Fröhlich Interaction Based Parametric Amplification in a Degenerate Polar Semiconductor Magnetoplasma

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Abstract: Principal aim of this paper is to explore amplification characteristics of a semiconductor magnetoplasma by determining threshold pump field required for the onset of parametric amplification and absorption coefficient of the medium. Propagation of intense laser beam in doped semiconductor plasma has been investigated by considering induced second order optical nonlinearity. Hydrodynamic model for one component plasma is used to allow suitable wave propagations and nonlinear interactions resulting into three wave mixing phenomenon namely optical parametric amplification. Coupled mode theory is applied to analyse coupling of electron and Longitudinal Optical (LO) phonons and further pump, polaron and signal modes in the medium. Theoretical model is developed with the help of rotating wave approximation method. Numerical estimations have been done by assuming n-type GaAs crystal shined by CO₂ laser in the presence of an external magnetostatic field under Voigt geometry. The results indicate that the second order optical susceptibility can be tailored by varying magnitudes of carrier concentration and external magnetic field. It is found that proper selection of doping profiles reduces threshold pump field and enhances gain profile of the amplifier. This study could be utilized in the fabrication of parametric amplifier and design of optical switches.

Keywords – Fröhlich interaction, III-V polar semiconductor, Polaronic effects, Magnetoplasma excitations.

I. Introduction

An interaction between the plasma particles and the existing force fields corresponds to fundamental properties of plasma, where the fields may be externally applied or internally generated ones [1]. At low electric field, electrons will not gain enough kinetic energy, but at higher electric field sufficiently energetic electrons would be able to transfer energy to LO phonons due to coupling between them. Fröhlich interaction originates from the coupling of one itinerant electron with the macroscopic electric field generated by any LO phonon. It is considered to be the strongest exciton–phonon coupling mechanism in polar semiconductor crystallites for incoming photon energy in resonance with excitonic states [2].

Polaronic effects based on the Fröhlich interaction were earlier studied by Lee and Pines [3], Feynman [4] and Landau [5] on theoretical grounds recognised as pioneering work in this field. Polaronic effects arising due to Fröhlich interaction between electrons and the longitudinal-optical (LO) phonon of a polar semiconductor have been the object of many studies [6-9] and have grown as an explosive area of research. Electron-phonon interaction in a single modulation-doped GaInAs has been studied by taking Fröhlich interaction mechanism by Orlita et al. [10]. The electron-phonon interaction in confined systems has been studied by Rink et al. [11]. They reported that the strength of the electron phonon coupling increases with decreasing particle size. Resonant enhancement of Raman modes of nano structured copper oxide showed the existence of strong electron–phonon coupling mediated by Frohlich interaction [12]. Blocking of polaron interaction has been studied by Wu et al. [13]. It was observed by them that due to electron-phonon interaction large corrections are produced on Cyclotron Resonance (CR) frequency not only in the resonant region (ℏω_e ∥ ℏω_ρ), but also in the off-resonant region. Faugeras et al. [14] tested the concept of Fröhlich interaction and polaron mass in doped GaAs quantum well structures by infrared magneto absorption measurements. A study of confined phonon modes for a four layers Ga₁₋ₓAlₓAs/GaAs Quantum Cables (QC) system yields that the high frequency phonon modes have more significant contributions to the coupling of the electron-phonon interaction [15]. The polaron effect well above the LO phonon energy was studied through cyclotron resonance measurements in compound semiconductor in presence of ultra-high magnetic fields [16] and reported that the resonant polaron effect manifests itself when the cyclotron frequency approaches the LO phonon energy at sufficiently high magnetic fields.
Fröhlich interaction remains absent in covalent materials such as Ge and Si, but it predominantly affect the mobility of III-V compound semiconductors. For this type of studies semiconductors happens to be obvious choice because of the possibility of rendering them p-type or n-type conductors through doping. At smaller electron densities, the interaction remains unscreened, if density is increased then plasmons and phonons will not remain decoupled any longer. Rather the system will exhibit oscillations at coupled phonon-plasmon modes with frequency

\[ \omega^2_p = \frac{1}{2} \left( \omega^2_p + \omega^2_{LO} \right) \pm \left( \left( \omega^2_p + \omega^2_e \right)^2 - 4 \omega_p \omega_e \omega_{LO} \right)^{1/2} \]  

Hence, it would be interesting to find out the carrier concentration favourable for the coupling between plasmon-LO phonon in a degenerate polar semiconductor medium.

