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A Quaternary Wave Theory Of Electromagnetic Fields: A Theory On Propagation Of Electromagnetic Fields In Vacuum

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Abstract:
There are still some unclarified points in physics such as propagation of electromagnetic waves in free medium (vacuum), negative energy states emerging from Dirac's relativistic solution of particle position and initial formation of universe, which was hypothesized by Hawking as formation from zero (nothing) as absolutely equal positive and negative energy mediums. In this paper, by using Poynting theorem and a new vector calculus conditional identity, energy flow in a simple electromagnetic energy conversion system will be defined in an alternative equal form, where, in this alternative form two new fields can possibly be defined. Later, new fields will be described in the form of modified Maxwell's equations and it will be shown that in this modified form of electromagnetic theory, the wave behaviour of electromagnetic fields become consistent with the fundamental mechanical wave concept of simple harmonic motion such that in this case the electromagnetic energy oscillates between positive and negative energy during propagation in vacuum. The concept of negative energy, emerging in the electromagnetic waves, and the possible related research areas are evaluated finally.

Keywords: *Maxwell's equations, electromagnetic waves, time-harmonic fields, propagation in vacuum*

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I.Introduction

Similar to mechanical waves electromagnetic waves transport and deliver energy but unlike mechanical waves electromagnetic waves do not require a physical medium to propagate, they also propagate well in vacuum. In the end of 1800's it was considered that a medium in vacuum should exist for the propagation of electromagnetic waves, and it was hypothesized as "ether". However, it has never been observed experimentally, so far. On the other hand, vacuum, which is actually nothing, can be considered as a medium according to Hawking's hypothesis of the formation of the universe. In [1], Hawking assumes that nothingness (zero) can be decomposed into two forms of energy that cancel each other out, positive and negative energy. In this idea, positive energy corresponds to the matter of the universe and negative energy exists in the space surrounding the matter. Although there can be different interpretations of what Hawking actually meant according to different advanced physics concepts, the simplest interpretation of the idea, as done above, seems to be the most acceptable scientific explanation for the initial formation of matter for an ordinary people. Otherwise, the only scientific explanation that matter has existed forever would make the subject unconceivable in terms of eternity of time.

The concept of negative energy had previously appeared in relativistic quantum mechanics with the equation formulated by Dirac in 1928. In addition to positive energy solutions, the equation has also negative energy solutions, corresponding to negative energy particles that are not defined in classical physics. Since its initial formulation, there have been several interpretations of negative energy solutions; from Dirac's hole theory suggesting the particle-antiparticle duality to Feynman-Stuckelberg interpretation of negative energy particles, propagating backward in time. However, none of them has been fully proven so far. Recently, there have been modern interpretations that proposes particles propagating backwards in space with negative energy [2]. In fact, Hawking's negative energy hypothesis and results of Dirac Equation are related such that if there is a particle with positive energy there should be a particle with complementary negative energy. What makes the Dirac equation interesting is that it may describe the motion of positive and negative energy particles relative to each other as commented in [2]. However, these are all advanced quantum physics subjects and the details are left to the physician readers.

In this paper, a quaternary wave structure for electromagnetic fields is proposed, which can provide a scientific explanation for the propagation of electromagnetic fields in vacuum in terms of positive-negative energy approximation of vacuum. In this context, in addition to the classical electric and magnetic fields two additional fields that propagate together with electric and magnetic fields are defined. The definition of new fields is based on a new vector calculus identity, providing an alternative equivalent expression for electromagnetic energy flow. With Maxwell's equations modified accordingly, it can then be shown that the new fields can be attributed to the negative energy medium of the vacuum, and the whole behaviour of the electromagnetic waves becomes oscillatory between positive and negative energy. This is analogous to the simple harmonic motion of mechanical waves where energy oscillates between potential and kinetic energy while total energy remains constant. The rest of the paper is configured as follows, in Section 2 the classical electromagnetic theory and energy flow with electromagnetic fields, described by Poynting's theorem, will be reviewed. Then, in Section 3, the energy flow analysis of electromagnetic fields will be applied to a simple electromechanical system. At this point, using a conditional vector calculus identity, it will be shown that energy flow with electromagnetic fields can be defined in an alternative form and accordingly two additional fields will be defined. In Section 4, Maxwell's equations will be reconfigured according to the newly defined fields and modified equations will be applied to time harmonic fields to obtain the behaviour of new fields in the vacuum. Finally the results are discussed in Section 5.

