The Use of Autoregressive Moving Average and Artificial Neural Network as Short Term Wind Speed Forecasting Tools for Lagos, Nigeria

Joseph Aidan\(^1\) and Issa Abdourahamane Ide\(^2\)
\(^1\)Department of Physics, Modibbo Adama University of Technology, P. M. B. 2076, Yola, Adamawa State, Nigeria.
\(^2\)Department of Physics, Faculty of Sciences, Gombe State University, Gombe, Gombe State, Nigeria.

Abstract: Autoregressive moving average (ARMA) and Artificial neural network (ANN) models were applied to a univariate response series of hourly wind speed data to forecast future wind speeds. After model identification, parameter estimation and diagnostic checking; and according to Bayesian Information Criterion (BIC), ARMA (2, 2)(1, 1) model was found suitable. The ANN design and training sampling revealed that a 5, 5, 1 neuron structure with sigmoid transfer functions at both the hidden and the output layers was the most suitable for the wind speed forecast for Lagos. The models were compared and evaluated on the basis of their performance for 1 to 6 hours ahead forecast. The maximum values of MAE, RMSE and MRE were found to be 0.098, 0.131 and 0.100 respectively for the ANN model; and 0.318, 0.403 and 0.733 respectively for the ARMA model. Hence ANN model has proven a better forecasting model for Lagos than the ARMA model.

Keywords: Wind speed forecast, Autoregressive moving average, Artificial neural network.

I. Introduction

Owing to wind variability, forecasting has become a necessary tool to locate sites for optimum wind turbine performance. To this end, several forecasting methods have evolved over the years among them are the Auto Regressive Integrated Moving Average (ARIMA) and the Artificial Neural Networks (ANN). These forecasting models have been used severally by authors \[1, 2, 3, 4\] to forecast short and long term wind speeds for different locations. In Nigeria, these methods have not been made popular. Only one or few others \[5\] have used them for wind speed forecasting. What is lacking is a short term forecast. Short term (1 to 6 hours ahead) forecasting is a veritable tool that can be used to determine the reliability of candidate sites for wind power development. Sometimes, wind speed forecast do help power system operators to dispatch more economically.

However, some of the attempts for short term forecast though very few, lack detailed and proper interpretations of processes hence difficult to be understood and applied. In this work, ARMA and ANN models for short term hours ahead wind speed forecast have been developed and applied to the wind speed data of Lagos. Simple and more detailed interpretations of the processes involved have been made and results from the models compared using some goodness of fit tests.

II. Data And Data Analysis

2.1 Data source and processing

A six months (184 days) secondary wind speed data, recorded at 5 minute intervals at UNILAG CR1000 weather station in Lagos State has been used in this study. The data which was converted to hourly steps using Microsoft Excel were divided into two sets: 179 day for training (model fitting) and 5 day for validation.

Figures 1 (a & b) Shows the time series plots of the observed and quartic root wind speeds of Lagos for the first ten days. The wind speed series plot (Figure 1a) exhibited seasonal pattern consisting of one major peak and several minor peaks daily. The major peaks appeared to be bigger or smaller over the days, indicating the non constancy of the seasonal variation. Data preprocessing has been done to stabilize the seasonal variation and the quartic root of the wind speed plot (Figure 1 b) has indicated just that and has shown a more uniform seasonal variation with reduced difference in major peaks hence fitted for stationarity test.
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Figure 1(a & b): Time series plots of observed and quartic root wind speeds for the first ten days

The indicator plots (Figures 2 a & b) of the Quartic roots wind speed data (QRWSD) have shown how the autocorrelation function (ACF) dies out fairly and quickly both at the seasonal and non-seasonal levels and requiring no differencing (d=0, D=0) and the PACF shows a major spike at lag 1 and two minor spikes at lags 2 and 3 after which it cuts off. The ACF also gives a clear picture of the series seasonality and reveals the period length of 24 (i.e. \( s = 24 \)) as expected for hourly time series data.

