Run Or Walk In The Rain? (Orthogonal Projected Area of Ellipsoid)

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Abstract: Although there have been many articles about this problem, still many people want to know about the conclusion of it. That is because the previous articles were made with the too much complicated ways and there was no clear formulas for explaining well. In fact, this problem does not need a high level of mathematical knowledge. In this paper, it is used a simple method considering the various bodies types, the speed & angle of body and rain even the acceleration. And it is also considered in the cases; the body moves in a time as well as in the same distance. At least in theory, I expect this paper will show the final conclusion.

Keywords - Run or walk in the rain, orthogonal projected area of ellipsoid, cylinder and rectangular

I. Introduction

There have been many papers about the rain problem; Walk or Run? All the previous papers about this problem had been made by assumption that the object moved standing vertically in a given distance. The latest author [1] Bocci considered the slanted model but he only thought about the slated plane. Now we will consider the slanted bodies - rectangular, ellipse and cylinder. And then we will find the formulas and if we can, the optimal speeds too. And we will check how the best strategy in the rain can be different in the two cases; in a given distance and in a given time. In fact this problem is not limited only to the rain. For example, in the three-dimensional space, a space shuttle looked like an ellipsoid \((x^2/A^2+y^2/B^2+z^2/C^2=1)\), where \(A=52.07\,\text{m}, B=73.43\,\text{m}, C=254.31\,\text{m}\) travels through a straight line \((y\text{-axis})\). The space shuttle leans forward as much as 60.52° from \(y\)-axis to the direction of movement and its speed is 38.85 m/s. Very small rocks are evenly spread in the space and their speed is 57.85 m/s, their direction vector is \((-\cos40.33°, -\cos127.50°, -\cos77.30°)\) and the density is 7.7656/km³. If the space shuttle travels as much as 1053.90 km, how many do you expect the small rocks will hit the space shuttle during its trip? And if it travels for 2.16 days, how many? In the given angle of the space shuttle (60.52°), does an optimal speed exist? If it exists, find the optimal speed and the number of small rocks that will hit the space shuttle in two cases; when it travels in the given distance and in the given time.

II. Method

1. Orthogonal projection
1-1 Two-dimensional model

Figure 1 Rectangle

Figure 2 Ellipse

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The projected length to the line that is vertical to the vector \((a, b)\)

\[
L_r = \frac{A|b| + B|a|}{\sqrt{a^2 + b^2}} \quad \text{(Proof is omitted)} \quad (1)
\]

\[
L_e = \frac{2\sqrt{(Ab)^2 + (Ba)^2}}{\sqrt{a^2 + b^2}} \quad \text{(Proof is omitted)} \quad (2)
\]

1.2. Three-dimensional models

1-2-1. Rectangular

The projected area to the plane that is vertical to the vector \((a, b, c)\)

\[
S_1 = \frac{AB|c| + BC|a| + CA|b|}{\sqrt{a^2 + b^2 + c^2}} \quad \text{(Proof is omitted)} \quad (3)
\]

\[
1-2-2. \text{Ellipsoid}
\]

On the figure 4, the projected area is also ellipse.

\[
L_1 = \frac{\sqrt{(Ab)^2 + (Ba)^2}}{\sqrt{a^2 + b^2}} \quad K_1 = \frac{(a^2 + b^2)A^2B^2}{(Ab)^2 + (Ba)^2} \quad L_2 = \frac{2K_1C^2 + C^2(a^2 + b^2)}{\sqrt{a^2 + b^2 + c^2}} \quad S_e = \pi \quad L_1 \quad L_2 = \frac{\pi(ABc)^2 + (BCa)^2 + (CAb)^2}{\sqrt{a^2 + b^2 + c^2}} \quad (4)
\]

For making the formula (4), there can be another method, for example using matrix but I think the way in this paper is the easiest.
1-2-3 Cylinder

\[
S_1 = \frac{\pi AB |c|}{\sqrt{A^2 + B^2 + C^2}},
\]

\[
L_1 = 2\sqrt{(Ac)^2 + (Ba)^2}/\sqrt{a^2 + b^2} \quad L_2 = 2\sqrt{(Ba)^2 + (Ca)^2}/\sqrt{a^2 + b^2 + c^2}
\]

\[
S_c = S_1 + L_1 \quad L_2 = \frac{\pi AB |c| + 2\sqrt{(BCa)^2 + (CAB)^2}}{\sqrt{a^2 + b^2 + c^2}}
\]