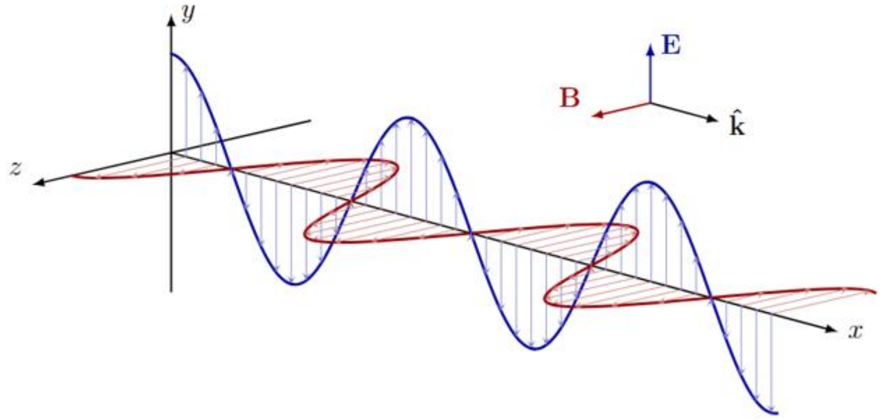

FIGURE 1. A snapshot of electromagnetic waves at $t = 0$.

II.Review Of Classical Electromagnetic Field Theory

The classical electromagnetic theory is formulated in the unified form by Maxwell's four equations. In the form as applied to free space, they can be presented as follows,

$$\nabla.\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad (1)$$

$$\nabla.\mathbf{B} = 0 \qquad (2)$$

$$\nabla\text{x}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad (3)$$

$$\nabla\text{x}\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} \qquad (4)$$

where, **E** and **B** are electric field intensity and magnetic field density vectors, and μ_0 and ϵ_0 are permeability and permittivity of free space, respectively . In free space, where $\rho = 0$ and $\mathbf{J} = 0$, (3) and (4) result in general travelling wave equations for **E** and **B**,

$$\nabla\text{x}(\nabla\text{x}\mathbf{E}) = -\mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} \qquad (5)$$

$$\nabla\text{x}(\nabla\text{x}\mathbf{B}) = -\mu_0\epsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2} \qquad (6)$$

For simplicity, if one dimensional electromagnetic waves, propagating in +*x* direction, are considered, (5) and (6) take the second order partial differential equations form as below,

$$\frac{\partial^2\mathbf{E}}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} \qquad (7)$$

$$\frac{\partial^2\mathbf{B}}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2} \qquad (8)$$

The simplest solutions to (7) and (8) are sinusoidal (time-harmonic) travelling waves that are functions of *x* and *t*, as follows,

$$\mathbf{E} = E_m \cos(kx - \omega t)\mathbf{a_y} \qquad (9)$$

$$\mathbf{B} = B_m \cos(kx - \omega t)\mathbf{a_z} \qquad (10)$$

where,

$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad (11)$$

and also,

$$\frac{E_m}{B_m} = c \qquad (12)$$

In (9) and (10), vector notation is used to show that **E** and **B** oscillate perpendicularly. And it should also be noted that *E* and *B* are in phase. A snapshot of the electromagnetic waves, given in (9) and (10), are shown in Figure 1, where $\hat{\mathbf{k}}$ denotes the direction of the propagation of the waves. The instantaneous electric and magnetic energy densities at any point are expressed as follows,