Figure 2 (a & b): Sample plots of ACF and PACF of QRWSD

Observations of both the plots of ACF and the PACF of the QRWSD have shown series correlation characterized by alternate positive and negative signs and a damping tendency at higher lags which suggested a mixed ARMA models for its forecast. Hence considering the principle of parsimony, candidate ARMA models that may fit the QRWSD include: ARMA (1,1)(1,1)\(_{24}\), ARMA (2,1)(1,1)\(_{24}\), ARMA (3,1)(1,1)\(_{24}\), ARMA (2,2)(1,1)\(_{24}\), ARMA (2,3)(1,1)\(_{24}\), ARMA (1,0,2)(1,0,1)\(_{24}\), ARMA (1,3)(1,1)\(_{24}\) and ARMA (3,3)(1,1)\(_{24}\).

2.2 Model parameters and diagnostic checking

The SPSS 17.0 EXPERT MODELER has been used for the estimation of the model parameters and their corresponding significance levels (Table 1). The null hypothesis \( H_0 \) that the model parameter is not significantly different from zero is rejected for all parameters, except for ARMA (1,1)(1,1)\(_{24}\), ARMA (2,2)(1,1)\(_{24}\) and ARMA (2,3)(1,1)\(_{24}\) which have significance levels (Table 1) less than 0.05. Further checking of the overall adequacy of the three ARMA models based on the BIC, the Ljung-Box model statistics and its Significance [6]. ARMA (1,1)(1,1)\(_{24}\), ARMA (2,2)(1,1)\(_{24}\) and ARMA (2,3)(1,1)\(_{24}\) have shown significance greater than the minimum required value of 0.05 (Table 2). These then follows that the null hypothesis cannot be rejected in either of the three cases, meaning that the residuals are random and not significantly different from zero for each of ARMA (1,1)(1,1)\(_{24}\), ARMA (2,2)(1,1)\(_{24}\) and ARMA (2,3)(1,1)\(_{24}\).
However, ARMA (1,1)(1,1)’s significance value for this test is 0.087 (Table 2) which is too close to the lower limit of 0.05 to be considered suitable compared to ARMA (2,2)(1,1) and ARMA (2,3)(1,1) with 0.289 and 0.561 significance values respectively. Finally, the model with the best BIC value (Figure 2) has been selected for the QRWSD. Out of ARMA (2,2)(1,1) with a BIC of -5.823 and ARMA (2,3)(1,1) with a BIC of -5.822, the former was chosen.

Table 2: BIC and Ljung-Box Q (18) statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>BIC</th>
<th>LB QS</th>
<th>DF</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARMA (1,1)(1,1)</td>
<td>5.826</td>
<td>21.631</td>
<td>14</td>
<td>0.087</td>
</tr>
<tr>
<td>ARMA (2,2)(1,1)</td>
<td>5.823</td>
<td>14.191</td>
<td>12</td>
<td>0.289</td>
</tr>
<tr>
<td>ARMA (2,3)(1,1)</td>
<td>5.822</td>
<td>9.662</td>
<td>11</td>
<td>0.561</td>
</tr>
</tbody>
</table>

2.3 ARMA forecasting theory

The moving average (MA) model is a form of ARMA model in which the time series is regarded as a moving average (unevenly weighted) of a random shock series \(e_t\). The first-order moving average, or MA (1), model [7] is:

\[ y_t = e_t + \theta_1 e_{t-1} \quad (1) \]

where \(e_t, e_{t-1}\) are the residuals at times \(t\) and \(t-1\), and \(\theta_1\) is the first-order moving average coefficient. MA models of higher order than one include more lagged terms. For example, the second order moving average model, MA (2) is:

\[ y_t = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} \quad (2) \]

The letter q is used for the order of the moving average model. The second-order moving average model is MA (q) with q = 2. It can be seen that the autoregressive model includes lagged terms on the time series itself, and that the moving average model includes lagged terms on the noise or residuals. By including both types of lagged terms, we arrive at what is called autoregressive-moving-average, or ARMA, model. The order of the ARMA model is included in parentheses as ARMA (p,q), where p is the autoregressive order and q the moving average order. The simplest ARMA model is first-order autoregressive and first-order moving average, or ARMA (1,1):

\[ y_t + \varphi_1 y_{t-1} = e_t + \theta_1 e_{t-1} \quad (3) \]

A mixed \(p^\text{th}\)-order autoregressive process and \(q^\text{th}\)-order moving average process, ARMA (p,q), is formally [8] given by:

\[ Y_t = C + \sum_{i=1}^{p} \varphi_i Y_{t-i} + \sum_{i=1}^{q} \theta_i e_{t-i} \quad (4) \]

In Equation (4), \(Y_t\) is the time series to be described, C is an internal constant value of the process, \((\varphi_1, \varphi_2, ..., \varphi_p)\) are the AR process parameters, \((\theta_1, \theta_2, ..., \theta_q)\) are the MA process parameters. The order p defines the number of previous observations with which the future wind speed correlates, and the parameters \(\varphi_i, i = 1,..., p\) describe how strong the correlations are.