Figure 7 Cylinder

2. Application

2-1 Terms and assumption

\(V_R\) = The speed of the rain, \(V_O\) = The speed of the object (\(V_R\) and \(V_O\) cannot be zero at the same time)

\(\theta_R\) = The angle between the horizon and the rain to the direction of movement \((0 \leq \theta_R \leq 180^\circ)\)

\(\theta_O\) = The angle between the horizon and the object to the direction of movement \((0 \leq \theta_O \leq 180^\circ)\)

\(\rho\) = The number of raindrops per unit area or volume \((1 \times 1\) or \(1 \times 1 \times 1)\)

\(Q_D\) = The swept area or volume in the rain field when the object moves in the distance \(D\)

\(Q_T\) = The swept area or volume in the rain field when the object moves in the time \(T\)

On the Figure 8, the outer side of the virtual object floating in the air represents the boundary of the last raindrops that will hit the object when it reaches the point \((1, 0)\). That is, the raindrops in the region between the object on the zero point and the floating one will hit the object. The position of the object floating in the air is determined by the ratio of the two speeds, \(V_R/V_O\) and the angle of the rain; \(\theta_R\).

The key is to find the area of the region. The area of the region is equal to the area of the rectangle that has the same width and height of the region. Because the object is slated, in order to simplify the problem we make the object stand vertically. In other words, we rotate the objects as much as \(90^\circ - \theta_O\). Using the length of orthogonal projection we found, formula (1), it can be found the area of the region.

The area of the rectangle:

\[
\text{Area} = Q_D = A \left[ \cos \theta_O + \frac{V_R}{V_O} \cos(\theta_O - \theta_R) \right] + B \left[ \sin \theta_O + \frac{V_R}{V_O} \sin(\theta_O - \theta_R) \right]
\]

2-2 Rain Formulas

In the rectangle model,

\[
\text{Area} = Q_D = A \left[ \cos \theta_O + \frac{V_R}{V_O} \cos(\theta_O - \theta_R) \right] + B \left[ \sin \theta_O + \frac{V_R}{V_O} \sin(\theta_O - \theta_R) \right]
\]

The total number of raindrops that will hit the object when it moves in the distance \(D\) is

\[
\text{Total}_D = \rho D \left[ A \left| \cos \theta_O + \frac{V_R}{V_O} \cos(\theta_O - \theta_R) \right| + B \left| \sin \theta_O + \frac{V_R}{V_O} \sin(\theta_O - \theta_R) \right| \right]
\]
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The total amount of rain that will hit the object is obtained by multiplying the volume of a raindrop to the TotalD.

How it will be if the object moves in a given time?
When it moves with the speed \( V_0 \) in the time 1, the distance of movement is \( V_0 \), so we can find \( QT \) as,

\[
QT = V_0 \left( A \left| \cos \theta_0 \right| + B \left| \sin \theta_0 \right| \right)
\]

By applying the same way, in the two-dimensional elliptic model,

\[
Q_D = \frac{2}{\pi} \left( A^2 + B^2 \right)
\]

Optimal speed \( V_0 \) =

\[
\frac{-V_R \cos \alpha + \cos \beta \cos \theta_0 + \cos \gamma \sin \theta_0 + \sin \theta_0 \cos \beta \cos \theta_0 + \cos \gamma \sin \theta_0}{A^2 \cos^2 \theta_0 + B^2 \sin^2 \theta_0}
\]

The three-dimensional models are thought as the same way of two-dimensional models. The object moves from zero point by one to the positive direction of y-axis. And we find the swept volume between the object on the origin and the virtual object floating in the air. In order to simplify, we make the object stand vertically as we did in the two-dimensional models. If we rotate the object as much as \( 90^\circ - \theta_O \), the point \( \frac{V_R}{V_0} \cos \alpha, \frac{V_R}{V_0} \cos \beta, \frac{V_R}{V_0} \cos \gamma \) will be converted into

\[
\frac{V_R}{V_0} \cos \alpha, \frac{V_R}{V_0} \cos \beta \sin \theta_0 - \cos \gamma \cos \theta_0, \frac{V_R}{V_0} \cos \beta \cos \theta_0 + \cos \gamma \sin \theta_0 + \cos \theta_0
\]

We substitute the point to the formulas; (3), (4), (5) and then we multiply them to the distance between two figures.

