

Elucidation of Time Symmetry Predicted by the Special Theory of Relativity

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Abstract: In the thought experiment in this paper, we considered inertial frames M and A moving at a constant velocity relative to each other. A light signal emitted from inertial frame M , when time of a clock in inertial frame M was 1(s), arrived at inertial frame A when time of a clock in inertial frame A was 2(s). Conversely, the light signal emitted from inertial frame A , when time of a clock in inertial frame A was 1(s), arrived at inertial frame M when time of a clock in inertial frame M was 2(s). These results show that symmetry exists between the two inertial frames. However, the logic of explaining the arrival time of the light signal differs between observers M and A . Einstein regarded all inertial frames as equivalent, but it is not the case that all inertial frames are equivalent.

Keywords: Special Theory of Relativity; Minkowski Diagram; Relativistically Stationary System; Velocity Vector.

I. Introduction

In the era of classical physics, as exemplified by Newtonian mechanics, it was thought that physical laws exist independently of the existence of human beings. The role of physics was to discover physical laws, and describe them in the language of mathematics. In classical physics, "observation" was the task of checking the value of physical quantities which have their own real existence independent of us, without disturbing the object. In classical physics, the value of a physical quantity with its own objective reality was equal to the measured value, and there was no need to distinguish between the two.

However, with the advent of quantum mechanics, physicists realized that the classical worldview does not carry over to the micro world. According to the uncertainty principle, micro particles are affected by observation, and it is not possible to accurately know their state prior to measurement. Therefore, quantum mechanics is not a theory which searches for physical laws present in the natural world. Quantum mechanics is a theory where order is found from data obtained through observation, and that is then mathematically systematized.

Now, what sort of theory is Einstein's special theory of relativity (STR)? In order to understand the essence of the STR, this paper examines the symmetry of time. Before that, it confirms the "principle of constancy of light speed" introduced by Einstein.

II. The "Principle of Constancy of Light Speed E" Introduced by Einstein

2.1 The "Principle of Constancy of Light Speed"

When Einstein developed the STR, he assumed the "principle of relativity" and the "principle of constancy of light speed." The latter includes the following two principles [1].

"Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c , whether the ray be emitted by a stationary or by a moving body."

"Let a ray of light start at the "A time" t_A from A towards B, let it at the "B time" t_B be reflected at B in the direction of A, and arrive again at A at the "A time" $t_{A'}$.

In agreement with experience we further assume the quantity

$$\frac{2AB}{t_{A'} - t_A} = c,$$

to be a universal constant — the velocity of light in empty space."

In this paper, we distinguish between the former principle as the "principle of constancy of light speed I" and the latter principle as the "principle of constancy of light speed II." The "principle of constancy of light speed I" asserts that the light speed in vacuum does not depend on the speed of the light source. The "principle of constancy of light speed II" asserts that the light speed calculated from the round-trip travel time is constant. Let there be a given stationary rigid rod of length L_0 as measured by a ruler which is stationary, and assume that the rod is placed along the positive direction of the stationary system's x -axis.

Assume that clocks A and B of the same type are set up at points A and B on the rear and front end of this rod.

Here clock A will be abbreviated as C_A , and clock B as C_B .

Suppose a ray of light is emitted in the direction of B from A at time t_A of C_A , reaches and is reflected at B at time t_B of C_B , and then returns to A at time $t_{A'}$ of C_A . Einstein determined that if the following relationships hold between these two times, then the two clocks represent the same time by definition [1].

$$t_B - t_A = t_{A'} - t_B. \tag{1}$$

$$\frac{1}{2}(t_A + t_{A'}) = t_B. \tag{2}$$

If the relationship in Eq. (1) does not hold for the times of C_A and C_B , then it is necessary to adjust the time of C_B so that the relationship in Eq. (1) holds. (Actually, either clock can be adjusted.) However, Einstein did not include actual adjustment time in his formulation of the problem.

Next, assume that the stationary rod has been accelerated, and has attained the constant velocity v . (see Fig. 1) Suppose a ray of light is emitted in the direction of B from A at the time t'_A of C_A , reaches and is reflected at B at time t'_B of C_B , and then returns to A at time $t'_{A'}$ of C_A (The ' mark on t' signifies a moving system.)

