Supersymmetric E₆ Models with Low Intermediate Scales

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Abstract: We propose supersymmetric E₆ models with intermediate Left-right symmetry as a result of spontaneous compactification of E₆ theory in a ten dimensional space. We show that much lower value of Left-right symmetry breaking scale and consistent unification scale can be achieved in presence of some appropriate light multiplets resulting from spontaneous compactification of higher dimensions at the Planck scale. In the model we could successfully lower the intermediate Left-right symmetry breaking scale M₉ up to 10⁷ GeV. With such a lower value of M₉, we can easily accommodate low scale leptogenesis in tune with gravitino constraint. The model can also predict desired value of neutrino mass that can be tested at LHC.

Keywords: Exceptional groups, Left-right symmetry, Renormalization group equation, Supersymmetry.

I. Introduction

Exceptional groups play a key role in particle physics i.e. E₃ = SU(3) ⊗ SU(2), E₄ = SU (5) , E₅ = SO(10). E₆ and E₇ are the known non-abelian gauge groups of unification models. Specifically the Grand-Unification Theories (GUT) based on an E₆ gauge group turn out to be very promising candidates for unification that have no gauge anomalies. E₆ [1] is the only exceptional Lie group that has complex representations and therefore the only exceptional group that can be used as a GUT in four dimensions. Further its Super symmetric (SUSY) version is inspired by E₆⊗E₇ string theory [2], which is the theory of everything. In the present paper, we consider super-symmetric E₆ model with two possible intermediate symmetries, with an attempt to correlate it with the well-known cosmological problem of matter-antimatter asymmetry through the possibility of Leptogenesis [3]. The discovery of neutrino masses makes leptogenesis a very attractive scenario for explaining the puzzle of the baryon asymmetry of the universe. In its simplest version, Leptogenesis is dominated by the CP violating interactions of the lightest heavy Majorana neutrinos, thus relate the observed baryon asymmetry in the Universe to the low-energy neutrino data. The original GUT-scale leptogenesis scenario, however, runs into certain difficulties within super symmetric models. In particular, potential problem arises from the over closure of the universe by the thermally produced gravitinos. To avoid overproduction of gravitinos, the reheat temperature of the Universe should be lower than 10⁹ – 10⁷ GeV. Since leptogenesis takes place just below the scale of Left-right symmetry breaking, one has to search for a model with low scale Left-right symmetry breaking. On the other hand, when the mass scale of the right-handed neutrinos is low, it has been shown that sufficient amount of baryon asymmetry in the universe can be generated through Resonant Leptogenesis [4]. In tune with the above requirements, in the present paper, we propose super symmetric E₆ models with two possible breaking chains, such that much lower value of Left-right symmetry breaking scale and a consistent unification scale can be achieved in presence of some additional light Higgs multiplets. We address the issues of neutrino mass, low-scale leptogenesis consistent with the gravitino constraint, manifest unification of gauge couplings through renormalizable interactions, which can be testable at the Tevatron, LHC or ILC. The paper is organized as follows. In the next section, we discuss the models along with the patterns of symmetry breaking.

Section–3 is devoted to obtain the mass scales at different stages through the Renormalization Group calculations including the one-loop beta function. We shall then conclude in the last section with a remark on the possibility of a light neutrino and leptogenesis.