Since semiconductors represent universally recognized materials with high optical nonlinearities which can be easily controlled by externally applied electric and magnetic fields [17]. In the presence of magnetic field, the coupling of collective cyclotron excitations with LO phonons, via the macroscopic longitudinal electric field gives rise to modified normal modes. Under Voigt configuration frequencies of modified normal modes at the center of Brillouin zone (zero wave vector mode) is given by [18]

\[ \omega^2_{\epsilon n} = \frac{1}{2} \left( \omega^2_p + \omega^2_{LO} + \omega^2_{c LO} \pm \left( \left( \omega^2_p + \omega^2_e + \omega^2_{LO} \right)^2 - 4 \omega_p \omega_e \omega_{c LO} \right)^{1/2} \right) \]  

where \( \omega^2_{\epsilon n} \) is the modified normal mode frequency corresponding to the magnetoplasma excitations. It is clear that presence of magnetoplasma excitations modifies the coupling of electron-LO phonon quiet distinctly.

Hence, motivated by the present state of art, a study on the effect of magnetoplasma excitations in compound semiconductor medium via Fröhlich interaction has been carried out. Study of interdependence of coupled phonon-plasmon modes and magnetoplasma excitations during Fröhlich interaction in the presence of an intense laser beam will be the main focus of our studies. Nonlinear response of this medium will result in coherent collective mode in the medium as a result of coupling between pump wave and these optically excited coherent collective mode (such as acoustical and optical phonon modes, polaron mode). This three wave mixing process could be considered as parametric amplification process. We derive analytical expressions for the nonlinear polarisation and second order susceptibility arising from the nonlinear induced current density. Data of III-V direct band gap n-GaAs semiconductor subjected to pulsed CO\textsubscript{2} laser is used for the numerical estimations.

II. Theoretical formulation

A theoretical depiction for evaluating the expression for second order susceptibility, threshold pump field and absorption coefficient has been developed. The model used in the analysis is the well-known hydrodynamic model of the homogeneous one component (viz., n-type) semiconducting-plasma. We consider the propagation of electric pump wave \( \tilde{E}_p = \tilde{x}E_{0p} \exp(-i\omega_p t) \) applied along x-direction in homogeneous semiconductor medium immersed in an external magnetostatic field \( \tilde{B}_0 \) along the y-axis. This considered field geometry is known as Voigt geometry [19]. We also assume that the energy transfer between the pump, polaron and signal waves satisfy phase matching conditions \( \omega_k = \omega_0 - \omega_{pl} \) and \( k = k_0 - k_{pl} \) under spatially uniform pump field \( \left| \left| k \right| \right| = 0 \) hence \( k \parallel k_{pl} = k \) (say).

The optical phonons are active only in IR region [20], thus we consider here an infrared pulsed laser with pulse duration much larger than the acoustic damping time, thus one may very safely treat the interaction as steady-state type.

2.1. Effective second-order polarization

The basic equations describing parametric interactions of a pump with the medium are as follows:

\[ \frac{\partial n_0}{\partial t} + n_0 \frac{\partial \delta_e}{\partial x} = 0 \]  

\[ \frac{\partial \mu}{\partial x} = -\frac{n_e}{\varepsilon_0} + \left( \frac{N q}{\varepsilon_0} \right) \frac{\partial R}{\partial x} \]  

(3) is continuity equation representing the conservation of charge in which \( n_0 \) and \( n_e \) are the equilibrium and perturbed electron densities, respectively. \( \delta_e \) and \( \delta_t \) are the oscillatory fluid velocities of electrons of effective
mass $m_e$, $E_p$ is effective polaron electrostatic field arising due to the induced electronic and lattice polarizations and may be determined from Poisson's equation (4) in which $\varepsilon_0$ is the permittivity of free space.

The induced depolarizing field strongly couples the longitudinal and transverse degrees of freedom of the medium and shifts the natural frequency away from the cyclotron frequency and hence induces the collective cyclotron excitation with the resonance frequency $\omega_{c^*} = \omega_c^2 + \omega_1^2$, where $\omega_c = (-eB/m_e)$ is the electron cyclotron frequency, $\omega_p = (n_ec^2/m_ec)$ is the electron plasma frequency and $\varepsilon = (\varepsilon_0\varepsilon_c)$ is the dielectric constant in which, $\varepsilon_c$ is the high frequency dielectric constant of the medium.