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{\epsilon_0}{2} E_m^2 cos^2(kx - \omega t) \qquad (13)$$

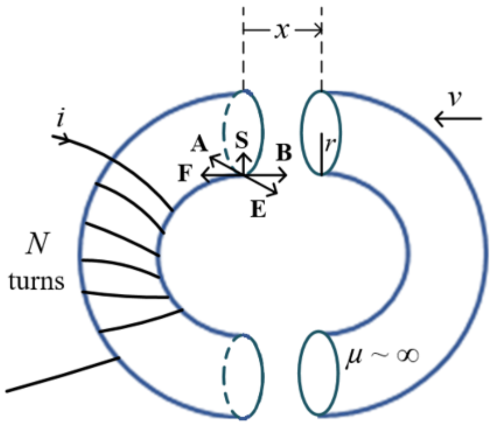

FIGURE 2. A simple electromechanical system with a stationary coil part and a moving iron part.

$$u_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0} B_m^2 cos^2(kx - \omega t) \qquad (14)$$

As seen from (13) and (14), two energy forms of the electromagnetic waves are also in phase, meaning they change simultaneously without any energy exchange between them. The Poynting vector gives rate of change (power) and direction of total electromagnetic energy flow per unit area, perpendicular to the direction of wave motion, as follows,

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\mathrm{x}\mathbf{B} = \frac{1}{\mu_0} E_m B_m cos^2(kx - \omega t)\mathbf{a_x} \qquad (15)$$

Equation (15) tells us that according to the classical electromagnetic wave theory, the instantaneous total rate of change of the energy of electromagnetic waves passing through per unit plane surface oscillates between zero and a maximum. This form of the classical theory explains the apparent face of the related natural events successfully. But what if different phenomena also happen in the unperceived face of the nature in conjunction with the apparent ones. In the rest of this paper, an alternative quaternary wave theory will be presented for the electromagnetic waves. In the following section, by using Poynting theorem, the energy flow in a simple electromechanical system will be analysed. After that, by using a novel vector calculus identity, it will be shown that the energy flow can also be described in an alternative form with complementarily defined two new fields. Then, with these new fields and corresponding carried energy, the total instantaneous energy of the electromagnetic waves will be shown to be oscillating between two forms, positive and negative energy.

III.Energy Flow Analysis In A Simple Electromechanical System

Poynting theorem is frequently used to analyse energy flow analysis of electromagnetic waves in electromagnetic field theory but finds very few applications in other fields, an example can be given as energy flow analysis in a conducting wire [3, p. 300-301]. Especially, it is not applicable to the energy flow analysis of electrical machines, this is because most of them operate based on Lorentz's force, and it is hard to identify electric and magnetic field regions. However, Poynting theorem can be used for an electromechanical system that is based on reluctance force rather than Lorentz's force. Such a simple system is shown in Figure 2. In Figure 2, a toroidal iron core is divided into two pieces and while one piece with an *N* turn current carrying coil is kept stationary, the other piece is free to move with a speed of *v* at a distance *x* at any time instant. The force on the moving iron part is totally due to the stationary electromagnet part. The coil current can be assumed to be constant. For the sake of simplicity, the permeability of the iron will be assumed infinite, therefore all the electromagnetic energy will be assumed to be contained in the airgap section between the iron parts. The fringing and leakage flux will also be neglected as they do not affect the intent of the energy flow analysis. For the analysis, the system will be described with only one airgap section using cylindrical coordinate system. However, effect of the other airgap section will be included in the equations equivalently. Then, for the current direction, seen in Figure 2, the corresponding magnetic flux density vector, **B** at a point on the periphery of the upper airgap section, becomes as in Figure 2. Taking this direction as +*z* and assuming uniform flux density in the airgap, the airgap flux density field can be described as follows,

$$\mathbf{B} = \frac{Ni}{2RA}\,\mathbf{a_z} \qquad (16)$$

where, A is crossectional area and R is reluctance of the single airgap, defined as follows,