II. The Model

In the present paper, we take an E₆ gauge theory coupled with N =1 SUSY in four dimension. This E₆ gauge model may be viewed as a remnant of supersymmetric E₆ group in a ten dimensional theory with compact six dimensional coset space ( G₂/SU(3) ). It has been shown in [5] that, as a result of Coset Space Dimensional Reduction one can obtain a E₆ model with Higgs {27+27+650} [6]. In the conventional superstring inspired E₆ models [2], the Higgs sector is confined to only 27 and 27. However here we shall take an additional Higgs belonging to {650} representation of E₆ to lower the left-right symmetry breaking scale. To allow low scale left-right symmetry breaking scale we take account of contribution to beta function from light chiral multiplets belonging to {650} representation of E₆. We shall now consider the symmetry breaking pattern from E₆ to low energy as given by the following two models.
Model -I
E_{6} \otimes \text{SUSY} \rightarrow \text{SU}(2)_{L} \otimes \text{SU}(2)_{R} \otimes \text{SU}(4)_{c} \otimes \text{U}(1)_{\psi} (G_{2241}) \otimes \text{SUSY}
M_{\psi} \rightarrow \text{SU}(2)_{L} \otimes \text{SU}(2)_{R} \otimes \text{SU}(3)_{c} \otimes \text{U}(1)_{B-L} \otimes \text{U}(1)_{\psi} (G_{2231}) \otimes \text{SUSY}
M_{R} \rightarrow \text{SU}(2)_{L} \otimes \text{SU}(3)_{c} \otimes \text{U}(1)_{Y} \otimes \text{U}(1)_{X} (G_{3241}) \otimes \text{SUSY}
M_{S}=M_{R} \rightarrow \text{SU}(2)_{L} \otimes \text{SU}(3)_{c} \otimes \text{U}(1)_{Y} (G_{2341})
M_{Z} \rightarrow \text{SU}(3)_{c} \otimes \text{U}(1)_{Q} (G_{31})
(1)

In the above breaking channel, the first step of symmetry breaking takes place at Planks scale. Here the
exceptional gauge group E_{6} is broken down to the Left – Right Pati-Salam group (G_{2241}) extended by an
additional U(1)_{\psi} by the vacuum expectation value (VeV) of (1,1,1)_{\psi} (G_{2211b}) contained in 54_{0} representation
of SO(10) \otimes U(1)_{\psi} \leq 650 of E_{6}. In the first step, the effect of 54_{0} containing G_{2241-singlets breaks SO(10)
keeping D-parity intact [6]. Then in the second step, the SU(4) symmetry is broken down to SU(3)_{c} \otimes U(1)_{b-L}
by the VeV of (1,1,1)_{\psi} contained in the Gut scale M_{U}. Here we may note that within the mass scale M_{U}
and M_{\psi} there is a surrogate SUSY GUT [7] based on (G_{2241}) \otimes \text{SUSY} which resolves the proton decay as well as
the doublet-triplet splitting problem of SUSY GUTs, with a Planck scale unification in a more natural way. This
is also expected, as E_{6} may be viewed as remnant of E_{6} superstring theory at the Planck scale. In the next step
the SU(2)_{L} \otimes U(1)_{Y} \otimes U(1)_{b-L} symmetry is broken down to U(1)_{Y} \otimes U(1)_{b-L} symmetry at the M_{R}
scale, by the VeV of (1,2,4)_{1/2} \otimes (1,2,4)_{1/2} of 16_{0} \otimes 16_{1/2} (SO(10) \otimes U(1)_{Y}) \leq 27 \otimes 27
representation of E_{6}. Then the U(1)_{Y} symmetry is spontaneously broken at the super symmetry scale M_{S}
which is assumed to occur at the TeV scale. This is achieved by the G_{2211} singlet S(1,1,1)_{\psi} contained in the {27}
representation. Finally the electro-weak symmetry breaking is achieved by the VeV of the bi-doublet (2,2,1,1) \leq 10 \leq 27, at M_{Z}.

Model-II:
E_{6} \otimes \text{SUSY} \rightarrow \text{SU}(2)_{L} \otimes \text{SU}(2)_{R} \otimes \text{SU}(3)_{C} \otimes \text{U}(1)_{B-L} \otimes \text{U}(1)_{\psi} (G_{2231}) \otimes \text{SUSY}
M_{R} \rightarrow \text{SU}(2)_{L} \otimes \text{SU}(3)_{C} \otimes \text{U}(1)_{Y} \otimes \text{U}(1)_{X} (G_{3241}) \otimes \text{SUSY}
M_{S}=M_{R} \rightarrow \text{SU}(3)_{C} \otimes \text{U}(1)_{Q} (G_{31})
(2)