We proceed with the following equations of motion for electron and polaron mode under one-dimensional configuration (along x-axis) for n-type moderately doped semiconductor magnetoplasma

$$\frac{\partial^2 \vec{r}}{\partial t^2} + \left(\omega_p^2 + \omega_c^2\right)\vec{r} + 2\Gamma_e \frac{\partial \vec{r}}{\partial t} = -\frac{e}{m_e} \left(\vec{E}_0 + \frac{\partial \vec{E}}{\partial t} \times \vec{B}\right)$$ \hspace{1cm} (5)

$$\frac{\partial^2 \vec{R}}{\partial t^2} + \left(\omega_p^2 + \omega_c^2\right)\vec{R} + 2\Gamma_p \frac{\partial \vec{R}}{\partial t} = \frac{q}{M_{pl}} \vec{E}_0$$ \hspace{1cm} (6)

$\vec{E}_0$ and $\vec{B}$ are the pump electric fields and applied magnetostatic field respectively. $\Gamma_e$ represents electron-electron collision frequency and $\Gamma_p = \Gamma_e + \Gamma_{ph}$ in which $\Gamma_{ph} (= 10^{-2} \omega_p)$ is the optical phonon decay constant. $q$ is the effective charge given by

$$q = \omega_p \left[\frac{M}{N}\varepsilon_0\varepsilon_c \left(\varepsilon_c^{-1} - \varepsilon_e^{-1}\right)\right]^\frac{1}{2}$$ \hspace{1cm} (7)

where $M$ and $N (= \alpha^{-3})$ are the reduced mass of the diatomic molecule and number of unit cells per unit volume, respectively, and $\alpha$ is the lattice constant of the crystal.

$M_{pl}$ depicts the mass of polaron. If electron-phonon coupling in a solid is strong enough to form polarons and/or bipolarons, one will expect a substantial isotope effect on effective mass of carriers. Moving electron drags the lattice distortion with it, by creating a larger inertia, results in slight increment in polaron mass. For weak coupling limit the polarization can be considered as a small perturbation [21] and quantum mechanical perturbation theory yields polaron mass [22] as

$$M_{pl} \approx m_e \left(1 + \frac{\alpha}{6}\right), \hspace{1cm} \text{(For } \alpha << 1 \text{)}$$ \hspace{1cm} (8)

here $\alpha$ is Fröhlich coupling constant which is always positive. Half of $\alpha$ gives the average number of ‘virtual phonons’ carried along by electron [23].

The components of oscillatory electron fluid velocity in presence of pump and magnetic fields are obtained from (5) as

$$v_{ox} = -\frac{\vec{E}}{2\Gamma_e - i\omega_p} \hspace{1cm} \text{and} \hspace{1cm} v_{oy} = \frac{\omega_p \vec{E}}{\omega_0} \frac{\omega_0^2}{\omega_0^2 - \omega_e^2}$$

where, $\vec{E} = \left(-\frac{e}{m}, -\frac{\omega_0^2}{\omega_0^2 - \omega_e^2}\right) \vec{E}_0$

Following the procedure adopted by Neogi and Ghosh [24] and using (3), (5) and (6), we obtain,

$$\frac{\partial^2 n_1}{\partial t^2} + 2\Gamma_e \frac{\partial n_1}{\partial t} + \frac{\omega_p^2}{\omega_p^2 - \omega_e^2} n_1 A_1 Z - \omega_p^2 \omega_{p,pl} n_0 A_1 \frac{\partial R}{\partial x} = -2\Gamma_{ph} T = -i\kappa n_1 A_2 E$$ \hspace{1cm} (9)

where,

$$\frac{\omega_p^2}{\omega_p^2 - \omega_e^2} Z = \frac{q m}{e M_{pl} \varepsilon_0} \hspace{1cm} \omega_{p,pl} = \frac{N q^2}{M_{pl} \varepsilon_0} \hspace{1cm} T = n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x}$$

$$A_1 = \frac{\omega_0^2}{\omega_0^2 - \omega_p^2 - \omega_e^2} \hspace{1cm} \text{&} \hspace{1cm} A_2 = \frac{\omega_0^2}{\omega_0^2 - \omega_p^2 - \omega_e^2}$$

Now on applying rotating wave approximation (RWA), (9) yields two coupled equations in terms of density perturbations. After some mathematical simplifications, we get slow component of density perturbation as
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\[ n_s = i k n_0 \left( \frac{q}{M_{pl}} \right) Q G E_{pl} \omega_{p,pl}^2 \]  

(10)

where, 
\[ Q = \frac{A_1}{-\alpha_e - \omega_e^2 - 2i \omega_e \Gamma_{pl}} \]  
\[ G = \left[ \frac{\delta_2^2 - 2i \omega_e \Gamma_{pl}}{\delta_1^2 + 2i \Gamma_{pl} \omega_e} \right] \]  
\[ \delta_1 = \left( \frac{A_1}{-\alpha_e} - \omega_e^2 \right)^{1/2} \]  
\[ \delta_2 = \left( \frac{A_1}{-\alpha_e} - \omega_e^2 \right)^{1/2} \]  