To estimate the parameters of the ARMA models, a least squares minimization is employed. This mathematical procedure finds the best-fitted curve to the set of values of the time series by minimizing the sum of the squares of the residuals – the differences between the values of the time series and the values reproduced by the model.

The model selected was used to forecast the QRWSD which was in turn retransformed back to wind speed data. The mathematical form of the ARMA (p,q)(P,Q)S model [9] is:

\[ \phi(B) \Phi(B^S) Y_t = \theta(B) \Theta(B^S) e_t \quad (5) \]

where \(Y_t\) stands for the series term, \(e_t\) is the noise at time \(t\) and \(S\) is the period length of the season, \(\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p\) is the non-seasonal autoregressive polynomial of order \(p, B\) is the...
backward shift operator (defined as \( B^p Y_t = Y_{t-p} \)).

\[ \Phi(B^p) = 1 - \Phi_1 B^p - \Phi_2 B^{2p} - \cdots - \Phi_p B^{pq} \]

\[ \theta(B) = 1 - \theta_1 B - \theta_2 B^{2} - \cdots - \theta_q B^{q} \]

\[ \Theta(B^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \cdots - \Theta_q B^{qs} \]

\[ \Theta_0 \]

\[ \Theta_1 \]

\[ \Theta_2 \]

Equation (5) can then be written for the ARMA (2,2)(1,1) model as:

\[ Y_t = (\Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} - \Theta_1 \epsilon_{t-1} - \Theta_2 \epsilon_{t-2}) \]

Equation (6) gives:

\[ Y_{t+k} = Y_{t-(k)} = B^{-k} Y_t \]

Substitution of the expression of \( Y_t \) from Equation (7) in Equation (8) yields:

\[ Y_{t+k} = \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \Theta_1 \epsilon_{t-1} + \Theta_2 \epsilon_{t-2} \]

Applying the backward shift operator \( B \),

\[ Y_{t+k} = \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \Theta_1 \epsilon_{t-1} + \Theta_2 \epsilon_{t-2} \]

Taking conditional expectations at time \( t \), as follows:

Expected value of \( Y_{t+k} \) at time \( t \), on condition on series \( Y_t \) is \( E(\epsilon_{t+k} | Y_t) = \delta_{t+k} \) if \( k > 0 \).

Expected value of \( Y_{t+k} \) at time \( t \), conditional on series \( Y_t \) is \( E(\epsilon_{t+k} | Y_t) = \epsilon_{t-q} \) if \( q > 0 \).

Forecasts for 1, 2, 3, 4, 5 and 6 hours ahead can be generated using Equation (10):

\[ \delta_{t+1} = \Phi_1 Y_{t} + \Phi_2 Y_{t-1} + \Theta_1 \epsilon_{t-2} + \Theta_2 \epsilon_{t-2} \]

\[ \delta_{t+2} = \Phi_1 Y_{t} + \Phi_2 Y_{t-1} + \Theta_1 \epsilon_{t-2} + \Theta_2 \epsilon_{t-2} \]

\[ \delta_{t+3} = \Phi_1 Y_{t} + \Phi_2 Y_{t-1} + \Theta_1 \epsilon_{t-2} + \Theta_2 \epsilon_{t-2} \]

\[ \delta_{t+4} = \Phi_1 Y_{t} + \Phi_2 Y_{t-1} + \Theta_1 \epsilon_{t-2} + \Theta_2 \epsilon_{t-2} \]

\[ \delta_{t+5} = \Phi_1 Y_{t} + \Phi_2 Y_{t-1} + \Theta_1 \epsilon_{t-2} + \Theta_2 \epsilon_{t-2} \]

\[ \delta_{t+6} = \Phi_1 Y_{t} + \Phi_2 Y_{t-1} + \Theta_1 \epsilon_{t-2} + \Theta_2 \epsilon_{t-2} \]