Here the SUSY E_{6} symmetry is broken down to the Left–Right symmetry
SU(2)_{L} \otimes SU(2)_{R} \otimes SU(3)_{C} \otimes U(1)_{B-L} extended by an additional U(1)_{\psi}, i.e. (G_{2231}), by the vacuum expectation
value (VeV) of (1,1,1)_{\psi} (G_{2211b}) \leq 210_{0} \otimes (SO(10) \otimes U(1)_{\psi}) \leq 650, near the Planck scale M_{R}. In the next step, as in
the previous model, the SU(2)_{L} \otimes U(1)_{Y} \otimes U(1)_{b-L} symmetry is spontaneously broken to U(1)_{Y} \otimes U(1)_{b-L}
symmetry at the M_{R} scale. This U(1)_{Y} symmetry is broken spontaneously at a mass scale M_{Z} of the order of 10^{7} GeV, by
the G_{3241} singlet S(1,1,1)_{\psi} contained in the {27} representation. This may provide a heavy neutral Z-boson along
with the conventional Z-boson. Finally the electro-weak symmetry breaking occurs at (M_{Z}). For simplicity, we
consider that the Supersymmetry scale (M_{3}) lie at the electroweak symmetry breaking scale (M_{Z}).

III. Gauge Coupling Unification through Renormalization Group Analysis.
Now we discuss the Renormalization Group equations, including one-loop beta function contributions
to calculate the corresponding mass scales at different stages. In the minimal supersymmetric models, it has
been observed that, the inter mediate scale M_{R} is very close to the Grand Unification scale M_{U} \sim 10^{16} GeV at the
one-loop level, which is inconsistent with the accommodation of leptogenesis in the model. In the present case,
we show that the result can be better and consistent with leptogenesis, if some additional light multiplets are
taken into account. As has been mentioned before, to implement leptogenesis, the intermediate mass scale
(where the Left-Right symmetry breaks) must be much lower (M_{R} < 10^{6} GeV). Therefore to lower the M_{R} scale
consistently, we consider some appropriate additional light multiplets, belonging to \{650\} representation of E_{6}
which get their respective masses at the M_{R} scale. The effect of these multiplets will be visible through
renormalization group equation, which we shall discuss in detail for Model-I and Model-II respectively.

We shall now consider the Renormalization Group (R.G.) equations involved at different mass scales
involved in the symmetry breaking pattern of Model-I. Between the mass scales M_{Z} and M_{S} the R.G. equations run as,
\alpha_{i}^{-1}(M_{Z}) = \alpha_{i}^{-1}(M) + \frac{\ln(M_{Z})}{2\pi} \ln\left(\frac{M}{M_{Z}}\right)
\Rightarrow \alpha_{i}^{-1}(M) = \alpha_{i}^{-1}(M_{Z}) + \frac{\ln(10) \log_{10}(M)}{2\pi} \ln(M_{Z})
(3)
where, \( \alpha_i = (g_i^2/4\pi) \), \( g_i \) being the coupling constant for the corresponding gauge interaction. Here 'i' stands for SU(3)C, SU(2)L and U(1)Y gauge groups and \( b_i \) is the one loop beta function. The formula for non-supersymmetric beta function values involved in one-loop calculations for SU(N) group are given as:

\[
b_i = \frac{-11}{3} N + \frac{4}{3} N_g + \frac{1}{6} \sum T_i^2 .
\]

Here \( N_g \) is the number of generation, \( T_i \) is the contribution from Higgs. The electro-weak symmetry breaks by the VeV of the bi-doublet \( \phi(2,2,1) \) and also the Left-handed doublet \( H_L(2,1,4)_{1/2} \subset 16_{1/2} \subset 27 \). Therefore the beta function value for this stage are given as

\[
b_y b_{2L} b_{3C} = \frac{21}{5} - \frac{3}{5} - \frac{7}{5} \]

At the \( M_0 \) scale the MSSM particles get mass, therefore the formula for beta function values gets changed to,

\[
b_i^s = -3N + 2N_g + \sum T_i, \quad (\text{for } i=SU(3)_C, SU(2)_L, U(1)_Y)
\]