Here the effect of transition dipole moment has been neglected with an aim to concentrate only to the contribution of nonlinear current density on the induced polarization of the medium. The induced nonlinear current density is given by

\[ J(\omega) = -e v_i n_s^* \]  

(11)

where \( * \) represents the complex conjugate of the quantity. Substitutions of corresponding expressions in (11) yields,

\[ J(\omega) = -\frac{q k e_0}{M_{pl}} \left( \omega_0 \omega_{p,pl}^2 \omega_{p,pl}^2 \right) Q^* G^* E_0 E_{pl}^* \]  

(12)

2.2 Threshold pump amplitude

The time integral of (11) yields the expression for nonlinear induced polarization due to the perturbed carrier density as

\[ P_{nl}(\omega) = \int J(\omega) dt \]  

(13)

By using (12) in the above equation, the expression of polarization is obtained as

\[ P_{nl}(\omega) = -\frac{q k e_0}{M_{pl} \omega_0} \left( \omega_0 \omega_{p,pl}^2 \omega_{p,pl}^2 \right) Q^* G^* E_0 E_{pl}^* \]  

(14)

To incite parametric interactions in the medium, the pump amplitude should exceed certain threshold value \( E_{0,th} \) necessary to supply minimum required energy to the medium. This may be obtained from (14) by setting \( P_{nl} \) equals to zero as

\[ E_{0,th} = \left| \frac{m_0 \left( \omega_0^2 - \omega_e^2 \right) \delta_1 \delta_2}{e k A_2 \alpha_0^3} \right| \]  

(15)

The above equation reveals that \( E_{0,th} \) is strongly influenced by external parameters: wave vector (k), magnetic field \( (\omega_0) \), and carrier concentration \( (\omega_e) \). It is evident that threshold pump field is inversely proportional to wave vector k therefore lower threshold could be achieved for higher values of wave vector.

2.3 Second order optical susceptibility and Absorption coefficient of the magnetoplasma medium

Nonlinear induced polarization is given by

\[ P_{nl}(\omega) = \epsilon_0 \chi_{eff}^{(2)} E_0 E_{pl}^* \]  

On comparing with (14), we get effective nonlinear susceptibility as

\[ \chi_{eff}^{(2)} = -\frac{q k}{M_{pl} \omega_0} \left( \omega_0 \omega_{p,pl}^2 \omega_{p,pl}^2 \right) Q^* G^* \]  

(16)

The effective nonlinear susceptibility characterizes the steady state optical response of the medium and governs the nonlinear wave propagation through the medium in presence of transverse magnetostatic field.

In order to investigate the amplification characteristics in a doped semiconductor, we employ the relation

\[ \alpha_{eff} = \frac{k}{2 \epsilon_0} \text{Im} \left( \chi_{eff}^{(2)} \right) |E_0| \]

Substituting value from (16), the expression of absorption coefficient becomes,
\[ \alpha_{\text{eff}} = \frac{k_0}{2\varepsilon_0} \left( -i\omega \right) \left( \frac{\omega_0^2 - \omega_{\text{p,pl}}^2}{\omega_0^2 - \omega_i^2} \right) Q' G' |E_0| \]  

(17)

The nonlinear parametric gain of signal as well as idler waves can be possible only if absorption coefficient \( \alpha_{\text{eff}} \) is negative.

### III. Results and discussion

In order to establish the validity of present model, we have chosen a weakly-polar narrow band-gap semiconductor crystal (n-GaAs) at 77 K as the medium; it is assumed to be irradiated by a 10.6 µm pulsed CO₂ laser. The physical parameters chosen are: \( m_e = 0.601 \times 10^{-33} \text{kg} \), \( \varepsilon_{\text{re}} = 12.9 \), \( \alpha_b = 1.78 \times 10^{14} \text{s}^{-1} \), \( \omega_b = 5.1 \times 10^{13} \text{s}^{-1} \), \( \omega_0 = 5.548 \times 10^{13} \text{s}^{-1} \) and \( \alpha = 0.068 \).

