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Determination Of Threshold Density Of Some Astrophysical Bodies To Reduce To Schwarzschild Black Hole.

Ishiyaku Ibrahim Babayo, Ahmadu Muhammad Aliyu,
Hamza Abubakar Hamza, Aliyu Sisa Aminu, Aminu Muhammad
Department Of Pure And Applied Physics, Faculty Of Science, Gombe State University, Gombe State Nigeria

Abstract
Gravitation, electromagnetism, and nuclear interactions constitute the fundamental forces governing physical phenomena in nature. Among these forces, gravitation exhibits a unique and universal property which is the function of density. In this work threshold density of some astrophysical bodies to reduce to black hole was determined. It was found that Jupiter has a lower threshold density of while Mercury has higher threshold density of both of the mean density of order 10^2-10^3 kgm^{-3} which means Jupiter has less mass to size ratio and gravity is less making it harder for anything to escape when it reduce to Schwarzschild black hole compared to other planets. in the case of moons, Earth moon (Luna) has a lower threshold density of while Phobos has higher threshold density of .which means lunar has less mass to size ratio and gravity is less, making it harder for thing to escape when it reduces to Schwarzschild's black hole compared to other moons. In the case of Asteroids Psyche has higher threshold density of while ceres lower threshold density of .Which means Ceres has less mass to size ratio and gravity is less, making it harder for thing to escape when it reduces to Schwarzschild black hole compared to asteroid. In general the results obtained show that most of the astrophysical bodies are gravitationally stable

***Keywords**: Astrophysical bodies, electromagnetism, Gravitation, nuclear interactions, Schwarz child black hole*

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I. Introduction

The nature of gravitation and its role in shaping the structure and evolution of the universe has remained a central theme in physics for centuries (Adams, 1886). The gravitational force brings together any two objects that have mass. This force is called 'attractive' because it consistently seeks to draw masses closer together rather than pushing them apart. Every object, including ourselves, exerts a pull on all other objects throughout the universe (Tippler & Llewellyn.,2012), (Kerr, 1963).

From the classical formulation of gravitational attraction by Isaac Newton to the revolutionary geometric interpretation introduced by Albert Einstein, our understanding of gravity has undergone profound transformation (Einstein, 1916), (Carroll 2004). Newtonian gravity successfully explains a wide range of phenomena, from planetary motion to terrestrial mechanics, through the inverse-square law of attraction. However, it fails to accurately describe extreme gravitational environments where spacetime curvature becomes significant. This limitation led to the development of General Relativity, which provides a more complete framework for understanding gravitational phenomena, particularly in regimes involving very high mass densities and strong gravitational fields (Einstein, 1916; Misner *et al*., 1973).

Several studies have been made to understand astrophysical bodies' example, the work of Herbert and Ahmadu (2024). That Study of Radiation emission by some Astrophysical bodies using the laws of black Body radiation. Ahmadu and Sambo,(2021) that Determine the Schwarzschild's Radius of some Planetary Bodies in the Solar System Using Newtonian Mechanics

The study of threshold density is therefore essential for bridging the gap between classical astrophysical objects and relativistic compact objects. Although planets, moons, and asteroids are far from the conditions required for natural black hole formation, calculating their threshold densities allows for a deeper understanding of how far these objects are from gravitational collapse and highlights the role of geometric constraints in determining their stability.

Despite its importance, the concept of threshold density is often underrepresented in standard treatments of black hole physics, which tend to emphasize dynamical collapse processes rather than static conditions for collapse. As a result, there is a need for systematic studies that explicitly quantify the relationship between size and density for a variety of astrophysical bodies. Such analyses not only enhance conceptual understanding but also provide a basis for further theoretical and computational investigations.

The present study here aims to address this gap by determining the threshold density required for selected astrophysical bodies including planets, moons, and asteroids to collapse into Schwarzschild black holes. By employing analytical methods grounded in both Newtonian mechanics and General Relativity, the study derives a general expression for critical density and applies it to a range of celestial objects using observed physical parameters. The results are used to establish trends and comparisons that highlight the dependence of gravitational collapse on size and geometry.