For the mass scale lying between Minimal Supper Symmetry breaking scale and the intermediate Left-Right Symmetry breaking Mass scale (\( M_0 < M < M_R \)), the minimal super symmetric standard model one-loop beta function coefficients are given as:

\[
b_y b_{2L} b_{3C} = \frac{33}{5} - \frac{1}{5} - 3 \]

The corresponding R.G. equations is given as,

\[
\alpha_{\alpha}^{-1}(M) = \alpha_{\alpha}^{-1}(M_0) - \frac{b_i^s}{2\pi} [ \ln(10) \log_{10}(M) - \ln(M_0) ]
\]

Then for the mass scale between the intermediate Left-Right symmetry breaking scale and GUT scale (\( M_R < M < M_{GUT} \)), the renormalization group equations run as:

\[
\alpha_{\alpha}^{-1}(M) = \alpha_{\alpha}^{-1}(M_R) - \frac{b_i^s}{2\pi} [ \ln(10) \log_{10}(M) - \ln(M_R) ],
\]

Here i = SU(3)_C, SU(2)_L,U(1)_Y, \( \text{and } U(1)_Y \).

Now we shall consider some additional light multiplets, which get their respective masses at the intermediate Left-Right Symmetry breaking mass scale (\( M_R \)) as,

\[\begin{align*}
\sigma(1,1,3)_{1/3,1} \oplus \bar{\theta}(1,1,\bar{3})_{1/3,\bar{1}} & \subset 10_0 \subset 27 \\
\eta(1,1,3)_{2/3,0} \oplus \bar{\eta}(1,1,\bar{3})_{2/3,\bar{1}} & \subset 45_0 \subset 650 \\
H_L(2,1,1)_{1/2,3} \oplus \bar{H}_R(1,2,1)_{1/2,3} & \subset 144_3 \oplus 144_{3} \subset 650 \text{ of } E_6
\end{align*}\]

So, all these extra light multiplets contribute to the beta function value for the mass scale greater than \( M_R \). Therefore the one loop beta function contributions between mass scales \( M_R \) and \( M_0 \) are given as,

\[
\left( \begin{array}{c}
b_0 \\
b_{2L=2R} \\
b_{3C}
\end{array} \right) = \left( \begin{array}{c}
18 \\
3 \\
0
\end{array} \right)
\]

Finally in between the Grand Unification Mass scale \( M_U \) and Plank scale (\( M_U < M < M_P \)), the R.G. equation runs as:

\[
\alpha_{\alpha}^{-1}(M) = \alpha_{\alpha}^{-1}(M_U) - \frac{b_i^s}{2\pi} [ \ln(10) \log_{10}(M) - \ln(M_U) ]
\]

(For \( i= SU(4)_C, SU(2)_L,U(1)_Y \)).
The beta function contributions in this stage are given as:

\[
\begin{pmatrix}
\frac{b_Y}{\Delta R}
\frac{b_2}{\Delta R}
\end{pmatrix} = \begin{pmatrix}
11.3
9
4
\end{pmatrix}
\] (13)

Using the R.G. equations (3), (8), (9) and (12), we can observe the running couplings of SU(4)C, SU(2)L and SU(2)R unify at a very high scale close to the Plank scale (M_P=10^{19} GeV). We assume the total unification of all the forces at that mass scale (M_R). Therefore the gauge coupling for U(1)Y runs down from that scale to the M_R scale (with the beta function value 11.3). Then at M_R the values of the gauge coupling constant for U(1)Y [8] is determined by the linear combination of values of coupling constant of groups U(1)Y, SU(4)C and SU(2)R. At the M_R scale, the SU(2)R\otimes U(1)Y \otimes U(1)_B is symmetry is broken to U(1)\otimes SU(1)_Y symmetry by the Vevs of (1, 2, 4)_{1/2}H_R \oplus (1, 2, 4)_{1/2}H_{BR}. The gauge bosons associated with diagonal SU(4) generator is \( T_4^\gamma \), the diagonal SU(2)_R generator \( T_2^\gamma \) and the U(1)Y generator \( T_\gamma \) are related by the Higgs bosons to create two light mass less gauge bosons associated with U(1)_Y and U(1)_\chi.