If the following relation holds between the times of the two clocks at this time, then the times of the two clocks are the same by definition.

$$t'_B - t'_A = t'_{A'} - t'_B. \tag{3}$$

$$\frac{1}{2}(t'_A + t'_{A'}) = t'_B. \tag{4}$$

Therefore, if a rod which was stationary begins moving at a constant velocity, then the time C_B must be adjusted again so that the relationship in Eq. (3) holds between the times C_A and C_B . Due to this operation, the light speed on the outward and return paths measured in the moving system of the rod is measured as c on both paths. Considered classically, an inertial frame in which light propagates isotropically is a stationary system, and an inertial frame in which light propagates anisotropically is a moving system.

2.2 The "Principle of Constancy of Light Speed E" Introduced by Einstein

If clock time is adjusted according to the requirements of Einstein, light propagates isotropically at the same speed in all inertial frames. Also, all inertial frames become stationary systems in the sense of the theory of relativity. In this paper, the principle introduced by Einstein is called the "principle of constancy of light speed E." (where "E" stands for Einstein.) That is,

Principle of constancy of light speed E: In all inertial frames, light speed of the outward path and return path is constant (c).

In another paper, the author has presented thought experiments enabling discrimination of two types of inertial frames. (see Appendix)

Therefore, in this paper, it should be permissible to carry out thought experiments using an inertial frame in which light propagates isotropically. (In this paper, an inertial frame in which light propagates isotropically will be defined as "Michelson's stationary system.")

III. Thought experiment

Rocket A is moving at a constant velocity of $3c/5$ in the x -axis direction of "Michelson's stationary system." (In the following, "Michelson's stationary system" may be indicated as S_M , and the coordinate system of rocket A as S'_A . The "M" in S_M is the M in "Michelson".)

There is an observer M at the origin O of the x -axis of S_M , and M has a stopwatch W. In addition, there is an observer A at the origin O'_A of the x'_A -axis of S'_A , and A has a stopwatch W_A . (In the following "stopwatch W" may be abbreviated as W, and "stopwatch W_A " as W_A .)

Now, when rocket A passes in front of observer M in S_M , observer M starts W, and observer A starts W_A .

According to the STR, an observer in S_M , finds the following relationship between the time t which elapses on W and the time t'_A which elapses on W_A .

$$t'_A = \frac{t}{\gamma} = t \left(1 - \frac{v^2}{c^2} \right)^{1/2}. \tag{5}$$

Here, when 1(s) is substituted for t ,

$$t'_A = \frac{4}{5} \text{ (s)}. \tag{6}$$

Here, this thought experiment is explained using Minkowski diagram. (see Fig. 2)

3.1 Minkowski diagram

The following explanation in this section is an excerpt from another paper [2].

Point O indicates both origins: $x = 0, t = 0$ and $x'_A = 0, t'_A = 0$. The point event M_0 of the point light source O and the point event A_0 of the point light source O'_A are at the origin O. (Here, the subscripts "0" of the point events M_0 and A_0 mean, respectively, $t = 0$ and $t'_A = 0$.)

The x -axis indicates the x -axis of the inertial frame S_M when $t = 0$. In addition, the x'_A -axis indicates the x'_A -axis of the inertial frame S'_A when $t'_A = 0$.

The ct -axis is the path for $x = 0$. Put another way, it is the world line of the origin of S_M . The ct'_A -axis is the world line of the origin of S'_A .

In addition, the straight line extending at a 45° angle from the origin O indicates the light signal emitted from the two light sources at the instant that O and O'_A pass by each other.

OE is the distance over which the light signal emitted from O propagates in the x -axis direction while 1(s) elapses on the stopwatch W in S_M .

OE' is the distance over which the light signal emitted from O'_A propagates in the x'_A -axis direction while 1(s) elapses on the stopwatch W_A in S'_A .

Oe is the value when an observer in S_M measures the distance OE', and Oe' is the value when the distance OE is measured by an observer in S'_A . However, Ee' is parallel to the ct -axis, and eE' is parallel to the ct'_A -axis.

Therefore, the relationship between OE, OE', Oe and Oe' is as follows.

$$\frac{Oe}{OE} = \frac{Oe'}{OE'} = \frac{1}{\gamma}, \quad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \quad (7)$$

Here, when the position of the point E is determined, it is possible to determine the positions of the points e', e and E' based on the relationship in Eq. (7).

Furthermore, if a point is plotted on the ct -axis at a distance equal to OE from O, that is the point event M_1 for O at $t = 1(s)$.

Also, if a point is plotted on the ct'_A -axis at a distance equal to OE' from O, that is the point event A_1 for O'_A at $t'_A = 1(s)$.