In the rectangular parallelepiped model,

\[
Q_D = AB \left| \frac{V_R}{V_0} \left( \cos \beta \cos \theta_0 + \cos \gamma \sin \theta_0 \right) + \cos \theta_0 \right| + BC \left| \frac{V_R}{V_0} \cos \alpha \right| + CA \left| \frac{V_R}{V_0} \cos \beta \sin \theta_0 - \cos \gamma \cos \theta_0 \right| + \sin \theta_0
\]

In the elliptic model,

\[
Q_D = \frac{\pi}{2} \left( A^2 + B^2 \right)
\]

Optimal speed \( V_0 \) =

\[
\frac{-V_R \cos \alpha + \cos \beta \cos \theta_0 + \cos \gamma \sin \theta_0}{A^2 \cos^2 \theta_0 + B^2 \sin^2 \theta_0}
\]

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\[ Q_1 = \pi A^2 B^2 (V_R \cos \beta \cos \theta_O + V_R \cos \theta_O)^2 + B^2 C^2 (V_R \cos \alpha)^2 + C^2 A^2 (V_R \cos \beta \sin \theta_O - \cos \gamma \cos \theta_O + V_R \sin \theta_O)^2 \]

\[ V_O = -V_R \frac{\pi A^2 B^2}{A^2 B^2 \cos^2 \theta_O + C^2 A^2 \sin^2 \theta_O} \]

In the cylindrical model with the elliptic bottom,

\[ Q_0 = \pi AB \left[ \frac{V_R}{V_O} (\cos \beta \cos \theta_O + \cos \gamma \sin \theta_O) + \cos \theta_O \right] + 2 \left( B^2 C^2 \left( \frac{V_R}{V_O} \cos \alpha \right)^2 + C^2 A^2 \left( \frac{V_R}{V_O} (\cos \beta \sin \theta_O - \cos \gamma \cos \theta_O) + \sin \theta_O \right) \right) \]

\[ Q_1 = \pi AB \left[ \frac{V_R}{V_O} (\cos \beta \cos \theta_O + \cos \gamma \sin \theta_O) + \cos \theta_O \right] + 2 \sqrt{B^2 C^2 (V_R \cos \alpha)^2 + C^2 A^2 (V_R \cos \beta \sin \theta_O - \cos \gamma \cos \theta_O) + V_O \sin \theta_O}^2 \]

If the object does not move in a constant speed but with acceleration, how can we find the formulas?

![Figure 11](https://via.placeholder.com/150)

Figure 11: The object moves with a constant acceleration; a

When it moves in a distance \( D \)

\[ S_k = \frac{D}{n} \left[ A \left[ \cos \theta_O + \frac{V_R}{\sqrt{2aDk/n}} \cos (\theta_O - \theta_R) \right] + B \left[ \sin \theta_O + \frac{V_R}{\sqrt{2aDk/n}} \sin (\theta_O - \theta_R) \right] \right] \]

\[ Q = \sum_{k=1}^{\infty} S_k = D \int_0^1 A \left[ \cos \theta_O + \frac{V_R}{\sqrt{2aDx}} \cos (\theta_O - \theta_R) \right] + B \left[ \sin \theta_O + \frac{V_R}{\sqrt{2aDx}} \sin (\theta_O - \theta_R) \right] \]

Total number of raindrops = \( pQ \)

When it moves in a time \( T \), \( D = \frac{1}{2} aT^2 \)

\[ Q = \frac{1}{2} aT^2 \int_0^1 A \left[ \cos \theta_O + \frac{V_R}{\sqrt{2ax}} \cos (\theta_O - \theta_R) \right] + B \left[ \sin \theta_O + \frac{V_R}{\sqrt{2ax}} \sin (\theta_O - \theta_R) \right] \]

Other models (2, 3-dimensional) are the same.