\[ U(1)_{B-L} \otimes U(1)\gamma \otimes U(1)\gamma \rightarrow U(1)\gamma \otimes U(1)\tau \] (14)

Therefore at M_R, we can write,
\[ \chi = T_\gamma + T_2^\gamma - C_{24}^2 \chi \] (15)

Where \( C_{12} = \cos \theta_{12} \), for \( \theta_{12} \) is the mixing angle such that, \( \tan \theta_{12} = \frac{\alpha_{2\gamma}}{g_{B-L}} \). Here Y and \( \chi \) are not normalized, such that the normalized charges are, Y \( \Rightarrow (Y/N_Y) \) and \( \chi \Rightarrow (\chi/N_\chi) \), where \( N_Y^2 = 3/5, N_\chi^2 = 7-2C_{24}^2 + \left(\frac{3}{5}\right)C_{12}^4 \). Thus at M_R we can write that,
\[ \alpha^{-1}_{B-L} = \frac{1}{2} \alpha_Y^{-1} - \frac{3}{2} \alpha_{2\gamma}^{-1} \] (16)

Now the value of the gauge coupling constant of U(1)_\chi at M_R scale, can be calculated from the following equation.
\[ \alpha^{-1}_\chi = \frac{1}{N_\chi^2} \left[ 6 \alpha_Y^{-1} + \frac{1}{\alpha_Y^{-1} + \alpha_{2\gamma}^{-1}} \right] \] (17)

Here, \( N_\chi^2 \) is the normalization constant and is related to the cosine of the mixing angle \( C_{12} \), as has been mentioned before. In between the mass scales M_R and M_S, the gauge coupling of U(1)_\chi runs down according to the renormalization group equation
\[ \alpha^{-1}_\chi (M_S) = \alpha^{-1}_\chi (M_R) + \frac{b_\chi}{2\pi} \int \ln (M_R) - \ln (M) \] (18)

Here \( b_\chi \) is the beta function value for U(1)_\chi, which also varies with the cosine of the mixing angle \( C_{12} \) as in the formula bellow
\[ b_\chi = 6 + \sum T_2^2 = 6 + \frac{1}{N_\chi^2} \sum \chi_i^2 \] (19)

Where \( \chi_i \) can be calculated by equation (15). The values of \( \chi_i^2, N_\chi \) and \( b_\chi \) for different M_R scale are noted in the Table-1.

### Table-1: Beta function value of U(1)_\chi corresponding to different M_R

<table>
<thead>
<tr>
<th>M_R(GeV)</th>
<th>C_{12}</th>
<th>( \sum \chi_i^2 )</th>
<th>N_\chi^2</th>
<th>b_\chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^5</td>
<td>0.2518</td>
<td>8.5598</td>
<td>6.602</td>
<td>7.296533</td>
</tr>
<tr>
<td>10^6</td>
<td>0.2619</td>
<td>8.4478</td>
<td>6.5905</td>
<td>7.296529</td>
</tr>
<tr>
<td>10^7</td>
<td>0.2731</td>
<td>8.5283</td>
<td>6.578</td>
<td>7.29648</td>
</tr>
<tr>
<td>10^8</td>
<td>0.2857</td>
<td>8.5101</td>
<td>6.5645</td>
<td>7.29637</td>
</tr>
</tbody>
</table>

Using the evolution equations (3), (8), (9), (12) and (18) along with the values of \( b_\chi \) as given in Table-1, we obtain the mass scales M_1 and M_P graphically for M_R in the range 10^5 - 10^8 GeV. Here we have used the input values of Standard Model coupling measured on the Z-pole at LEP as \( a_t(M_Z) = 0.016947, a_\chi(M_Z) = 0.033813 \) and \( a_{\tau}(M_Z) = 0.1187 \) to calculate the couplings. We observe that consistent M_1 and M_P are obtained even with low M_R as given in Figure 1-4.
Gauge coupling Unification for different \( M_R \).