$$R = \frac{x}{\mu_0 A} \qquad (17)$$

where, *x* is length of the airgap sections. Then, airgap flux density becomes,

$$\mathbf{B} = \frac{Ni\mu_0}{2x}\,\mathbf{a_z} \qquad (18)$$

With the uniform airgap flux density assumption, this flux density is the same everywhere in the airgaps including airgap periphery. The corresponding magnetic flux intensity, **H** vector, from $B = \mu H$, is,

$$\mathbf{H} = \frac{Ni}{2x}\,\mathbf{a_z} \qquad (19)$$

It should be noted that the magnetic field intensity vector defined in (19) is valid only in the airgap sections. In the core sections it is zero due to infinite permeability. On the other hand, because of the motion of the moving iron part the flux density in the air gaps is not constant, leading to the induction of electric field within the airgaps according to Faraday's law given by (3). Electric field intensity vector, **E**, in any point in the airgaps can be found by using (3). But, since we are dealing with the power flow analysis using Poynting's Theorem, the electric field intensity vector at the periphery of the airgaps is enough for us. This can be obtained by applying Stoke's Theorem to (3) at the periphery of the upper airgap, shown in Figure 2, as follows,

$$El = \frac{-\partial B}{\partial t}A \qquad (20)$$

where, *l* is peripheral length and A is crossectional area of the airgap sections, respectively, defined as follows,

$$l = 2\pi r \text{ and } A = \pi r^2 \qquad (21)$$

Then, the peripheral electric field intensity vector equation becomes,

$$\mathbf{E} = \frac{Ni\mu_0 r}{4x^2}\,v\,(-\mathbf{a}_\phi) \qquad (22)$$

Note that, in writing (22), Chain Rule has been applied in the form of,

$$\frac{\partial B}{\partial t} = \frac{\partial B}{\partial x}\frac{\partial x}{\partial t} \qquad (23)$$

where,

$\frac{\partial x}{\partial t} = -v$ (24)

v is the speed of the moving iron with the direction shown in Figure 2.

The direction of the electric field intensity vector can also be found with right-hand-rule and Lenz's law. Then, the Poynting vector, which is power flow per unit airgap surface area can be found as below,

$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\mathrm{x}\mathbf{B} = \mathbf{E}\mathrm{x}\mathbf{H} = \frac{N^2 i^2 \mu_0 r}{8x^3} v\ (-\mathbf{a}_\rho)$ (25)

The direction of power flow is into the airgap as expected (see Figure 2). Total power input to the system can be found by multiplying Poynting vector magnitude with the airgap side surface area (twice of it as there exist two airgaps), as follows,

$P_{in} = 2\int (ExH)dS = 2.2\pi r x \,.\frac{N^2 i^2 \mu_0 r}{8x^3} v = \frac{N^2 i^2 \mu_0 \pi r^2}{2x^2} v$ (26)

Note that the power expression in (26) is the total power input to the system including magnetic stored energy and mechanical power. For linear systems without magnetic saturation they are equal to each other and so the mechanical input power becomes half of the power expression in (26), as follows,

$P_m = \frac{N^2 i^2 \mu_0 \pi r^2}{4x^2} v$ (27)

In mechanical systems power is expressed as force times speed, therefore the electromagnetic force on the moving part is expressed as,

$f_{em} = \frac{N^2 i^2 \mu_0 \pi r^2}{4x^2}$ (28)

The force expression in (28) would be the same as that could be obtained using conventional energy-coenergy method described in [4, p.112]. Note also that the Poynting method can also be applied to the N-turn coil as in [3], resulting in a power vector that is directed away from the coil surface by the amount given in (26), confirming the conservation of energy. However, what we are interested in is description of energy flow in an alternative way. This can be achieved by using Poynting expression and vector potential notation for magnetic field density vector, as follows,

$\mathbf{B} = (\nabla \mathrm{x} \mathbf{A})$ (29)

Then,
$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\mathrm{x}\mathbf{B} = \frac{1}{\mu_0}\mathbf{E}\mathrm{x}(\nabla \mathrm{x}\mathbf{A})$ (30)

where, **A** is vector potential of magnetic field density vector, **B**, which is conventionally named as vector magnetic potential in the literature. At this point, the following vector calculus theorem will enable us the required identity to write the equivalent alternative form of the electromagnetic energy flow expression.