IV. Discussion

Now when W in S_M is at 1(s), a light signal is emitted from O to O'_A in S'_A . That light propagates isotropically with respect to O. Then it arrives at O'_A when W_A on rocket A is 2(s). (This light signal corresponds to the world line M_1A_2 .)

In the inverse case, when W_A on rocket A is 1(s), a light signal is emitted from O'_A to O. That light arrives at O when W of the stationary system is 2(s). (This light signal corresponds to the world line A_1M_2 .)

These results show that symmetry exists between the two inertial frames. The following elucidates the mechanism whereby this symmetry holds.

In the case of this paper, where S_M has been introduced, S_M is always the stationary system. Here, let us express the situation of the propagation of the light signals (M_1A_2 and A_1M_2) as follows.

$$t = 1 (s) \rightarrow t'_A = 2 (s), \quad (8a)$$

$$t = 2 (s) \leftarrow t'_A = 1 (s). \quad (8b)$$

In contrast, in the STR which regards the two inertial frames as equivalent, the expression for Eq. (8b) changes to the following:

$$t = 1 (s) \rightarrow t'_A = 2 (s), \quad (8a)$$

$$t_A = 1 (s) \rightarrow t' = 2 (s). \quad (9a)$$

In the case of Eqs. (8a) and (8b), S_M is the stationary system, and in the case of Eq. (9a), the coordinate system of rocket A becomes the stationary system. Now, let's continue the discussion further regarding Eqs. (8) and (9a).

4.1 Explanation 1

Light propagation M_1A_2 seen from observer in S_M . (explanation of this paper and the STR)

$$t = 1 (s) \rightarrow t'_A = 2 (s). \quad (8a)$$

First, from Eq. (6), the time t'_A of W_A is 0.8 (s) when the time t of W is 1(s).

Next, if x is taken to be the distance which O'_A moves while 1(s) elapses on W in S_M ,

$$x = vt = \frac{3}{5}c \times 1 = \frac{3}{5}c. \tag{10}$$

Now, a light signal is emitted from O to O'_A when t=1(s). If the time t required for that light signal to reach O'_A is measured with W in S_M, then the following equation holds.

$$ct = \frac{3}{5}c(1+t). \tag{11}$$

If this is used to find t,

$$t = 1.5 \text{ (s)}. \tag{12}$$

Equation (5) is used to find the time t'_A which elapses in S'_A while 1.5(s) passes in W. Here, if 1.5 is substituted for t and 0.6c for v in Eq. (5),

$$t'_A = 1.2 \text{ (s)}. \tag{13}$$

Therefore, the time when the light signal emitted from O (when the time of W was 1(s)) arrives at O'_A is found by totaling (6) and (13). That is,

$$t'_A = \frac{4}{5} + \frac{6}{5} = 2 \text{ (s)}. \tag{14}$$

The time t which has passed on W during this interval is:

$$t = 1 + \frac{3}{2} = 2.5 \text{ (s)}. \tag{15}$$

The following world line in the diagram corresponds to Eq. (14).

$$A_0A_2 = A_0A_{4/5} + A_{4/5}A_2. \tag{16}$$

Up to this point, the predictions of this paper and the STR agree.

Incidentally, the light signal emitted from rocket A when W_A on rocket A was 1(s) arrives at O when W is 2(s). This propagation situation A₁M₂ is interpreted as follows by observer M and observer A.

4.2 Explanation 2

Light propagation A₁M₂ seen from observer in S_M. (explanation of this paper)

$$t'_A = 1 \text{ (s)} \rightarrow t = 2 \text{ (s)}. \tag{8b}$$

The observer M in S_M predicts the time t which elapses on W when 1(s) passes on W_A as follows based on Eq. (5).

$$t = \gamma = \frac{5}{4} \text{ (s)}. \tag{17}$$

Now if x is taken to be the distance O'_A moves while 1.25(s) passes in the stationary system,

$$x = \frac{3}{5}c \times \frac{5}{4} = \frac{3}{4}c. \tag{18}$$

The observer in S_M applies the "principle of constancy of light speed I" to the propagation of the light signal emitted from O'_A.

According to this principle, the light speed does not depend on the velocity of rocket A. A light signal emitted from O'_A propagates toward O at the same speed as light emitted when O'_A was stationary.