Utilising these magnitudes, the numerical estimations depicting threshold and amplification characteristics are plotted in figures 1-4. Fig. 1 depicts the dependence of threshold pump field on carrier concentration. Nature of variation of threshold field for both the cases (in absence and in presence of magnetic field) is identical. It is observed that in both the cases threshold pump field decreases on increasing carrier concentration upto \( n_0 \approx 1.5 \times 10^{23} \text{m}^{-3} \) (in presence of magnetic field) and \( n_0 = 3.1 \times 10^{23} \text{m}^{-3} \) (in absence of magnetic field) and attains minima due to resonance between plasma frequency and modified normal and normal mode frequencies respectively. Beyond this value, plasma frequency becomes greater than modified normal mode frequency, responsible for enhancement in the value of threshold pump field. Further increment in carrier concentration reduces threshold pump field. Presence of magnetic field reduces the magnitude of pump field required to incite parametric interactions.

**Figure 1.** Variation of threshold pump field with carrier concentration at \( k = 2 \times 10^9 \text{m}^{-1} \)

**Figure 2.** Variation of threshold pump field with magnetic field at \( k = 2 \times 10^9 \text{m}^{-1} \)

Fig. 2 illustrates variation of threshold pump field with external magnetic field. It can be inferred from the Fig. that higher magnetic field reduces the pump amplitude required to incite parametric processes. Thus, one may significantly reduce the threshold pump field by increasing the external magnetic field even at different doping concentrations in the semiconductor.

**Figure 3.** Variation of absorption coefficient with pump field at \( k = 2 \times 10^9 \text{m}^{-1}, n_0 = 1.5 \times 10^{23} \text{m}^{-3} \)

\[ E_0 = 1.3 \times 10^7 \text{Vm}^{-1} \]

**Figure 4.** Variation of absorption coefficient with carrier density at \( k = 2 \times 10^9 \text{m}^{-1} \) and \( B_0 = 11 \text{ tesla} \)
Variation of absorption coefficient $\alpha_{eff}$ with pump field is shown in Fig 3. Intense pump is the prime requirement for the significant gain, this fact is again established. Highest gain is achieved at pump field $E_0 \approx 1.27 \times 10^7 Vm^{-1}$, beyond this magnitude gain decreases which may be attributed to intensity dependent refractive index of the medium.

In Fig. 4 absorption coefficient $\alpha_{eff}$ is plotted as a function of doping concentration. It is found that the nature of variation is almost similar in both the cases but presence of magnetic field enhances magnitude of gain by a factor of 10. Initially $\alpha_{eff}$ is negative exhibiting parametric gain, which increases with increasing doping concentration till a particular carrier density $(n_0 = 1.5 \times 10^{23} m^{-3})$ at which maximum gain is achieved. On elimination of magnetic field higher doping concentration $(n_0 = 3.1 \times 10^{23} m^{-3})$ is needed for the same. The maxima are due to resonance between plasma frequency and coupled plasmon-phonon mode frequency. Beyond this doping level a sharp increment shows absorption due to sign reversal of $\alpha_{eff}$. So we realize that presence of magnetic field leads higher gain at lower doping level.

First cross over point signifies a particular carrier concentration $(n_0 \approx 1.5 \times 10^{23} m^{-3})$, upto which the presence of magnetic field leads to gain whereas in the absence of magnetic field gain begins at this concentration. Therefore $n_0 = 1.5 \times 10^{23} m^{-3}$ is found to be favourable for the efficient coupling between phonon-plasmon modes in the absence of magnetoplasma excitations. On the other hand coupling between plasma frequency and modified normal mode frequency resulting into amplification initiates at quite lower doping and this continues till $(n_0 = 1.5 \times 10^{23} m^{-3})$. Favourable doping concentrations reported in this figure are the same which are already interpreted in Fig. 1 for both the cases. Beyond this concentration an abrupt decrease of gain is observed. Such behaviour can be utilised in the construction of optical switches.

IV. Conclusions

Present work deals with the analytical investigations of parametric amplification in weakly polar narrow band-gap magnetized semiconductors duly shined by a pulsed 10.6 μm CO$_2$ laser. The role of cyclotron excitations and coupled plasmon phonon modes has been examined thoroughly. Detailed analysis enables one to draw the following conclusions:-

(1) It is found that the correction to the parametric amplification/absorption coefficient due to electron-phonon interaction and magnetoplasma excitations depends strongly on electron density.
(2) Presence of external magnetic field modifies the carrier dynamics effectively. Higher magnetic field is favourable for the onset of parametric interactions and also for significant amplification coefficient.
(3) Coupled collective magnetoplasma excitations tend to enhance parametric gain and lower threshold pump intensity. At higher magnetic fields, these favourable parameters shift towards lower doping profiles.
(4) Present study could be utilised for designing optical switch and for the fabrication of cost effective optical parametric amplifier based on Fröhlich interaction in polar semiconductors.

References


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