1. When it moves in a distance \( D \)
   \[ Q = D \int_0^1 Q_{Dx} \, dx \quad (\text{in } Q_{Dx} V_O \to \sqrt{2aDx}) \]

2. When it moves in a time \( T \)
   \[ Q = \frac{1}{2} aT^2 \int_0^1 Q_{Dx} \, dx \quad (V_O \to aT \sqrt{x}) \]

As we see until now, once we know the orthogonal projection area or length, the rain formulas can be made easily by a simple principle. It will be more complicated if we make a formula by the vector sum of the speeds of rain and object from the point of view of the object. But when a third person looks at the rain and the object, their directions and speeds will not be changed. And it is no need to divide the direction of rain into “the speed of cross wind”, “the speed of head wind” etc.

The main parameters are the ratio of speeds between the rain and the body, the sharp and size of the body and the angles of rain and object. The distance or time of travel, the volume of a rain drop and the density are just proportional constants.
III. Result And Conclusion

Previous conclusions

1. Generally, to a slim body, running with the maximum speed is not always the best option while a fat body should run as fast as possible. That is, a slim one has a higher probability of taking an optimal speed. [1]Bocci,[2]Bailey

2. If the rain comes from the back of a rectangular object, the optimal speed is the horizontal speed of the rain. [2]Bailey

3. If the rain comes from the back of an elliptic object, the optimal speed is bigger than the horizontal speed of rain. [3]Hailman&Torrents

4. With a headwind (when it rains from ahead), the object also can have the optimal speed. [1] Bocci

The previous conclusions are excellent, but they are not enough for the secret of rain. Let us find new conclusions looking at the following Excel tables.

And we assume that the body’s width is shorter than the height in the following examples to think simply.

1. Two-dimensional rectangle model

<table>
<thead>
<tr>
<th>Table 1-1-1</th>
<th>A=0.5m, B=1.7m, ( \theta_R=60^\circ ), D=1m, ( V_R=8m/s ), ( \rho=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>11.96  6.83  5.12  4.84  4.27  4.11  3.75  3.41  3.17  2.98  2.73  2.56  2.38  2.28  2.21  1.99  1.91  1.80  1.71</td>
</tr>
<tr>
<td>80</td>
<td>10.17  5.97  4.56  4.33  3.86  3.74  3.44  3.16  2.96  2.81  2.60  2.46  2.32  2.32  2.18  2.00  1.93  1.85  1.77</td>
</tr>
<tr>
<td>70</td>
<td>8.07   4.92  3.87  3.69  3.34  3.25  3.03  2.82  2.67  2.56  2.40  2.29  2.19  2.19  2.08  1.95  1.89  1.83  1.77</td>
</tr>
<tr>
<td>60</td>
<td>5.72   3.72  2.06  2.94  2.72  2.66  2.52  2.39  2.29  2.22  2.12  2.06  1.99  1.99  1.92  1.84  1.80  1.76  1.73</td>
</tr>
<tr>
<td>50</td>
<td>5.32   2.41  2.15  2.11  2.02  1.99  1.94  1.89  1.85  1.82  1.78  1.76  1.73  1.73  1.70  1.67  1.66  1.64  1.63</td>
</tr>
<tr>
<td>43,61</td>
<td>6.86   3.03  1.75  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53  1.53</td>
</tr>
<tr>
<td>40</td>
<td>7.70   3.50  2.09  1.86  1.39  1.27  1.30  1.33  1.35  1.36  1.39  1.40  1.42  1.42  1.43  1.45  1.46  1.47  1.47</td>
</tr>
<tr>
<td>30</td>
<td>9.85   4.72  3.00  2.72  2.15  1.99  1.64  1.29  1.09  0.87  0.87  0.95  1.01  1.06  1.06  1.12  1.19  1.22  1.25  1.28</td>
</tr>
<tr>
<td>20</td>
<td>11.69  5.79  3.82  3.50  2.84  2.66  2.25  1.86  1.57  1.36  1.07  0.87  0.87  0.95  1.01  1.06  1.06  0.87  0.77  0.77  0.77  0.94  0.99  1.05</td>
</tr>
<tr>
<td>10</td>
<td>13.19  6.69  4.53  4.17  3.44  3.25  2.80  2.36  2.05  1.82  1.50  1.28  1.06  1.06  0.87  0.77  0.77  0.77  0.63  0.71  0.78</td>
</tr>
<tr>
<td>0</td>
<td>14.28  7.39  5.09  4.71  3.94  3.74  3.26  2.80  2.47  2.22  1.88  1.65  1.42  1.42  1.19  0.89  0.78  0.64  0.51</td>
</tr>
</tbody>
</table>

