Calculation the Energy Levels and Energy Bands and Energy Ratios for even-even$^{154, 156, 158}$Dy Isotopes

Sallama Sadiq Hummadi (*)
(*) Physics Department, Science College, Al-Mustansiriyah University, Baghdad, Iraq

Summary: Interacting boson model (IBM-1) was used in the present work to study some of nuclear structures for selected Dysprosium isotope of even mass number Dy(A=154-158). The energy levels and energy ratios of these isotopes were investigated for there are experimental. The calculated results were compared with the available experimental data, the results were in general good agreement.

I. Introduction

The Interacting Boson Model (IBM), initially introduced by Arima and Iachello has been rather successful in describing the collective properties of several medium and heavy nuclei. We note that in the interacting boson model-1 (IBM-1) one describes an even-even nucleus as a system of N bosons able to occupy two levels, one with angular momentum restricted to zero (s boson) and one with angular momentum 2 (d boson). (1)

II. The interacting Boson model

In the simplest version of the interacting boson model (IBM-1), its assumed that low-lying collective states in even-even nuclei away from closed shells are dominated by excitation of the valence protons and the valence neutrons (particles outside the major closed shell) while the closed shell core is inert. Furthermore, it's assumed that the particle configurations which are most important in shaping the properties of the low-lying states are these in which identical particles are coupled together forming pairs of angular momentum 0 and 2. (1)

The interacting Boson model (IBM-1) is used in the present work, this model represents very important step formed in the description of collective nuclear excitations. The underlying U(6) group structure of model basis leads to a simple Hamiltonian which is capable of describing the three specific limits of collective structure vibrational U(5), rotational SU(3) and gamma unstable O(6). (2)

The interacting boson model offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, based on general algebraic group theoretical techniques which have also recently found application in problems in atomic, molecular, and high-energy physics (3,4).

III. Theoretical Basis:

Hamiltonian operator function according to IBM-1 is written in terms of creation and annihilation operators as follows : (5,6)

\[ \hat{H} = \hat{n}_d + a_0 (\hat{p}, \hat{p}) + a_1 (\hat{1}, \hat{I}) + a_2 (\hat{Q}, \hat{Q}) + a_3 (\hat{T}_3, \hat{T}_3) + a_4 (\hat{T}_4, \hat{T}_4) \]

Where \(E = E_d - E_s\) is the boson energy.

Where \(c, a_0, a_1, a_2, a_3, a_4\) are parameters used in IBM-1 to determine the Hamiltonian function.

The operators:

\[ \hat{n}_d = (\hat{d}^\dagger \hat{d}) \] the boson number operator

\[ \hat{p} = \frac{1}{2} (\hat{d}^\dagger \hat{d}) - \frac{1}{2} (\hat{s}^\dagger \hat{s}) \] the pairing bosons operator

\[ \hat{I} = \sqrt{10} \left[ \hat{d}^\dagger \hat{d} \right]^{(1)} \] the angular momentum operator

\[ \hat{Q} = \left[ \hat{d}^\dagger \hat{s} \right] + \left[ \hat{s}^\dagger \hat{d} \right] \] the quadrupole operator
3.1.1 Rotational Limit SU(3):
The electromagnetic transition rates, B(E2) values of this chain and the quadrupole moments (Q) are described by [6]:

\[
B(E2; I^+ \rightarrow I) = \alpha_2^2 \left( \frac{I}{2} + \frac{2N - I}{2} \right) \]

----- (3)

\[
Q_i = \beta_2 \left( \frac{16\pi}{70} \right) \frac{I}{2}
\]

In particular, for \( I=0, \) or \( 2 \)

\[
B(E2; 2^+_1 - 0^+_1) = \alpha_2^2 N \\
Q_{2^+_1} = \beta_2 \left( \frac{32\pi}{35} \right)^{1/2}
\]

The basic condition for the observation of a SU(5) symmetry in the electromagnetic transition is [7]:

\[
\frac{B(E2; 4^+_1 - 2^+_1)}{B(E2; 2^+_1 - 0^+_1)} = \frac{B(E2; 2^+_2 - 2^+_1)}{B(E2; 2^+_2 - 0^+_1)} = \frac{2}{N} \left( \frac{N - 1}{N} \right) < 2 \]

----- (5)

Where the necessary conditions for the observation of the SU(3) symmetry are [8];

\[
\frac{B(E2; 4^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} = \frac{10(N-1)(2N+5)}{7N(2N+3)} \left( \frac{10}{7} \right) \]

----- (6)

\[
\frac{B(E2; 2^+_2 \rightarrow 2^+_1)}{B(E2; 2^+_2 \rightarrow 0^+_1)} = \frac{B(E2; 0^+_2 \rightarrow 2^+_1)}{B(E2; 2^+_2 \rightarrow 0^+_1)} = 0 \\
\]

----- (7)

3.1.2. The Vibrational SU(5) Limit
The electromagnetic transition rates, B(E2) values of this chain and the quadrupole moments (Q) are described by (8).

\[
B(E2; 1^+ \rightarrow 1) = \alpha_2^2 \left( \frac{I}{2} + \frac{2N - I}{2} \right) \\
Q_{1^+} = \beta_2 \left( \frac{16\pi}{70} \right)^{1/2}
\]

----- (8)

----- (9)

In particular, for \( I=0, \) or \( 2 \)

\[
B(E2; 2^+_1 - 0^+_1) = \alpha_2^2 N \\
\]

----- (10)
\[ Q_{2i} = \beta_2 \left[ \frac{32\pi}{35} \right]^{\frac{1}{2}} \]  \quad (11)

The basic condition for the observation of a SU(5) symmetry in the electromagnetic transition is[8]:

\[
\frac{B(E2;A^+_1 - 2^+_1)}{B(E2;^2_1^+ - 0^+_1)} = \frac{B(E2;A^+_2 - 2^+_2)}{B(E2;^2_2^+ - 0^+_1)} = \left( \frac{N - 1}{N} \right)^2 < 2 \quad (12)
\]

**IV. Transitional Regions in IBM-1**

**4.1.1 SU(3)-SU(5) transitional dynamical symmetry**

This transitional region includes the two groups, SU(3) and SU(5). The SU(3) has to be broken with \( \in n_d \) term.

The general form of Hamiltonian operator of this region can be given as\( (9,10) \).

\[
\hat{H} = \in \hat{n}_d + a_1 \hat{l} \cdot \hat{l} + a_2 \hat{Q} \cdot \hat{Q} \quad (13)
\]

The solution of the equation (13) depends on the ratio of \( (\in/a_3) \), when the ratio \( (\in/a_2) \) is large the eigenfunction of \( \hat{H} \) are those appropriate to the limiting SU(5). Also the B(E2) values are affected by the ratio \( (\in/a_2) \). The B(E2) ratios (Branching Ratios) \( R \) can be given by (10).

\[
R = \frac{B(E2;^2_2^+ \rightarrow 0^+_1)}{B(E2;^2_2^+ \rightarrow 2^+_1)} \quad (14)
\]

Where:

- \( R=0 \) in SU(5) region
- \( R=7/10 \) in SU(3) region

**4.1.2 SU(3)-O(6) transitional dynamical symmetry**

The breaking of SU(3) symmetry in the direction of O(6) symmetry can be treated in this transitional region by adding the term \( \hat{P}^+ \cdot \hat{P} \), so that the Hamiltonian form can be written as (11).

\[
\hat{H} = a_9 \hat{P}^+ \cdot \hat{P} + a_1 \hat{l} \cdot \hat{l} + a_2 \hat{Q} \cdot \hat{Q} + a_3 \hat{T} \cdot \hat{T} \quad (15)
\]

The solutions of equation (14) depends on the ratio \( (a_2/a_2) \), when it is large, the eigenfunction of the Hamiltonian \( \hat{H} \) are appropriate to O(6) Symmetry, but if it is small, the eigenfunction are appropriate to SU(3) symmetry. In this region the change in the electromagnetic rates can be seen from the branching ratios \( R \) which takes values

- \( R=7/10 \) in SU(3) symmetry
- \( R=0 \) in O(6) symmetry

**4.1.3 SU(5)-O(6) transitional dynamical symmetry**

The form of Hamiltonian in this region can be written as (12).

\[
\hat{H} = \in \hat{n}_d + a_9 \hat{P}^+ \cdot \hat{P} + a_1 \hat{l} \cdot \hat{l} \quad (16)
\]

The solution of the equation (15) depends on the ratio \( (\in/a_3) \). The B(E2) values show a smooth transition towards typical O(6) values, the branching ratios takes constant value (12,13).

- \( R=0 \) in SU(5) symmetry
- \( R=0 \) in O(6) symmetry

Also, when the ratio \( (\in/a_2) \) is large, the eigenfunction of \( \hat{H} \) are those appropriate to the limiting SU(5), while when it is small, the eigenfunctions are appropriate to symmetry O(6).

**V. Results & Discussion**

In this work we have studied the energy levels of even-even Dy (\( A=154 \text{-}158 \)) isotope transitions with change in number of neutrons observed when moving from the lighter to heavier isotopes, i.e. SU(5) - SU(3) transitional regions (Table 1). The even-even \( ^{154,156,158}\text{Dy} \) isotopes have (66) protons and (88,89,90) neutrons respectively. The core is taken at major closed shell \( (82) \) for protons and neutrons. Therefore, the number of bosons were determined for \( ^{154}\text{Dy}, ^{156}\text{Dy} \text{ and } ^{158}\text{Dy} \), is equal (11),(12) and (13) bosons respectively. The nuclear deformation increases with
the increasing of valance boson number. The interacting boson model version one (IBM-1) gives us a very closing value with the experiment.

5.1.2. Energy bands (g, β, γ):

Figures (1-3) show the energy levels and the energy bands arrangement compared with identical bands for the three limits SU(5) - SU(3) and O(6). The IBM-1 model in its group theoretical formulation, exhibits three dynamical symmetries, each corresponding to a particular way of breaking the degeneracy of the parent U(6) group.

The behavior of the structure of each nucleus considered in this work is deduced by studying the dynamical symmetry of deformed Dy (A = 154–158) and the energy spectrum according to the sequences of energy bands g, β, γ. The parameters of Eq.[1] fitted to the experimental data are used to calculate the eigenvalues and eigenvectors of Dy (A = 154–158) isotopes, which are tabulated in Table 1.

To carry out the calculation of 154–158 Dy using interacting boson model-1(IBM-1), which does not distinguish between the neutron- and proton- boson, we must evaluate the total number of bosons N (Table 2) and the dynamical symmetry. Levels energy belonging to the g, β, γ bands, energy Levels, and energy ratios are calculated. The calculated values are compared with the available theoretical and experimental data and show reasonable agreement.

The energy spectrum according to the sequences energy bands (g, β, γ) give agood agreement for the level sequence of each band with the typical sequence of ground band (0+, 2+, 4+,…….), (β– band) (0+, 2+, 4+,…….), and (γ – band) (2+, 3+,4+, 5+,…….). There are good agreement between the present results and the experimental data.

5.1.1. Energy ratios

Table(3) show the theoretical energy ratios compared with the experimental data for chosen even-even isotopes Dy. The energy ratios of Dy(A=154–158) is for the 154–158 Dy isotopes under study have been determined. Which is found to be 154 Dy which has transitional motion of the SU(5)-O(6) translation region 156 Dy which has transitional motion of the O(6)-SU(3) dynamical symmetry and 158 Dy which has rotational motion of the SU(3) dynamical symmetry.

Table(1) Theoretical energy ratios compared with the experimental data for chosen even-even isotopes

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$^{154}$Dy</th>
<th>$^{156}$Dy</th>
<th>$^{158}$Dy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{11}$</td>
<td>$^{13}$</td>
<td>$^{14}$</td>
</tr>
<tr>
<td>$^{154}$Dy</td>
<td>$^{156}$Dy</td>
<td>$^{158}$Dy</td>
<td></td>
</tr>
<tr>
<td>Exp$^{(a)}$</td>
<td>IBM-1 (pw)</td>
<td>Exp$^{(a)}$</td>
<td>IBM-1 (pw)</td>
</tr>
<tr>
<td>$^{154}$Dy</td>
<td>$^{156}$Dy</td>
<td>$^{158}$Dy</td>
<td></td>
</tr>
<tr>
<td>$^{11}$</td>
<td>$^{13}$</td>
<td>$^{14}$</td>
<td></td>
</tr>
<tr>
<td>$^{0+}$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$^{2+}$</td>
<td>0.3346</td>
<td>0.33</td>
<td>0.3346</td>
</tr>
<tr>
<td>$^{4+}$</td>
<td>0.7467</td>
<td>0.842</td>
<td>0.7467</td>
</tr>
<tr>
<td>$^{6+}$</td>
<td>1.2237</td>
<td>1.537</td>
<td>1.2237</td>
</tr>
<tr>
<td>$^{8+}$</td>
<td>1.7473</td>
<td>2.415</td>
<td>1.7473</td>
</tr>
<tr>
<td>$^{10+}$</td>
<td>2.6608</td>
<td>0.66</td>
<td>2.6608</td>
</tr>
<tr>
<td>$^{12+}$</td>
<td>3.905</td>
<td>0.792</td>
<td>3.905</td>
</tr>
<tr>
<td>$^{14+}$</td>
<td>1.2519</td>
<td>1.458</td>
<td>1.2519</td>
</tr>
<tr>
<td>$^{16+}$</td>
<td>1.659</td>
<td>2.197</td>
<td>1.659</td>
</tr>
<tr>
<td>$^{18+}$</td>
<td>2.1632</td>
<td>3.075</td>
<td>2.1632</td>
</tr>
<tr>
<td>$^{20+}$</td>
<td>0.0577</td>
<td>1.21</td>
<td>0.0577</td>
</tr>
<tr>
<td>$^{22+}$</td>
<td>1.3904</td>
<td>1.452</td>
<td>1.3904</td>
</tr>
<tr>
<td>$^{24+}$</td>
<td>2.52</td>
<td>2.052</td>
<td>2.52</td>
</tr>
<tr>
<td>$^{26+}$</td>
<td>2.747</td>
<td>3.625</td>
<td>2.747</td>
</tr>
<tr>
<td>$^{28+}$</td>
<td>3.625</td>
<td>0.99</td>
<td>3.625</td>
</tr>
<tr>
<td>$^{30+}$</td>
<td>1.3343</td>
<td>1.429</td>
<td>1.3343</td>
</tr>
<tr>
<td>$^{32+}$</td>
<td>1.4426</td>
<td>1.502</td>
<td>1.4426</td>
</tr>
<tr>
<td>$^{34+}$</td>
<td>1.74</td>
<td>2.26</td>
<td>1.74</td>
</tr>
<tr>
<td>$^{36+}$</td>
<td>1.8586</td>
<td>2.34</td>
<td>1.8586</td>
</tr>
<tr>
<td>$^{38+}$</td>
<td>2.218</td>
<td>3.282</td>
<td>2.218</td>
</tr>
<tr>
<td>$^{40+}$</td>
<td>3.339</td>
<td>1.5076</td>
<td>3.339</td>
</tr>
<tr>
<td>$^{42+}$</td>
<td>1.782</td>
<td>2.089</td>
<td>1.782</td>
</tr>
<tr>
<td>$^{44+}$</td>
<td>1.8</td>
<td>2.218</td>
<td>1.8</td>
</tr>
</tbody>
</table>

---

**www.iuosjournals.org** 4 | Page
Calculation the Energy Levels and Energy Bands and energy ratios for even-even $^{154}, 156, 158\ldots$

<table>
<thead>
<tr>
<th>$^\pi$</th>
<th>2.924</th>
<th>$^\pi$</th>
<th>2.094</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^\nu$</td>
<td>3</td>
<td>$^\nu$</td>
<td>2.329</td>
</tr>
<tr>
<td>$^\nu$</td>
<td>3.9416</td>
<td>$^\nu$</td>
<td>2.71</td>
</tr>
<tr>
<td>$^\nu$</td>
<td>4</td>
<td>$^\nu$</td>
<td>3</td>
</tr>
</tbody>
</table>

Table (2) The parameters of Hamiltonian function operator for Dy ($A = 154$-$158$) isotopes.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>$N_\pi$</th>
<th>$N_\nu$</th>
<th>$N_{\text{tot}}$</th>
<th>$\hat{p}<em>\pi \hat{p}</em>\pi$ MeV</th>
<th>$\hat{I}<em>\pi \hat{I}</em>\pi$ MeV</th>
<th>$\hat{Q}<em>\pi \hat{Q}</em>\pi$ MeV</th>
<th>$(\hat{T}<em>\pi, \hat{T}</em>\pi)$ MeV</th>
<th>CHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{154}<em>{66}Dy</em>{88}$</td>
<td>8</td>
<td>3</td>
<td>11</td>
<td>0.0000</td>
<td>0.0550</td>
<td>0.0190</td>
<td>0.0000</td>
<td>0.1540</td>
</tr>
<tr>
<td>$^{156}<em>{66}Dy</em>{90}$</td>
<td>8</td>
<td>4</td>
<td>12</td>
<td>0.3250</td>
<td>0.0100</td>
<td>-0.0090</td>
<td>-0.0142</td>
<td>0.1691</td>
</tr>
<tr>
<td>$^{158}<em>{66}Dy</em>{92}$</td>
<td>8</td>
<td>5</td>
<td>13</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0195</td>
<td>-0.0089</td>
<td>0.0019</td>
</tr>
</tbody>
</table>

Table (3) Theoretical energy ratios compared with the experimental data for chosen even-even isotopes.

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>$\frac{E(4^+_1)}{E(2^+_1)}$</th>
<th>$\frac{E(6^+_1)}{E(2^+_1)}$</th>
<th>$\frac{E(8^+_1)}{E(2^+_1)}$</th>
<th>EXP.</th>
<th>IBM-1 (pw)</th>
<th>EXP.</th>
<th>IBM-1 (pw)</th>
<th>EXP.</th>
<th>IBM-1 (pw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{154}<em>{66}Dy</em>{88}$</td>
<td>2.2356</td>
<td>2.5515</td>
<td>3.6637</td>
<td>4.6575</td>
<td>5.2314</td>
<td>7.3181</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{156}<em>{66}Dy</em>{90}$</td>
<td>2.9325</td>
<td>3.2945</td>
<td>5.5899</td>
<td>8.8527</td>
<td>8.8150</td>
<td>11.6356</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{158}<em>{66}Dy</em>{92}$</td>
<td>3.2072</td>
<td>3.3333</td>
<td>8.4469</td>
<td>7.000</td>
<td>10.5571</td>
<td>11.9895</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig (1) Comparison between calculated IBM (pw) and experimental energy bands states $g, \beta, \gamma$ in isotope $^{154}_{66}Dy_{88}$ of the dynamical symmetry SU(5) – O(6).
Calculation the Energy Levels and Energy Bands and energy ratios for even-even $^{154}, 156, 158$.

**Fig 2** Comparison between calculated IBM (pw) and experimental energy bands states ($g, \beta, \gamma$) in isotope $^{156}_{66}Dy_{92}$ of the dynamical symmetry SU(3) – O(6).

**Fig 3** Comparison between calculated IBM (pw) and experimental energy bands states ($g, \beta, \gamma$) in isotope $^{158}_{66}Dy_{94}$ of the dynamical symmetry SU(3) – O(6).
Calculation the Energy Levels and Energy Bands and energy ratios for even-even $^{154}, 156, 158 \ldots$.

The relation between the energy ratios as a function of number of neutron $N$ for the even-even Dy ($A=154-158$) isotopes.

References