Here, if the time required for the light signal to propagate over the distance 3c/4 is measured with W and taken to be t,

$$t = \frac{3c}{4} \div c = \frac{3}{4} \text{ (s)}. \tag{19}$$

If the time t required for light to propagate over the interval O'_AO is measured with W, the result is 0.75(s).

Therefore, the time when light emitted from O'_A (when the time of W_A was 1(s)) arrives at O can be found by totaling (17) and (19). That is,

$$t = \frac{5}{4} + \frac{3}{4} = 2 \text{ (s)}. \tag{20}$$

The following world line of diagram corresponds to this time.

$$M_0M_2 = M_0M_{5/4} + M_{5/4}M_2. \tag{21}$$

4.3 Explanation 3

Light propagation A₁M₂ seen from observer in S_A. (explanation of the STR)

$$t_A = 1 \text{ (s)} \rightarrow t' = 2 \text{ (s)}. \tag{9a}$$

According to the STR, the coordinate systems of S_M and rocket A are equivalent. In this case, observer A in

rocket A regards his own coordinate system as a stationary system (that is, $S'_A \rightarrow S_A$, or $S_M \rightarrow S'_M$). Therefore, observer A determines that $t' = 0.8$ (s) when $t_A = 1$ (s). Observer A applies the "principle of constancy of light speed E" to propagation of the light signal emitted from O_A . With this principle, light propagates isotropically from O_A .

Also, observer A predicts that the time which elapses on W in S'_M while the light signal propagates over $O_A O'$ will be 1.2(s). This yields:

$$t' = \frac{4}{5} + \frac{6}{5} = 2 \text{ (s)}. \tag{22}$$

The logic for deriving Eq. (22) is the same as the logic for deriving Eq. (14). The logic of explaining the arrival time of the light signal differs between observers M and A.

V. Conclusion

A light signal emitted from light source O when the time on W in S_M was 1(s) arrived at rocket A when the time on W_A was 2(s). Also, conversely, a light signal emitted from a light source on rocket A when W_A was 1(s) arrived at S_M when W was 2(s).

Propagation of these light signals was described as follows by observer M in S_M .

$$t = 1 \text{ (s)} \rightarrow t'_A = 2 \text{ (s)}, \tag{8a}$$

$$t = 2 \text{ (s)} \leftarrow t'_A = 1 \text{ (s)}. \tag{8b}$$

Observer M explained Eqs. (8a) and (8b) as follows.

$$t'_A = \frac{4}{5} + \frac{6}{5} = 2 \text{ (s)}. \tag{14}$$

$$t = \frac{5}{4} + \frac{3}{4} = 2 \text{ (s)}. \tag{20}$$

When explaining (20) observer M applied the "principle of constancy of light speed I" to propagation of light. In contrast, observer A in rocket A described this situation as follows.

$$t_A = 1 \text{ (s)} \rightarrow t' = 2 \text{ (s)}, \tag{9a}$$

$$t_A = 2 \text{ (s)} \leftarrow t' = 1 \text{ (s)}. \tag{9b}$$

Taking his own coordinate system to be a stationary system, observer A applied the STR to explain the time for a light signal to arrive from A to M. That is,

$$t' = \frac{4}{5} + \frac{6}{5} = 2 \text{ (s)}. \tag{22}$$

When making this explanation, observer A applied the "principle of constancy of light speed E" to the propagation of light.

Observer M explained (8a) with (14) and (8b) with (20). In contrast, observer A who applied the STR believed that (8b) was wrong and that (9a) was correct. Also, observer A explained (9a) as indicated in (22).

In contrast, in the STR which regards the two inertial frames as equivalent, the expression for Eq. (8b) changes to the following:

$$t = 1 \text{ (s)} \rightarrow t'_A = 2 \text{ (s)}, \tag{8a}$$

$$t_A = 1 \text{ (s)} \rightarrow t' = 2 \text{ (s)}. \tag{9a}$$

In the case of Eqs. (8a) and (8b), S_M is the stationary system, and in the case of Eq. (9a), the coordinate system of rocket A becomes the stationary system.

Judging from the observed time, symmetry exists between the two inertial frames. However, the observers in inertial frames M and A provided different explanations by applying different principles to the propagation of light from rocket A to S_M . In this paper, the description in (8b) is regarded as correct for the propagation in (8b) and (9a). Also, this paper concludes that the explanation (20) of observer M is correct, and the explanation (22) made by observer A applying the STR is a mistake. The author has already pointed out in another paper the violation of the STR.