On the table the vertical axis represents the angle of the object \( \theta_o \) and the horizontal axis represents the speed of the object \( V_o \). The values on the table represent the number of rain drops that will hit the object. And in the table 1-1-2, using the time 1/8 instead of 1second is for reducing the number size.

Maybe some people will be more interested in this case. Because usually they experiment in real; a man moves in the rain, the direction of the rain is nearly vertical (“when it rains vertically” is also included in this case I-1). In the TV programs; Korean television “Heaven of the Curiosity (2002)” and American television “Mythbusters (2006)”, they experimented about this problem. In Heaven of the Curiosity, the conclusion was running in order to get wet less and in the Mythbusters, the first conclusion was walking and later they modified as running. Let us look at this case mathematically.

When it moves in a given distance, there is a certain section in which the amount of wetting is constant regardless of the moving speed of the object, where \( \theta_o(\text{critical angle})= \theta_R - \tan^{-1}(A/B) \). This means that if you are thin and it rains nearly vertically, you have a higher possibility of being in the critical angle even if you
If the rain comes from the back, it is more complicated. In reality, it is almost impossible but we know that the direction of the rain is 170°. On the horizontal axis, $V_O=7.88m/s$ represents the horizontal speed of the rain to the direction of body’s movement ($V_O=-V_R\cos \theta_R$, they called it “speed of tail wind” in another article). Let us look at the Figure 8 and we move the floating object by changing the speed of $V_O$.

If an optimal speed exists, there are in following two cases.

1. The rain hits the object only on the top part.
   **Optimal speed:** $V_O=-V_R\frac{\sin(\theta_O-\theta_R)}{\sin \theta_O}$

2. The rain hits the object only on the back part or the front part (when it leans backward)
   **Optimal speed:** $V_O=-V_R\frac{\cos(\theta_O-\theta_R)}{\cos \theta_O}$

(That means an optimal speed can exist when the rain hits only the larger part of the body as well as when it hits only the shorter part.) But note that even the rain hits only the top or back front, an optimal speed does not always exist.

For example $\theta_O=90^\circ$, an optimal speed can exist if $90^\circ + \tan^{-1}(A/B) \leq \theta_R \leq 180^\circ$. This means that if it is slim and the angle of the rain is high, it has a bigger possibility of being in an optimal speed. And there is its least value and it's value is $A\sin \theta_R$ where $\theta_R-\theta_O=90^\circ$.

<table>
<thead>
<tr>
<th>$\theta_O$</th>
<th>$\theta_R$</th>
<th>$\frac{\sin(\theta_O-\theta_R)}{\sin \theta_O}$</th>
<th>$\frac{\cos(\theta_O-\theta_R)}{\cos \theta_O}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>170</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>90</td>
<td>180</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

If it rains from the back, it is more complicated. In reality, it is almost impossible but we know that the direction of the rain is 170°. On the horizontal axis, $V_O=7.88m/s$ represents the horizontal speed of the rain to the direction of body’s movement ($V_O=-V_R\cos \theta_R$, they called it “speed of tail wind” in another article). Let us look at the Figure 8 and we move the floating object by changing the speed of $V_O$. If an optimal speed exists, there are in following two cases.

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From now, let us consider only the case when it rains from the back. The horizontal speed of the rain is 4m/s. Where $\theta_R=90^\circ$, the optimal speed is bigger than the horizontal speed of the rain. As the shape of ellipse is more similar to a round, the gap between the optimal speed and the horizontal speed of the rain is bigger. And the condition of getting wet the least is where $\theta_R= \theta_O=90^\circ$, to make the area minimum, it is the same of rectangle model.