In summary, this work seeks to deepen our understanding of gravitational collapse by focusing on the fundamental role of density in black hole formation. By extending the analysis beyond stellar objects to include a broader class of astrophysical bodies, it provides a comprehensive perspective on the conditions required for the formation of Schwarzschild black holes and reinforces the predictive power of General Relativity in describing extreme physical phenomena.

II. Methodology

To determine the Critical Density () the density at which an object of radius R would become a black hole we assume the object is compressed until its actual radius equals its Schwarzschild radius

Determination of Volume and Density of Planetary Bodies

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Determination of Volume and Density of Planetary Bodies

We can calculate a planet's volume from its radius using the formula

. (1)

Where is radius of the body.

By doing so we assume the planet is spherical.

The physical density of any object is simply its mass divided by its volume; density is measured in units such as pounds per cubic foot, grams per cubic centimeter or kilograms per cubic meter. When calculating the density of a planet, look up its mass and radius, the latter of which is the distance from the surface to the center. Because planets are roughly spherical, calculate the volume of a sphere using the radius. Then divide the mass by the volume of the sphere to get the density.

The density of a planet (or anything else) is simply calculated as

(2)

Density = (), M= Mass of the planet, V= Volume of the planet.

Determination of Schwarzschild's Radius

The distance from the centre of a non-rotating black hole to the event horizon is known as the Schwarzschild radius. This is like taking a ruler and measuring the distance from the dark well's edge to its centre. The Schwarzschild radius depends only on the mass of the object that creates the black hole (Hobson *et.al*. 2006)

We can derived the same result by considering how the escape velocity from the surface of the star depend upon the size of the star.

Newtonian expression for escape velocity is given by

(3)

Where M is the mass of star with radius R If light is retarded by gravity it will be just fail to escape from the surface. When the escape velocity has the same value as .

Therefore

(4)

The above equation can be re-arrange to give the radius which for a given mass must be compressed to form a black hole.

(5)

Where is called a schwarzschild's radius. Substituting value of constant parameters

Then

(m/kg) (6)

And

(7)

And by substituting equation (7) in (5)

(8)

To get the Threshold Density ( ), the reduction ratio must be equals to one

Threshold Condition: Setting = 1, we solve for

From equation (8) we have

(9)

From equation above, for a black hole to be formed

Which Implies

Therefore equation (9) becomes

(10)

By substituting the value of $c=3\text{x}10^{8}\text{ms}^{-1}$, $G=6.67\text{x}10^{-11}\text{Nm}^{2}\text{kg}^{-2}$ and into the above,

The above equation it becomes

(11)

The above equation express the relation between threshold density and inverse-square radius dependance

(15)

III. Result And Discussion

Results

The threshold density for any astrophysical body (Planets, Asteroids and Moons) can be computed using equation (11)

Table 1: show the critical density of planets in the solar system.

Table 1 Threshold density () for planets in the solar system.

Planet	Radius (km)	Threshold density () (kgm^{-3})
Mercury	2439	
Venus	6052	
Earth	6378	
Mars	3396	
Jupiter	69911	
Saturn	60268	
Uranus	25559	
Neptune	2476	
Pluto (dwarf planet)	1185	

Table 2: Threshold density for Moons of Earth and Mars

Name of planet	Name of moon	Radius (km)	Threshold density () **(kgm^{-3})**
Earth	lunar	1738	
Mars	Phobos	5.55	
	Deimos	6.25	

Table 3: Threshold density for Moons of Jupiter

Name of moon	Radius (km)	Threshold density () (kgm^{-3})
Metis	20	
Andrastea	10	
Amalthe	86	
Thebe	50	
IO	1818	
Eropah	174	
Leda	8	
Ganymede	2634	
Sinope	14	
Himalia	85	
Lysithea	12	
Elera	40	
Ananka	10	
Pasphae	18	