The k-hours ahead forecast, \( \delta_{t+k} \), for \( k > 6 \), is obtained as follows:

\[ \delta_{t+k} = \Phi_1 Y_{t+k-1} + \Phi_2 Y_{t+k-2} + \Theta_1 \epsilon_{t+k-2} + \Theta_2 \epsilon_{t+k-2} \]

2.3 ANN model forecasting methodology

Based on the ACF of the wind speed time series data obtained from the execution of the ARMA methodology and having shown that the current value of QRWSD, though less significantly, depends on the immediate four lag values in the series up to the fifth one (Figure 2a); therefore, the five most recent pass values of the wind speeds have sufficiently proven the need for a five input units at the input layer of the ANN model. The network selected based on least sum of squares errors (SSE) has the following description: One input layer, one hidden layer, each consisting of five units (excluding the bias), and one output layer containing one unit (Figure 3). The ANN was then used to forecast one to six hours ahead wind speed starting from the end of the training and testing sets. The parameters of the ANN model is given in Table 3.
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2.4 Performance evaluation

Model comparison which were based on the Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Relative Error (MRE) results of forecasts from time series of different orders of magnitude were evaluated using equations (13,14 and 15):

\[
MAE = \frac{1}{N} \sum_{i=1}^{N} |Y_i - Y_i^*| \quad \text{(13)}
\]

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} (Y_i - Y_i^*)^2}{N}} \quad \text{(14)}
\]

\[
MRE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{Y_i - Y_i^*}{Y_i} \right| \quad \text{(15)}
\]

where \(Y_i\) and \(Y_i^*\) are the observed and the forecast wind speed values respectively, at time \(t\).

III. Results And Discussion

In terms of the correlation of the forecast and the observed wind speeds (Figures 4), The ANN with \(R^2\) value of 0.972 is better than ARMA which has \(R^2\) value of 0.887.
The comparison of the trend line slope and intercept is also to the advantage of ANN with line slope and intercept of 0.8679 and 0.1091 respectively than to the ARMA model with 0.512 and 0.379 respectively for its slope and intercept. Table 4 gives the evaluation criteria: the Mean Absolute Error (MAE), the Root Mean Square Error (RMSE), and the Mean Relative Error (MRE) respectively, of ARMA (2,2)(1,1)24 and ANN Models, which were computed for one hour ahead through six hours ahead, using Equations (13), (14) and (15). From Table 4 ANN has lower MAE, RMSE and MRE than the ARMA model at the 1-6 hrs time horizons.

### Table 4: MAE, RMSE and MRE for ARMA and ANN Models

<table>
<thead>
<tr>
<th>Time Ahead (hr)</th>
<th>MAE</th>
<th>ARMA RMSE</th>
<th>MRE</th>
<th>MAE</th>
<th>ANN RMSE</th>
<th>MRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.220</td>
<td>0.220</td>
<td>0.733</td>
<td>0.030</td>
<td>0.030</td>
<td>0.100</td>
</tr>
<tr>
<td>2</td>
<td>0.145</td>
<td>0.163</td>
<td>0.418</td>
<td>0.020</td>
<td>0.022</td>
<td>0.057</td>
</tr>
<tr>
<td>3</td>
<td>0.147</td>
<td>0.159</td>
<td>0.328</td>
<td>0.047</td>
<td>0.061</td>
<td>0.071</td>
</tr>
<tr>
<td>4</td>
<td>0.205</td>
<td>0.235</td>
<td>0.311</td>
<td>0.043</td>
<td>0.055</td>
<td>0.058</td>
</tr>
<tr>
<td>5</td>
<td>0.318</td>
<td>0.403</td>
<td>0.324</td>
<td>0.084</td>
<td>0.122</td>
<td>0.071</td>
</tr>
<tr>
<td>6</td>
<td>0.310</td>
<td>0.384</td>
<td>0.295</td>
<td>0.098</td>
<td>0.131</td>
<td>0.075</td>
</tr>
</tbody>
</table>

The ARMA (2,0,2)(1,0,1)24 and the ANN designed in this study could either way be used as prediction models for Lagos not minding the fact that the Box-Jenkins form of ARMA model has generally been outperformed by ANN. The maximum values of MAE, RMSE and MRE are 0.098, 0.131 and 0.100 respectively for ANN and 0.318, 0.403 and 0.733 for ARMA respectively.