Theorem (A conditional vector calculus identity): Let **U** and **V** be vector fields defined over all R^3.

If **U**//**V**, then $\mathbf{U}\mathrm{x}(\nabla \mathrm{x}\mathbf{V}) = \mathbf{V}\mathrm{x}(\nabla \mathrm{x}\mathbf{U})$ (31)

It should be noted that the required condition for the given identity is **U**//**V**.

Proof: For simplicity Cartesian coordinates will be used. In Cartesian coordinates vectors **U** and **V** are defined as follows,

$\mathbf{U} = U_x\mathbf{a_x} + U_y\mathbf{a_y} + U_z\mathbf{a_z}$ and $\mathbf{V} = V_x\mathbf{a_x} + V_y\mathbf{a_y} + V_z\mathbf{a_z}$ (32)

If two vectors are parallel their cross product is zero. Then, from definition of the cross product following equalities can be written,

$$U_x V_y = U_y V_x \quad (33)$$

$$U_y V_z = U_z V_y \quad (34)$$

$$U_z V_x = U_x V_z \quad (35)$$

Besides, a well-known vector calculus identity given below provides us the other required equalities [5, p. 735],

$$\nabla.(\mathbf{U}\mathrm{x}\mathbf{V}) = \mathbf{V}.(\nabla\mathrm{x}\mathbf{U}) - \mathbf{U}.(\nabla\mathrm{x}\mathbf{V}) \quad (36)$$

Since the cross product **U**x**V** is zero,

$$\mathbf{V}.(\nabla\mathrm{x}\mathbf{U}) = \mathbf{U}.(\nabla\mathrm{x}\mathbf{V}) \quad (37)$$

Leading to,

$$V_x\left(\frac{\partial U_z}{\partial y} - \frac{\partial U_y}{\partial z}\right) = U_x\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) \quad (38)$$

$$V_y\left(\frac{\partial U_x}{\partial z} - \frac{\partial U_z}{\partial x}\right) = U_y\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) \quad (39)$$

$$V_z\left(\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right) = U_z\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) \quad (40)$$

Now we can examine $\mathbf{U}\mathrm{x}(\nabla\mathrm{x}\mathbf{V})$. Not to make equation crowd, let's have a look at only x-axis component, which is written as follows,

$$\left[U_y\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) - U_z\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\right]\mathbf{a_x} \quad (41)$$

From (40) and (39) obtain $\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$ and $\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)$ respectively and substitute in (41), resulting in,

$$\left[\frac{U_y V_z}{U_z}\left(\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right) - \frac{U_z V_y}{U_y}\left(\frac{\partial U_x}{\partial z} - \frac{\partial U_z}{\partial x}\right)\right]\mathbf{a_x} \quad (42)$$

And finally from (34), (42) reduces to,

$$\left[V_y\left(\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right) - V_z\left(\frac{\partial U_x}{\partial z} - \frac{\partial U_z}{\partial x}\right)\right]\mathbf{a_x} \quad (43)$$

Note that, (43) is the x-axis component of $\mathbf{V}\mathrm{x}(\nabla\mathrm{x}\mathbf{U})$. Similarly, it can be shown that the y-axis and z-axis components of $\mathbf{U}\mathrm{x}(\nabla\mathrm{x}\mathbf{V})$ and $\mathbf{V}\mathrm{x}(\nabla\mathrm{x}\mathbf{U})$ are equal to each other.