Table 4: Threshold density for Moons of Saturn

Name of moon	Radius (km)	Threshold density () (kgm^{-3})
Epimetheus	60	
Janus	90	
Mimas	199	
Encaladus	249	

Table 5: Threshold density for Moons of Uranus

Name of moon	Radius (km)	Threshold density () (kgm^{-3})
Coradeli	13	
Ophelia	16	
Bianca	22	
Cresida	33	
Desdemona	29	
Juliet	42	
Portia	55	
Rosalind	27	
Belinda	29	

Table 6: Threshold density for Moons of Neptune

Name o moon	Radius (km)	Threshold density () (kgm^{-3})
Thalassa	40	
Naiad	29	
Despina	74	
Galatea	79	
Lraissa	193	
Proteus	105	
Triton	1352	

Table 7: Threshold density for Moons of Pluto

Name of moon	Radius (km)	Threshold density () (kgm^{-3})
Charon	586	
Nix	225	
Hydra	80	

ASTEROIDS

Table 8: Show the Threshold density for Asteroids in solar system.

Table 8: Threshold density for Asteroids in solar system

Name of asteroid	Radius (km)	Threshold density () (kgm^{-3})
Ceres	476	
Vesta	264.5	
Pallas	272	
Hygiea	265.5	
Remnia	163	
Davida	143	
Eunimia	134	
Juna	129	
Psyche	93	

IV. Discussion

Planetary Variation. The mean density range for all the planets is within 10^2-10^3 kgm^{-3}. Larger planets like Jupiter have a lower critical density () compared to smaller planets like Mercury (kgm^{-3}). This is because the larger initial radius of Jupiter makes the "gap" to its Schwarzschild state relatively easier to bridge in terms of density, compared to a small, compact body. Earth and Venus has value of order kgm^{-3} while Uranus has a critical density of order kgm^{-3}. Which means for all planets, the threshold density is astronomically larger than their actual density and that all planets in the solar system have densities many orders of magnitude lower than the critical threshold required for Schwarzschild collapse. The required increase (10^{13}–10^{1} times) is physically unattainable under natural conditions, which confirm that Planetary bodies are gravitationally stable and the inverse-square dependence of threshold density on radius strongly penalizes smaller bodies.

Moons: almost all the moons are in the range of mean density of 500 kgm^{-3}-3500 kgm^{-3}. The Earth's moon (Luna) requires a lower density to become a black hole () than the moons of Mars, such as Phobos () confirms that smaller bodies like Phobos are much further from becoming black holes than the larger Moon.

Metis,andrastea,amalthe,thebe,leda,sinope,himalia,lysithea,elera,ananka,pasphae are moons of Jupiter with the power of order and above while Eropah is having the average value with . IO and Ganymede has an order of . The least threshold density with the power of order .

Moons of Saturn (Epimetheus, Janus) are in same range of threshold density, while Mimas and Encaladus are also same range. This shows that the moons may attain equal threshold density to form Schwarzschild black hole.

Neptune with largest moon among the planet and even bigger than Pluto (Triton) and all rest have the critical density of order and above

In Asteroids, Ceres has less critical density followed by Pellas. Others have almost the same range of critical density.

It was found that among all astrophysical bodies in our solar system Jupiter has a lower threshold density while Phobos has higher threshold density. Which means Jupiter has less mass to ratio and gravity is less making it harder for things to escape when it reduce to Schwarzschild black hole compare to other astrophysical bodies.

V. Conclusion

The threshold density for some astrophysical bodies in the solar system were calculated where we avoid complex mathematical difficulty of General relativity. The relation was obtained for non-rotating, spherically symmetric bodies. Angular momentum, electric charge, and external gravitational field effects are ignored. The calculated results shows that not only the star can collapse to form a black hole rather anybody can form a black hole by attaining its threshold density it also shows that transformation of any astrophysical body in the solar system is strongly depend only on its radius.

In general none of these Astrophysical bodies that it critical density become closer to it mean density which confirm that they are gravitationally stable

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