 0, y^{''}(b) = 0$
ii. $y^{'''}(x) > G\left(x, y(x), y^{'}(x), y^{''}(x)\right)$ For Some $x \in \Psi_2$, For Some $G \in G_2$

Proof: Suppose There Exists $y \in C^3\left[[a,b], R^3\right]$ Satisfying The Hypothesis (I) And (Ii). Then Clearly $y(x) \neq 0$ On $[b,c]$.

Let Us First Consider The Case $y(c) - y(x_2) = 0, y(b) = 0$ And $y^{'}(b) = 0, y^{''}(b) > 0$

Then There Exists $r \in (x_2,\ c)$ Such That $y^{'}(r) = 0$ And Hence There Exists $p \in (r,b)$ Such That $y^{'}{'}(p) = 0$ And Since $y^{''}(b) > 0$ We Can Assume Without Loss Of Generality That $y^{''}(x) > 0\ on\ x \in [\,b,p)$.This Gives $\int_b^x y^{''}(x)dx = y^{'}(x) > 0$ Which Shows That $y^{'}(x) > 0 on\ x \in [b,\ p)$.

Again With The Same Logic And Using The Fact That $y(b) = 0$ We Can Show That

$y(x) > 0\ on\ x \in [b,\ p)$. This Means $y(x) and\ y^{'}(x)$Are Increasing On$[b,p)$ And Using The Fact That $y(b) = 0\ and\ y^{'}(b) = 0$ We Can Conclude That $y(p)y^{'}(p) > 0$. This Shows That $p \in \Psi_2$.

Hence By Our Assumption $y^{'''}(p) > 0$

However$y^{'''}(p) = \text{Lim}_{x \to p^-} \frac{y^{''}(x) - y^{''}(p)}{x - p} \leq 0$ Which Is A Contradiction.

Let Us Consider The Second Case
$y(c) - y(x_2) = 0, y(b) = 0$ And $y^{'}(b) > 0, y^{''}(b) = 0$
Now $y(c) - y(x_2) = 0$ Gives An $r \in (x_2, c)$ Such That $y^{'}(r) = 0$ And Since
$y^{'}(b) > 0$ We Can Assume Without Loss Of Generality That $y^{'}(x) > 0$ On $[b, r)$
With This We Can Get A $p \in [b, r)$ Such That $y^{''}(p) < 0$. For Otherwise If $y^{'}{'}(x) \geq 0$ On $[b, r)$ Then $\int_x^r y^{''}(x)dx = -y^{'}(x) \geq 0$ On (r, b) Which Is A Contradiction.
Since $y''(p) < 0$ And $y^{''}(b) = 0$ Gives That There Exists A $q \in [b, p)$ Such That
$y^{''}(q) = 0$ And $y^{''}(x) < 0$ On $(q, p]$.
Clearly Since $q \in (p, b], y^{'}(q) > 0$

Again Integrating The Fact That $y^{'(x)} > 0$ On $[b, r)$ And Using The Condition That $y(b) = 0$ Gives $y(x) > 0$ On $[b, r)$ And As $q \in [b, p) \in [b, r)$Hence $y(q) > 0$
Therefore $y(q)y^{'}(q) > 0$ Hence $q \in \Psi_2$.
Consequently Because Of Our Assumption (Ii), $y^{'''}(q) > G(q, y(q), y^{'}(q), y^{''}(q)) > 0$
But $y^{'''}(q) = \text{Lim}_{x \to q^+} \frac{y^{''}(x) - y^{''}(q)}{x - q} \leq 0$ Which Is A Contradiction.
Hence There Exists No $y \in C^3[[b, c], R^3]$ Satisfying The Hypothesis (I) And (Ii). This Proves Our Lemma.

Lemma 2.3: There Exists No $y \in C^3[[a, c], R^3]$ Satisfying

i. $y(a) - y(x_1) = 0, y(b) = 0, y(c) - y(x_2) = 0, y^{'}(b) \neq 0, \textit{and } y^{''}(b) \neq 0$
ii. $y^{'''}(x) > G(x, y(x), y^{'}(x), y^{''}(x))$ For Some $x \in \Psi_1 \cup \Psi_2$ For Some $G \in G_1 \cap G_2$