Now, let's continue with the examined electromechanical system. From (3) and (29), **E** and **A** can be related as follows,

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \quad (44)$$

Then, either from (22) and (44) or (18) and (29), **A** can be obtained as follows,

$$\mathbf{A} = \frac{Nir}{4x}\mathbf{a}_\phi \quad (45)$$

Comparing (22) and (45) it can be seen that **E** and **A** are parallel to each other. Then, using the above theorem, the energy flow into the system can be written in the equivalent alternative form as follows,

$\mathbf{S} = \frac{1}{\mu_0}\mathbf{A}\text{x}(\nabla\text{x}\mathbf{E})$ (46)

Now, let's define a fourth field, **F**, as follows (note that the third one is **A**),

$\mathbf{F} = (\nabla\text{x}\mathbf{E})$ (47)

Here, **F** should not be confused with the vector electric potential, defined in the literature as follows,

$\mathbf{E} = (\nabla\text{x}\mathbf{F})$ (48)

Then, from (3) and (47) it is apparent that

$\mathbf{F} = -\frac{\partial \mathbf{B}}{\partial t}$ (49)

In the examined electromechanical system **F** can be determined either using (22) and (48) or (18) and (49), as follows,

$\mathbf{F} = \frac{Ni\mu_0}{2x^2}v(-\mathbf{a_z})$ (50)

Now, from (25), (45) and (50) it can be shown that

$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\text{x}\mathbf{B} = \frac{1}{\mu_0}\mathbf{A}\text{x}\mathbf{F}$ (51)

In this section, we defined two vector fields, **A** and **F** that carry the same energy as the conventional **E** and **H** fields. The physical meanings of these fields can be interpreted better when Maxwell's governing equations are rearranged by taking into account the new fields, and the fields are defined as time-harmonic fields propagating in vacuum. In the following section, Maxwell's governing equations will be rearranged to take into account the newly defined fields, and it will be shown that the new fields propagate in a similar and complementary manner to the traditional fields.

IV.The Quaternary Wave Theory Of Electromagnetic Fields

To be able to determine source of a divergenceless field we need a curl expression of that field. For the vector magnetic potential, **A**, (29) satisfies the required equality, meaning that **A** is induced by the magnetic field density, **B** solely. In the literature vector magnetic potential is treated as a purely mathematical, imaginary quantity, used to simplify magnetic flux density/intensity field calculations [3]. However, a phenomenon, known as Aharonov-Bohm effect, showed that an electrically charged particle is effected by the vector magnetic potential [6, p. 273-277]. This phenomenon has given rise to the interpretations that the vector magnetic potential can be a real physical field like conventional electric-magnetic fields. On the other hand, the curl expression in (47) indicates that the electric field intensity is induced by the new field, **F**. This may seem to contradict Faraday's law. However, it can be seen that this is not the case when **F** is identified with its source. This can be done by taking curl of both side of (47) and substituting (3) and (4), resulting in,

$\nabla\text{x}\mathbf{F} = \frac{-\partial(\nabla\text{x}\mathbf{B})}{\partial t}$ (52)

$\nabla\text{x}\mathbf{F} = -\left(\mu_0\frac{\partial \mathbf{J}}{\partial t} + \mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}\right)$ (53)

Eqn. (53) states that the field **F** is induced by time varying currents (accelerating charges). It should be noted that from Ampere's law in (4), time varying currents also induces time varying magnetic fields, meaning that as a consequence Faraday's law is still valid. However, according to the quaternary wave approach, what actually induces electric fields is not time varying magnetic fields directly, but it is a two-step process dictated by (53) and (47), such that time varying currents induce **F** and then **F** induces electric fields. The resulting governing equations of the quaternary field theory can be summarized as follows,

$$\begin{cases} \nabla \mathrm{x}\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t} \ (1) \\ \mathbf{B} = (\nabla \mathrm{x}\mathbf{A}) \ (2) \\ \nabla \mathrm{x}\mathbf{F} = -\left(\mu_0\frac{\partial \mathbf{J}}{\partial t} + \mu_0\epsilon_0\frac{\partial^2 \mathbf{E}}{\partial t^2}\right) \ (3) \\ \mathbf{F} = (\nabla \mathrm{x}\mathbf{E})(4) \end{cases} \quad (54)$$