From the results, analysis and performance evaluation that preceded, ANN was easier model to design than the ARMA model. It is also the less complex and most accurate prediction tool of the model. It is therefore a better proposed model for short term wind speed forecasting in Lagos.

Mathematically, since the ANN has one input layer comprising of five units and according to its architectural diagram (Figure 3), wind speed values lag 2, lag 1, lag 3, lag 5 and lag 4 behind the current values were in unit 1, unit 2, unit 3, unit 4 and unit 5 respectively of the network.

The input combinations (or weighted inputs) C1,1, C1,2, C1,3, C1,4 and C1,5 are given by:

\[
C_{1,i} = w_{1,1}Y_{t-2} + w_{1,2}Y_{t-1} + w_{3,1}Y_{t-3} + w_{4,1}Y_{t-5} + w_{5,1}Y_{t-4} + b_{1,1} \\
C_{1,2} = w_{1,2}Y_{t-2} + w_{2,2}Y_{t-1} + w_{3,2}Y_{t-3} + w_{4,2}Y_{t-5} + w_{5,2}Y_{t-4} + b_{1,2} \\
C_{1,3} = w_{1,3}Y_{t-2} + w_{2,3}Y_{t-1} + w_{3,3}Y_{t-3} + w_{4,3}Y_{t-5} + w_{5,3}Y_{t-4} + b_{1,3} \\
C_{1,4} = w_{1,4}Y_{t-2} + w_{2,4}Y_{t-1} + w_{3,4}Y_{t-3} + w_{4,4}Y_{t-5} + w_{5,4}Y_{t-4} + b_{1,4} \\
C_{1,5} = w_{1,5}Y_{t-2} + w_{2,5}Y_{t-1} + w_{3,5}Y_{t-3} + w_{4,5}Y_{t-5} + w_{5,5}Y_{t-4} + b_{1,5}
\]

where \( w_{i,j} \) is synaptic weight linking the \( i^{th} \) input unit to the \( j^{th} \) hidden unit; \( i = 1,2,3,4,5 \) and \( j = 1,2,3,4,5 \).

\( Y_{t-4} \) has its usual meaning.

With sigmoid activation function at the hidden layer, the hidden layer unit contents are:

\[
H_{1,1} = \text{sig}(C_{1,1}) = \frac{1}{1 + \exp(-C_{1,1})} \\
H_{1,2} = \text{sig}(C_{1,2}) = \frac{1}{1 + \exp(-C_{1,2})} \\
H_{1,3} = \text{sig}(C_{1,3}) = \frac{1}{1 + \exp(-C_{1,3})} \\
H_{1,4} = \text{sig}(C_{1,4}) = \frac{1}{1 + \exp(-C_{1,4})}
\]
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\[ H_{1,5} = \text{sig}(C_{1,5}) = \frac{1}{1+\exp(-C_{1,5})} \]

With sigmoid activation function at the output layer, the final output is

\[ Y_t = \text{sig}(w_{1,0}H_{1,1} + w_{2,0}H_{1,2} + w_{3,0}H_{1,3} + w_{4,0}H_{1,4} + w_{5,0}H_{1,5} + b_0) \]

or

\[ Y_t = \text{sig}(C_0) = \frac{1}{1+\exp(-C_0)} \quad (18) \]

where \( C_0 = w_{1,0}H_{1,1} + w_{2,0}H_{1,2} + w_{3,0}H_{1,3} + w_{4,0}H_{1,4} + w_{5,0}H_{1,5} + b_0 \)

IV. Conclusion

It is possible to forecast 6 h ahead wind speed for Lagos with 0.8874 and 0.9727 coefficients of determination (R^2) values, using ARMA (2,2)(1,1) and ANN respectively. The ANN was found to be better and simpler to use of the models designed in this study. It has five input nodes (containing the most recent previous wind speed data), one hidden layer with five neurons and one output neuron (containing the wind speed to be forecast). The ANN is capable of forecasting the 6 h ahead wind speed with maximum mean absolute error of 0.098, a maximum root mean square error of 0.131 and a maximum mean relative error of 0.100. Hence ANN may be considered as a tentative tool for short term wind speed forecasting in Lagos.

References