Einstein regarded all inertial frames as equivalent, but there are cases where a velocity vector is attached to some inertial frame. Einstein overlooked this fact, and thus a discrepancy appeared in the values predicted by the two observers.

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Appendix: Time that is Actually Adjusted in Synchronization of the Two Clocks

First, times are set so that the relation in Eq. (1) holds for clock A (C_A) and clock B (C_B) at the two ends of a rod of length L_0 at rest on the x -axis in Michelson's stationary system S_M [3, 4]. That rod begins to move at constant velocity v relative to the stationary system. (see Fig. 3)

Now when the time required for the light signal emitted from point A at the rear of the rod to travel from point A to point B is measured with the clock in S_M , it is $(t_B - t_A)$ by the definition in Eq. (1). Also, if this time is measured with the clock in S' , it is expressed as $(t'_B - t'_A)$.

According to the STR, the rod seen from S_M contracts by $1/\gamma$ times in the direction of motion. Also, the observer in S_M applies the "principle of constancy of light speed I" to the propagation of light emitted from the moving system S' , and thus $(t_B - t_A)$ is given by the following equation.

$$t_B - t_A = \frac{L_0}{c - v} \left(1 - \frac{v^2}{c^2} \right)^{1/2} \quad (s). \quad (A.1)$$

Also, the time $(t'_A - t'_B)$ required for the light signal to return from point B to point A is given by the following equation.

$$t'_A - t'_B = \frac{L_0}{c + v} \left(1 - \frac{v^2}{c^2} \right)^{1/2} \quad (s). \quad (A.2)$$

However, the denominator on the right side of Eqs. (A.1) and (A.2) does not signify that the speed of light changes.

According to the STR, the relationship of $(t'_B - t'_A)$ and $(t_B - t_A)$ is:

$$(t'_B - t'_A) = (t_B - t_A) \left(1 - \frac{v^2}{c^2} \right)^{1/2}. \quad (A.3)$$

Here, if the right side of Eq. (A.1) is substituted for $(t_B - t_A)$ in Eq. (A.3),

$$t'_B - t'_A = \frac{L_0(c + v)}{c^2} \quad (s). \quad (A.4)$$

Similarly, if the time $(t'_A - t'_B)$ which passes on the clock in S' while the light signal returns from point B to point A is measured from S_M ,

$$t'_A - t'_B = \frac{L_0(c - v)}{c^2} \quad (s). \quad (A.5)$$

If we set $t'_A = 0$ to simplify the equation, t'_A becomes the time which passes in S' while the light signal makes a round trip between A and B. Thus, the observer in S' determines that the time of C_B when the light has arrived at B is $t'_A / 2$. This time can be found from Eqs. (A.4) and (A.5). That is,

$$\frac{1}{2} t'_A = \frac{1}{2} [(t'_B - t'_A) + (t'_A - t'_B)] \quad (A.6a)$$

$$= \frac{L_0}{c} \quad (s). \quad (A.6b)$$

If Eqs. (A.4) and (A.6b) are compared, $L_0(c + v) / c^2 > L_0 / c$ and thus to solve this problem it is necessary to delay the time of C_B . If this adjustment time is taken to be $\Delta t'$,

$$\Delta t' = (t'_B - t'_A) - \frac{1}{2} t'_A \quad (A.7a)$$

$$= \frac{L_0 v}{c^2} \quad (s). \quad (A.7b)$$

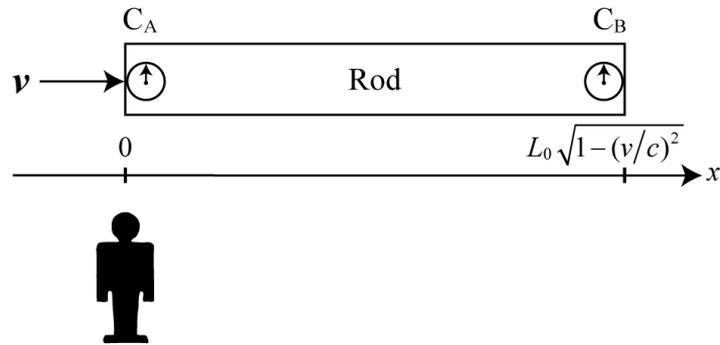
If the time of C_B is delayed by $L_0 v / c^2 (s)$, then a state is achieved where the times of C_A and C_B can be said to be simultaneous in S' .