Figure-1: \( M_R=10^5 \) GeV

Figure-2: \( M_R=10^6 \) GeV

Figure-3: \( M_R=10^7 \) GeV

Figure-4: \( M_R=10^8 \) GeV

From the graph, we have noted down the values of coupling constants and masses at the unification scale corresponding to the different values of the intermediate scales (\( M_R \)). The result is given in Table-2.

Table-2 : Mass scales \( M_U, M_P \) and \( \alpha_i^{-1} \) for different values of \( M_R \).

<table>
<thead>
<tr>
<th>( M_R ) in (GeV)</th>
<th>( M_U ) in (GeV)</th>
<th>( M_P ) in (GeV)</th>
<th>( \alpha_i^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^5 )</td>
<td>( 10^{18.229} )</td>
<td>( 10^{18.465} )</td>
<td>8.899</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>( 10^{18.226} )</td>
<td>( 10^{18.465} )</td>
<td>10.307</td>
</tr>
<tr>
<td>( 10^7 )</td>
<td>( 10^{18.226} )</td>
<td>( 10^{18.205} )</td>
<td>11.7</td>
</tr>
<tr>
<td>( 10^8 )</td>
<td>( 10^{18.225} )</td>
<td>( 10^{18.005} )</td>
<td>13.093</td>
</tr>
</tbody>
</table>

Similarly, we can achieve a low intermediate scale with Model-II (as given in eq. (2)) and a consistent Planck scale unification with an intermediate \( G_{22311} \) symmetry, when the contributions from some light multiplets are taken into account. Now we consider the R.G. equations at different mass scales as has been done in Model-I. Between the mass scales \( M_Z=S \) to \( M_\chi \) the RNGE are given as,

\[
\alpha_i^{-1}(M_Z) = \alpha_i^{-1}(M_\chi) + \frac{b_i}{2\pi} \left\{ \ln \left( \frac{M}{M_\chi} \right) \right\}, \quad i = 3c,2L,Y
\]  

(20)

Where the one loop beta function \( b_i \) includes the supersymmetric contributions as has been given in eq. (7). Then in the similar manner, as in the previous model, we can have Renormalization Group equations for subsequent stages of symmetry breaking. Between the mass scale \( M_\chi \) to \( M_R \) we follow the same evolution equations. Now we consider some appropriate additional light multiplets, belonging to \{650\} and \{27\} representation of \( E_6 \) which get their respective masses at the \( M_R \) scale in order to achieve low intermediate symmetry. We take the following light multiplets,
One copy of each

\[ \eta(1,1,6)_{23,0} (G_{22311}) \subset (1,1,20)_{G_{2241}} \subset (1,1,6)_{10} \subset (1,1,20)_{27} \]

\[ \phi(2,2,1)_{0,1} (G_{22311}) \subset (1,1,6)_{23} \subset (1,1,20)_{54} \subset (1,1,60)_{650} \]

All these extra light multiplets contribute to the beta function values. Therefore the one loop beta function contributions \( M_R \) to \( M_P \) are given as,

\[
\begin{pmatrix}
  b_{B-L} \\
  b_{3L=2R} \\
  b_{3C}
\end{pmatrix} = \begin{pmatrix}
  13.5 \\
  3 \\
  0
\end{pmatrix}
\]

Using the evolution equations we have done a graphical analysis in a similar manner as has been done in the Model-1. It is observed that the running couplings of \( SU(3)_C, SU(2)_L, SU(2)_R \) and \( U(1)_{B-L} \) get unified at a very high scale close to the Plank scale \( (M_P) \approx 10^{19} \) GeV, for an allowed value of \( M_R \) in the range \( 10^5 - 10^9 \) GeV.

Here the gauge coupling for \( U(1)_W \) run down from \( M_P \) to the \( M_R \) scale with beta function value 8.83. The corresponding graphs are given in Figure: 5-8.