How can we know the above conclusions intuitively without using complicated formulas we made? Let us look at the Figure 8 and assume that $\theta_R \leq \theta_O \leq 180^\circ$. In a given $\theta_O \theta_R$ and $V_R$, to make the area minimum, we can control the location of the floating object by changing the value $V_O$. That is, the two figures should be orthogonal projected area of ellipsoid.
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Usually the rectangle model can get an optimal speed when the rain hits only the top part (width) or just high if $\theta_R$ is big enough, as we saw the Table 1-2-1, and if the object is a fatter body, the optimal speed can exist even when the rain hits only the back part or the front part (when it leans backward). On paper, let's paint some pictures of ellipses (one is similar to a rectangle, the other is similar to a round) and rectangles (one is slim and the other is square) and compare them.

In the elliptic model, the floating object should not be situated on the parallel line to make the area minimum; it should be located slightly below the line. If the object is a perfect round, it should be located in a position which makes the distance between the two figures minimum. It means that the optimal speed of the ellipse is bigger than that of the rectangle model and if the body looks more like a round, it should run faster.

Now imagine that we can change even $\theta_R$ and we search the minimum value. In other words, now we can change the angle of object as well as the location of the floating object. In this case the minimum condition can change and we search the minimum value. $\theta_R$ and $\theta_O$ are equal. It should be mentioned that the body type and its value; $\theta_R - \theta_O = 90^o$, $V_O = \frac{1}{\cos \theta_R}$, which is not always the case. In this table, we will show the results.

### Table 2-2

<table>
<thead>
<tr>
<th>$\theta_R$</th>
<th>$\theta_O$</th>
<th>$\theta_R - \theta_O$</th>
<th>$\theta_R$</th>
<th>$\theta_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^o$</td>
<td>$0^o$</td>
<td>$0^o$</td>
<td>$0^o$</td>
<td>$0^o$</td>
</tr>
<tr>
<td>$90^o$</td>
<td>$90^o$</td>
<td>$90^o$</td>
<td>$90^o$</td>
<td>$90^o$</td>
</tr>
<tr>
<td>$180^o$</td>
<td>$180^o$</td>
<td>$180^o$</td>
<td>$180^o$</td>
<td>$180^o$</td>
</tr>
</tbody>
</table>

When it runs in the rain, the object should move standing vertically with the horizontal speed of the rain.

### 3. Three-dimensional rectangular model

<table>
<thead>
<tr>
<th>$A_{OX}$</th>
<th>$A_{OY}$</th>
<th>$A_{OZ}$</th>
<th>$V_O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.25m$</td>
<td>$0.85m$</td>
<td>$1.20m$</td>
<td>$8m/s$</td>
</tr>
</tbody>
</table>

When it rains from side and back, like a two-dimensional rectangle model, if there is an optimal speed, the condition should be one of the following two cases.

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1. When the rain hits only the upper side and part of the side of the body;  
Optimal speed; \( V_O = -V_R \frac{\cos \beta \cos \theta_O - \cos \gamma \sin \theta_O}{\sin \theta_O} \) and 

2. When it hits only the front side or the front and side part (when it leans backward) 
Optimal speed; \( V_O = -V_R \frac{\cos \beta \cos \theta_O + \cos \gamma \sin \theta_O}{\sin \theta_O} \)

But it does not always exist. If it is smaller (case 1), \( \alpha \) is closer to 90° and \( \beta \) is closer to 180°, it will have more possibility of being in an optimal speed. In the Figure 10, imagine a running or walking in the rain. Then we can easily understand this conclusion.

And when it rains from side view, there is also a section that makes the optimal speed differ depending on the vertical direction. 

Table 3-2: \( \theta_O = 68.61^{\circ} \), \( \beta = 150^{\circ} \), \( \gamma = 70^{\circ} \) 

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>70</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
</tbody>
</table>

Like other models, when it moves in a given time and it rains from the back, the object should move standing vertically with the horizontal speed of the rain.

And as we see the formulas of this model, when it moves in a given distance, the amount of wetting of the side part depends on the speed of the body (an author ignored this point) but when it moves in a given time, it is constant irrespective of the speed of body in a given rain condition.

4. Three-dimensional elliptic model

Table 4-1c: \( \alpha = 70^{\circ} \), \( \beta = 150^{\circ} \), \( \gamma = 70^{\circ} \), \( A = 0.5m \), \( B = 0.5m \), \( C = 0.85m \), \( D = 1m \), \( V_R = 8m/s \cdot \rho = 1 \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>70</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>150</td>
<td>90</td>
</tr>
</tbody>
</table>

Like two-dimensional elliptic model, if \( \theta_O = 90^{\circ} \), the optimal speed is bigger than the horizontal speed of the rain. In the given rain condition, the two-dimensional models (there is no side wind) will have the

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same minimum condition regardless of body type. However, in this example, the ellipse should move more slowly than the rectangular to get wet the least. On the table, \( \theta_O = 30^\circ \) and \( V_r = 8\text{m/s}, \rho = 1 \) are the minimum condition of rectangular.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( T )</th>
<th>( V_r )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>70.0</td>
<td>( 80.0 )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.85</td>
<td>1/8</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>110</td>
<td>150.0</td>
<td>( 110.0 )</td>
<td>0.5</td>
<td>0.5</td>
<td>1.7</td>
<td>1</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>70.0</td>
<td>150.0</td>
<td>( 70.0 )</td>
<td>0.5</td>
<td>1.7</td>
<td>1</td>
<td>1/8</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

Like other models, when it moves in a given time, the object should move standing vertically with the horizontal speed. In this example, the cylinder should move more quickly than the rectangular in the given rain condition in order to get wet the least. In this table, \( \alpha = 68.61^\circ \), \( \beta = 150^\circ \), \( \gamma = 70^\circ \), A = 0.5 m, B = 0.5 m, C = 0.85 m, T = 1/8 second, \( V_r = 8\text{m/s}, \rho = 1 \).

In this example, the cylinder should move more quickly than the rectangular to get wet the least.
When it rains from the back and the object should move in a given time, the rectangular, elliptic and cylindrical models have a best strategy when it moves standing vertically or totally leaning forward with the horizontal speed of the rain. To prove it by using formulas is not easy.

But let’s look at the Figures 8 and 10. When they move in a given time 1 with a speed $V_O$, the points in the sky will be $(V_O + V_R \cos \theta_R, V_R \sin \theta_R)$ and $(V_O \cos \alpha_O, V_O + V_R \cos \beta, V_R \cos \gamma)$ respectively. We can imagine how the floating objects move in the given speed and angle of rain. And we can find easily the condition to make the area or volume minimum.

The floating objects move on the parallel line to the x-axis and y-axis respectively. To make the swept area or volume minimum, they should stand vertically; $\theta_R=90^o$ or lean forward totally; $\theta_R=0^o$ and they should be located just above the zero point, it means that the optimal speeds are the horizontal speed of rain $V_O$, $V_O + V_R \cos \theta_R = 0$, $V_O + V_R \cos \beta = 0$.

### Solution of quiz in the introduction

When it travels 1053.90 km

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$V$</th>
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</thead>
<tbody>
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</tr>
<tr>
<td>10</td>
<td>2325.059</td>
</tr>
<tr>
<td>20</td>
<td>1109.892</td>
</tr>
<tr>
<td>30</td>
<td>716.9055</td>
</tr>
<tr>
<td>40</td>
<td>427.3903</td>
</tr>
<tr>
<td>50</td>
<td>275.2350</td>
</tr>
<tr>
<td>60</td>
<td>264.0700</td>
</tr>
<tr>
<td>70</td>
<td>258.7355</td>
</tr>
<tr>
<td>80</td>
<td>258.7352</td>
</tr>
<tr>
<td>90</td>
<td>300.1850</td>
</tr>
</tbody>
</table>

When it travels 2.16 days

<table>
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<tr>
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<th>$V$</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>10</td>
<td>4111.2207</td>
</tr>
<tr>
<td>20</td>
<td>3923.7300</td>
</tr>
<tr>
<td>30</td>
<td>3754.1729</td>
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<tr>
<td>40</td>
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<tr>
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<td>4270.1197</td>
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</tr>
<tr>
<td>120</td>
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### Reference


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