Also, at the time $\Delta t' = L_0 v / c^2 (s)$, it can be determined that the coordinate system where the rod was initially stationary was S_M where light propagates isotropically. On the other hand, at the time $\Delta t' \neq L_0 v / c^2 (s)$, it can be determined that the coordinate system where the rod was initially stationary was the coordinate system where light propagates anisotropically.

References

- [1]. A. Einstein, *The Principle of Relativity* (Dover Publication, Inc. New York, 1923)
- [2]. K. Suto, *Applied Physics Research*, **8**, 6. 70 (2016)
- [3]. K. Suto, *Physics Essays*, **23**, 3. 511 (2010)
- [4]. K. Suto, *Physics Essays*, **28**, 3. 345 (2015)

Figures and captions



“Stationary system”

Fig. 1 A rod is moving at constant velocity v relative to stationary system. Clock A and B are set up at A and B at each end of this rod, and the times of each of these clocks are synchronized while the system is stationary.

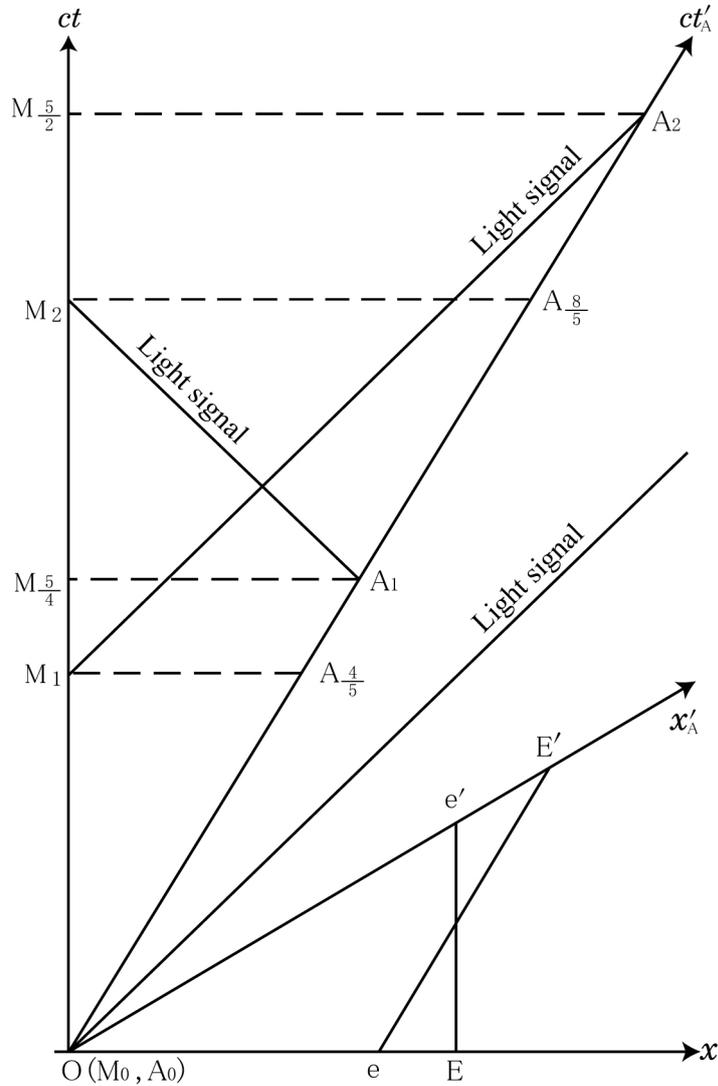
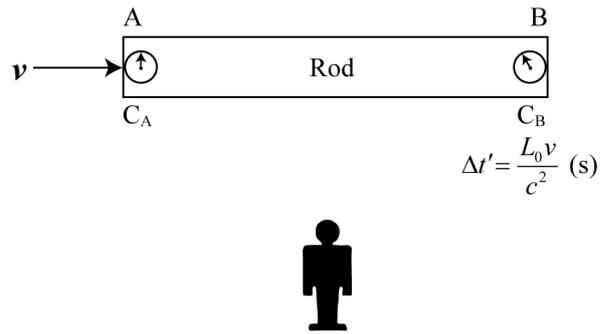


Fig. 2 Minkowski diagram: This diagram corresponds to thought experiment.



“Michelson’s stationary system”

Fig. 3 A rod is moving at the constant velocity v relative to the “Michelson’s stationary system.” In this case, if the time adjustment $\Delta t'$ performed with clock B of the rod is predicted by an observer in the stationary system, it will be $L_0 v / c^2$ (s).