**Gauge coupling Unification for different \( M_R \)**

**Figure-5: \( M_R = 10^5 \) GeV.**

**Figure-6: \( M_R = 10^6 \) GeV.**

**Figure-7: \( M_R = 10^7 \) GeV.**

**Figure-8: \( M_R = 10^8 \) GeV.**

From the graph, we have noted down the values of coupling constants and masses at the unification scale corresponding to the different values of the intermediate scales \( (M_R) \). The result is given in Table-3.

**Table-3 : Values of \( M_P \) and \( \alpha_{p}^{-1} \) for different \( M_R (M_R = M_S) \)**

<table>
<thead>
<tr>
<th>( M_R ) in GeV</th>
<th>( M_P ) in GeV</th>
<th>( \alpha_{p}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^5 )</td>
<td>( 10^{20.1} )</td>
<td>11.78</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>( 10^{19.76} )</td>
<td>12.9</td>
</tr>
<tr>
<td>( 10^7 )</td>
<td>( 10^{19.41} )</td>
<td>14.04</td>
</tr>
<tr>
<td>( 10^8 )</td>
<td>( 10^{19.1} )</td>
<td>15.09</td>
</tr>
<tr>
<td>( 10^9 )</td>
<td>( 10^{18.76} )</td>
<td>16.2</td>
</tr>
</tbody>
</table>
Here we note that, this model does not allow $M_R$ scale to lie below $10^5 \text{GeV}$, as the unification scale is shown to be high ($\sim 10^{20} \text{GeV}$). Thus it puts a lower bound on $M_R$ as compared to Model-I. We have also investigated for the case $M_Z \neq M_S$ and $M_S = M_\chi$ to lie at TeV scale, as has been done in Model-I. For the sake of completeness, we give the graph for evolution of couplings, for $M_R=10^5 \text{ GeV}$ ($M_Z \neq M_S$) in Figure-9. It is observed that, for $M_S$ at the TeV scale, the situation improves with consistent unification scale.

From the graph, we have noted down the values of coupling constants and masses at the unification scale corresponding to the different values of the intermediate scales ($M_R$). The result is given in below.

<table>
<thead>
<tr>
<th>$M_R$ in (GeV)</th>
<th>$M_P$ in (GeV)</th>
<th>$\propto (p-1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^5$</td>
<td>$10^{19.68}$</td>
<td>13.83</td>
</tr>
<tr>
<td>$10^6$</td>
<td>$10^{19.34}$</td>
<td>14.94</td>
</tr>
<tr>
<td>$10^7$</td>
<td>$10^{19.02}$</td>
<td>15.99</td>
</tr>
<tr>
<td>$10^8$</td>
<td>$10^{18.67}$</td>
<td>17.14</td>
</tr>
<tr>
<td>$10^9$</td>
<td>$10^{18.35}$</td>
<td>18.22</td>
</tr>
</tbody>
</table>

IV. Discussion

We have considered super symmetric $E_6$ models [10] with two possible intermediate scales, in which we can have a low Left-right symmetry breaking scale $M_R$ of the order of $10^5$-$10^8 \text{ GeV}$. In the given models, where we explore the possibility of low $M_R$, it is difficult to achieve consistent unification with minimal particle contents. This is achieved by introducing additional light multiplets belonging to {650} and {27} representation of $E_6$. Further the model can also predict a light left handed neutrino [11] through the double seesaw and type III seesaw mechanism, with the presence of the singlet $S (1,1,1)$. The mass can be obtained in the sub-eV range, even for low $M_R$ without fine-tuning of the Yukawa coupling. As far as leptogenesis is concerned, it allows resonant leptogenesis through the decay of the singlet fermions, which are the superposition of $S(1,1,1)$ and the right handed neutrino. These states lie below $10^8 \text{ GeV}$ constrained by the low $M_R$ to generate lepton asymmetry, consistent with the gravitino constraint. Thus the supersymmetric $E_6$ models with low intermediate scales can have nice phenomenological implication and can be testable in the future colliders.

References: