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String cosmology in Bianchi type-VI$_0$ space-time

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***Abstract**: In this paper we have constructed a spatially homogeneous and anisotropic Bianchi type-VI$_0$ string cosmological model in general relativity. This model is expanding, shearing, non-rotating and has no initial singularity. Here it is interesting to note that string tension density (λ) is greater than half of rest energy density (ρ) and particle density (ρ_p) is less than half of rest energy density. Further some physical and kinematical properties of the model are discussed.*
***Keywords:** Bianchi type-VI$_0$ space time, cosmic strings and general relativity.*

I. Introduction

In recent years cosmologists have been interested in constructing string cosmological models of the universe. The concept of the string is developed to describe events at early stages of the universe. Kibble [1], Zeldovich [2], and Vilenkin [3], believed that strings may be one source of density perturbations and that are required for the formation of large structure in the universe. The construction of string cosmological models was intiated by Vilenkin [4–8]. .The gravitational effects of strings were studied by several researchers [6, 9, 10] in four dimensional space-time and some others studied string cosmology in higher dimensional space time [11–15]. Therefore it is a subject of considerable interest of cosmologists to study cosmic string in the framework of general relativity.

The general relativistic formalism of cosmic strings are given by Letelier [16] and Stachel [17]. They considered the energy momentum tensor for string distribution in the form

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{1}$$

Where

$$u_i u^i = -x_i x^i = 1 \tag{2}$$

And

$$x^i x_j = 0 \tag{3}$$

Where ρ is the energy density for a cloud of strings with particles attached to them, λ the String tension density, the unit time like vector, u^i is the flow vector and the unit space like vector x^i specifies the direction of strings. The particle density and string tension density of the string cloud which are related by Letelier [5],

$$\rho_p = \rho - \lambda \tag{4}$$

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic from which the process of istropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generality than isotropic models. The simplicity of the field equations made Bianchi space time useful in constructing models of spacially homogeneous and anisotropic cosmologies. Today the universe is successfully described by maximally symetric models given by the Friedmann- Robertson -Walker (FRW) space time which is homogeneous and isotropic. However, in smaller scales the universe is neither homogeneous nor isotropic. Also we do not expect that the universe have these properties in its early stages. To get a realistic picture of the universe the homogeneous and anisotropic models have been studied in general relativity.

Einstein's himself pointed out that general relativity does not account satisfactorily for the inertial properties of matter; i.e. Mach's principle is not substantiated by general relativity. So, in recent years, there have been some inersting attempts to generalize the general theory of relativity by incorporating Mach's principle other desired features which are lacking in the original theory. Saez and Ballester [18] formulated a scalar tensor theory of gravitation in which metric is coupled with a dimensionless character of scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies [19–26]. Recently Rao et al. [26] constructed Bianchi type 11, V111 and 1X cosmological models which are free from initial singularity and expanding with time.

In this paper we have constructed the anisotropic Bianchi type-VI_0 cosmological model in general relativity and studied some physical and kinematical behaviors of the model.

II. The Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-VI0 metric of the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2m^2x}dy^2 - C^2e^{2m^2x}dz^2 \quad (5)$$

Where A, B, C are scale functions of t only and q is a non-zero constant. The Einstein field equations, in gravitational units (c=1 and 8πG = 1), are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad (6)$$

Where R_{ij} is the Ricci tensor, R is the Ricci scalar, and T_{ij} is the energy-momentum tensor string.

Using the equation (1), the field equation (6) for metric (5) can be written as

$$\frac{A'B'}{AB} + \frac{A'C'}{AC} + \frac{B'C'}{BC} - \frac{q^2}{A^2} = \rho \quad (7)$$

$$\frac{B''}{B} + \frac{C''}{C} + \frac{B'C'}{BC} + \frac{q^2}{A^2} = \lambda \quad (8)$$

$$\frac{A''}{A} + \frac{C''}{C} + \frac{A'C'}{AC} - \frac{q^2}{A^2} = 0 \quad (9)$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{A'B'}{AB} - \frac{q^2}{A^2} = 0 \quad (10)$$

$$\frac{B'}{B} - \frac{C'}{C} = 0 \quad (11)$$

Here afterwards the dots over field variable represent ordinary differentiation with respect to t.
Now solving equation (11), we get
B = kC (12)

Now to obtain an exact solution, one extra condition is needed. So we assume a relation between metric co-efficient given by

$A = B^n$ (13)

Where n (≠ 0) is a constant. With help of equation (12) and solving equation (10), we get

$A = qt + c_1$ (14)

With help of equation (12) and (14), we get

$C = c_2t + c_3$ (15)

Where $c_2 = \frac{q}{k}$ and $c_3 = \frac{c_1}{k}$

Thus the geometry of the universe descried by the line element is

$$ds^2 = -dt^2 + (qt + c_1)^2dx^2 + (qt + c_1)^2e^{-2qx}dy^2 + (c_2t + c_3)^2e^{2qx}dz^2 \quad (16)$$

Which represents Bianchi type-VI_0 cosmological model in general relativity.

III. Some Physical And Geometrical Properties Of The Model

The model (16) obtained in the previous section represents spatially homogeneous anisotropic string cosmological model in general relativity. The model has no intial singularity. As time increases, the model expands indefinitely along x, y and z axes. The properties of the physical and kinematical variables involved in this model are given as follows.

(a) Rest energy density,

$$\rho = \frac{2qc_2}{(qt + c_1)(c_2t + c_3)} \quad (17)$$

Which indicates that it tends to zero as time't' increases indefinitely. At the initial epoch t=0, this space time becomes flat. Hence it is interesting note that the model is free from initial singularity.

(b) String tension density,

$$\lambda = \frac{\rho}{2} + \frac{q^2}{qt + c_1} \tag{18}$$

And particle density,

$$\rho_p = \frac{\rho}{2} - \frac{q^2}{qt + c_1}$$

(19)

which indicates that both λ and ρ_p tend to zero as time 't' increases indefinitely and have no initial singularity.

(c). Spatial volume,

$$V = (qt + c_1)^2 (c_2 t + c_3) \tag{20}$$

From above equation (20), it is clear that at the initial epoch t = 0, the volume 'V' of the universe is zero (if c_1, c_2=0). The volume of the universe increases with increase of time't' and V →∞ as t→∞. Hence equation (16) represents an expanding model of the universe

(c) Scalar expansion,

$$\theta = \frac{2q}{qt + c_1} + \frac{c_2}{c_2 t + c_3} \tag{21}$$

From above equation (21), it is clear that at the initial epoch t = 0, the scalar expansion 'θ' is finite and θ →∞as t →∞. Hence there is a finite expansion in the model.

(d) Shear scalar,

$$\sigma^2 = \frac{q^2}{(qt + c_1)^2} + \frac{{c_2}^2}{2(c_2 t + c_3)^2} - \frac{2q}{3(qt + c_1)} - \frac{c_2}{3(c_2 t + c_3)} + \frac{1}{6} \tag{22}$$

Here it is clear that as't' gradually increases, the shear scalar σ^2 decreases and finally they vanish when t →∞. Since

$$\frac{\sigma}{\theta} \neq 0 \ ,$$

(23)

The anisotropic nature of the model is maintained throughout.

IV. Conclusion

Cosmologists believe in inflation even though there is no experimental evidence of its Existence. This is so because no other mechanism resolves the problem of FRW cosmology so well. The inflationary models propounded by Guth [27] resolve the flatness and horizon problems of the standard cosmological model. In this paper we have presented a spatially homogeneous Bianchi type-VI_0 string cosmological model which is expanding, shearing, no rotating, anisotropy and has no initial singularity. Here it is interesting to note that string tension density (λ) is greater than half of the rest energy density and particle density (ρ_p) is less than half of the rest energy density. It is also observed that λ, ρ and ρ_p are decrease with growth of cosmic time. When t→∞, cosmic string vanishes and the model leads to a vacuum universe.

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