Proof: Suppose There Exists A $y \in C^3[[a, c], R^3]$ Satisfying The Hypotheses.
Then Clearly $y(x) \neq 0$ On $[a, c]$. Without Loss Of Generality We Can Suppose That $y^{'}(b) > 0$.
Since $y(a) - y(x_1) = 0, y(c) - y(x_2) = 0$ There Exists $r_1 \in (a, x_1)$, $r_2 \in (x_2, c)$Such That $y^{'}(r_1) = 0$ And$y^{'}(r_2) = 0$ And As $y^{'}(b) > 0$ With Out Loss Of Generality We Can Take $y^{'}(x) > 0$ On (r_1, r_2). Then There Exists $q \in (r_1, b)$ Or $q \in (b, r_2)$ Such That $y^{''}(q) = 0$.
Consider The Case That $\in (r_1, b)$. Then Clearly $q \in (r_1, r_2)$ And Hence $y^{'}(q) > 0$. Now $y^{'}(x) > 0$ On (r_1, r_2)Hence $y^{'}(x) > 0$ On (r_1, b) Which Implies
$\int_x^b y^{'}(x)dx = -y(x) \geq 0$.
Hence $y(x) \leq 0$ On (r_1, b) And In Particular $(q) \leq 0$. Hence $y(q)y^{'}(q) < 0$. Therefore $q \in \Psi_1$.
Consequently It Follows That $y^{'''}(q) > G(q, y(q), y^{'}(q), y^{''}(q)) > 0$
But On The Other Hand$y^{'''}(q) = \text{Lim}_{x \to q^-} \frac{y^{''}(x) - y^{''}(q)}{x - q} \leq 0$ Which Is A Contradiction.
Hence There Exists No $y \in C^3[[a, c], R^3]$ Satisfying The Hypothesis (I) And (Ii). This Proves Our Lemma.

Lemma 2.4: Assume That $y \in C^3[[a, c], R^3]$ Satisfying

i. $y(a) - y(x_1) = 0, y(b) = 0, y^{''}(b) < 0$.
ii. $y^{'''}(x) > G(x, y(x), y^{'}(x), y^{''}(x))$ For Some $x \in \Psi_1$, For Some $G \in G_1$.

Then There Exists A $p \in [a, b)$ Such That $y^{'}(x) < 0$ On (p, b).
Proof:
Consider The Hypothesis (I) $y(a) - y(x_1) = 0, y(b) = 0, y^{''}(b) < 0$.
Then There Exists $r \in (a, x_1)$ Such That $y^{'}(r) = 0$ And $y^{'}(x) \neq 0$ On $(r, b]$ $\textit{and}$ Maintains The Same Sign In $(r, b]$.

Now We Will Show That $y^{'}(x) < 0$ On $(r, b]$.
To The Contrary If $y^{'}(x) > 0$ On $(r, b]$ Then There Exists $q \in (r, b]$ Such That $y^{'}(q)$ Is The Supremum Of $y^{'}(x)$ On $(r, b]$.Hence $y^{''}(q) = 0$ And Without Loss Of Generality We Can Take $y^{''}(x) < 0$ On $(q, b]$. Clearly As $q \in (r, b], y^{'}(q) > 0$.
Clearly As $y^{'}(x) > 0$ On $(r, b], y(x)$ Is Increasing On $(r, b]$ And Since $y(b) = 0$ Hence $y(x) < 0 \; on \; x \in (r, b]$. As $q \in (r, b], y(q) < 0$.
Therefore $y(q)y^{'}(q) < 0$.Hence $q \in \Psi_1$.
Consequently It Follows That $y^{'''}(q) > G(q, y(q), y^{'}(q), y^{''}(q)) > 0$
But On The Other Hand$y^{'''}(q) = \text{Lim}_{x \to q^+} \frac{y^{''}(x) - y^{''}(q)}{x - q} \leq 0$ Which Is A Contradiction.
This Proves Our Lemma.

Lemma 2.5: Assume That $y \in C^3[[b, c], R^3]$ Satisfying

i. $y(c) - y(x_2) = 0, y(b) = 0, y^{''}(b) > 0$.

ii. $y'''(x) > G(x, y(x), y'(x), y''(x))$ For Some $x \in \Psi_2$, For Some $G \in G_1$.

Then There Exists A $p \in (b,\ c]$ Such That $y'(x) < 0$ On $[b,\ p)$.

Proof: Proof Is Similar To The Proof Of Lemma 4 Hence Omitted.

II. Existence And Uniqueness By Matching Solutions

In This Section We Are Going To Establish The Existence And Uniqueness Of The Solution Of The Boundary Value Problem (1.1) Satisfying (1.2) By Matching The Solutions Of The Boundary Value Problems (1.1) Satisfying (1.4) And (1.1) Satisfying (1.6).

First In The Following Lemma We Will Show That The Four Boundary Value Problems Have At Most One Solution.

Lemma 3.1: Assume That

$G \in G_1 \cap G_2, f \in C\big[[a,c]XR^3\big]$ And f Satisfies

$f(x, y_1, y_2, y_3) - f(x, z_1, z_2, z_3) > G(x, y_1 - z_1, y_2 - z_2, y_3 - z_3)$ Whenever

i. $x \in [a, b),\ (y_1 - z_1)(y_2 - z_2) < 0$, And $y_3 = z_3$

ii. $x \in (b, c],\ (y_1 - z_1)(y_2 - z_2) > 0$, And $y_3 = z_3$

Holds.

Then The Four Boundary Value Problems (1.1) Satisfying (1.4) Or (1.5) Or (1.6) Or (1.7) Have At Most One Solution.

Proof: Consider The Bvp (1.1) Satisfying (1.4)

Suppose There Exists Two Solutions $y_1(x)\ and\ y_2(x)$Of The Bvp (1.1) Satisfying (1.4). On $[a.b]$.

Let Us Write $y(x) = y_1(x) - y_2(x)$. Then Clearly $y(a) - y(x_1) = 0, y(b) = 0, y'(b) = 0$

We Suppose That $y''(b) \neq 0$. Without Loss Of Generality Take $y''(b) < 0$

Let $x_0 \in \Psi_1$(Y) Then Clearly $then\ \ y''(x_0) = 0\ \ and\ \ y(x_0)y'(x_0) < 0$

Consequently $y'''(x_0) = f(x, y_1(x_0)y_1'(x_0), y_1''(x_0)) - f(x, y_2(x_0)y_2'(x_0), y_2''(x_0))$
$> G(x, y(x_0),\ \ y'(x_0),\ \ y''(x_0))$

Since All The Hypothesis Of The Lemma (2.1) Is Satisfied Hence There Exists No Such $y \in C^3\big[[a, b], R^3\big]$. Therefore $y''(b) = 0$. Then By The Uniqueness Of Solutions Of The Initial Value Problems Of (1.1) $y(x) \equiv 0$ On $[a,\ b]$ Which Implies The Bvp (1.1) Satisfying (1.4) Have At Most One Solution. Similarly We Can Prove The Uniqueness Of Solutions Of Other Boundary Value Problems.

Theorem 3.1 : Assume That

i. For Each $m \in \mathbb{R}$ There Exists Solutions Of Boundary Value Problems (1.1) Satisfying (1.4) Or (1.5) Or (1.6) Or (1.7).

ii. f And g Satisfies The Assumptions (I) Of Lemma 3.1.

Then There Exists A Unique Solution To The Boundary Value Problem (1.1) Satisfying (1.2).

Proof: Clearly By The Lemma 3.1 And Assumption (I), The Solutions Of Boundary Value Problems (1.1) Satisfying (1.4) Or (1.5) Or (1.6) Or (1.7) Are Unique.

Let $y_1(x,\ m)$ Be The Unique Solution Of (1.1) Satisfying (1.4).

Let $m_2 > m_1$ And Let $v(x) = y_1(x,\ m_1) - y_1(x,\ m_2)$.

Then $v(a) - v(x_1) = 0,\ v(b) = 0, v''(b) = m_1 - m_2 < 0$.

Let $x \in \Psi_1(v)$. Then $v''(x) = 0\ and\ v'(x)v''(x) < 0$ I.E $y_1''(x,\ m_1) = y_1''(x,\ m_2)$.

Now Using Assumption (Ii) We Get $v'''(x) > g(x, v(x), v'(x), v''(x))$ For $x \in \Psi_1$ And For $g \in G_1$.

Then By Lemma 2.4 Yields That There Exists A $p \in [a, b)$ Such That $y'(x) < 0$ On $(p, b]$ And In Particular $y'(b) < 0$ Which Implies $y_1'(b,\ m_1) - y_1'(b,\ m_2) < 0$.

Therefore $y_1'(b,\ m)$ Is Strictly Increasing Function Of m.

Similarly We Can Show That If $y_2(x,\ m)$ Be The Unique Solution Of The Boundary Value Problem (1.1) Satisfying (1.6), Then $y_2'(b,\ m)$ Is Strictly Decreasing Function Of m.

Since The Solutions Of (1.1) Satisfying (1.5) Or (1.7) Are Unique It Follows That The Ranges Of $y_1'(b,\ m)$ And $y_2'(b,\ m)$ Are The Set Of Real Numbers. Hence There Exists An $m_0 \in \mathbb{R}$ Such That $y_1'(b,\ m_0) = y_2'(b,\ m_0)$,

Hence $y(x)$ Defined By $y(x) = \begin{cases} y_1(x,\ m_0)\ if\ x \in [a, b] \\ y_2(b,\ m_0)\ \ if\ x \in [b, c] \end{cases}$ Is The Solution Of (1.1) Satisfying (1.2).

To Prove Uniqueness Of (1.1) And (1.2), Suppose There Are Two Solutions $y_1(x) and\ y_2(x)$ Of The Boundary Value Problem (1.1) Satisfying (1.2).

Set $v(x) = y_1(x) - y_2(x)$. Then $v(a) - v(x_1) = 0, v(b) = 0, v(c) - v(x_2) = 0$

If $v'(b) \neq 0$ And $v''(b) \neq 0$, Then It Is Easy To Check As Before That All The Assumptions Of Lemma (2.3) Which Implies That There Exists No Such $v(x)$ On$[a, c]$. Thus $v'(b) = 0$ Or $v''(b) = 0$.

Therefore By Lemma (3.1) $y_1(x) = y_2(x)$ Proving The Uniqueness.

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