Now we can make an initial interpretation about the physical meanings of fields **A** and **F** in terms of negative energy concept, as follows; The universe was initially formed from vacuum as positive and negative energy matters with equal but opposite amounts (in terms of sign and mass). The behaviour of charged positive and negative energy matter is interconnected, expressed by the governing equations in (54). This connection can be described as follows; moving positive energy (PE) charged particles (currents) induce positive energy magnetic field, **B** as in (54-1). As known, PE magnetic field governs the behaviour of moving PE charged particles w.r.t. each other. But, on the other hand, PE magnetic field induce field **A,** vector magnetic potential. Field **A** can be interpreted as negative energy (NE) electric field that governs the behaviour of negative energy particles accordingly to PE ones. In this respect, for example for the system given in Figure 2, the PE magnetic field density, B corresponds to magnetic dipoles in the flux section and so the induced NE electric field **A** governs the corresponding NE particles accordingly. At this moment two points should be paid attention; firstly, the field **A** is distributed through the space, so the NE matter may also be distributed through the space. Secondly, as seen from (45) and direction of current in Figure 2, the PE dipoles and NE electric field **A** are in the same direction. However, since the NE particles are expected to be with opposite charge, it can be interpreted that NE particles rotate in opposite direction w.r.t. PE ones.

On the other hand, acceleration has additional effect, accelerated PE particles (time varying currents) induce the field **F** as in (54-3). Field **F** can be interpreted as NE induced magnetic field. And finally, the field **F** induces PE electric field, **E** as in (54-4). Eqns. (54-3) and (54-4) describe the Farady's law in a two-step process, taking into account the NE medium in addition to PE.

Now, let's investigate behaviour of the new fields in vacuum. In vacuum all waves are divergenceless and as noted before $\mathbf{J} = 0$. Then, the wave equations of **A** and **F** can be obtained in a similar manner to that of **E** and **B**. Taking curl of both side of (29) results in,

$$\nabla \mathrm{x}\mathbf{B} = \nabla \mathrm{x}\nabla \mathrm{x}\mathbf{A} \quad (55)$$

Then, from Ampere's law in (54-1),

$$\nabla \mathrm{x}\nabla \mathrm{x}\mathbf{A} = \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t} \quad (56)$$

And, substituting (44) for **E** into (56),

$$\nabla \mathrm{x}\nabla \mathrm{x}\mathbf{A} = -\mu_0\epsilon_0\frac{\partial^2 \mathbf{A}}{\partial t^2} \quad (57)$$

Eqn. (57) is the equation of a wave travelling at the speed of light. Similarly, taking curl of both side of (54-3) and substituting (47), it can be shown that **F** has a travelling wave equation of the same form as others, as follows,

$$\nabla \mathrm{x}\nabla \mathrm{x}\mathbf{F} = -\mu_0\epsilon_0\frac{\partial^2 \mathbf{F}}{\partial t^2} \quad (58)$$

Also, what makes quaternary wave theory interesting is the similarity of electromagnetic waves to mechanical waves. In Simple Harmonic Motion, which is the basis of all mechanical waves, the energy of the system oscillates between potential and kinetic energies while the total energy is constant. And the two wave parameters, displacement, *x,* which is associated with potential energy and speed, *v,* which is associated with kinetic energy, are related to each other in differential form as follows [7, p. 413-422],

$$v(t) = \frac{dx(t)}{dt} \quad (59)$$

To see the similarity of electromagnetic waves with mechanical waves, let's define wave parameter pairs using time-derivative relations of the wave elements, **E**, **B**, **A** and **F**, given in (44) and (49), as summarized below,

$$\begin{cases}\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{F} = -\frac{\partial \mathbf{B}}{\partial t}\end{cases} \quad (60)$$

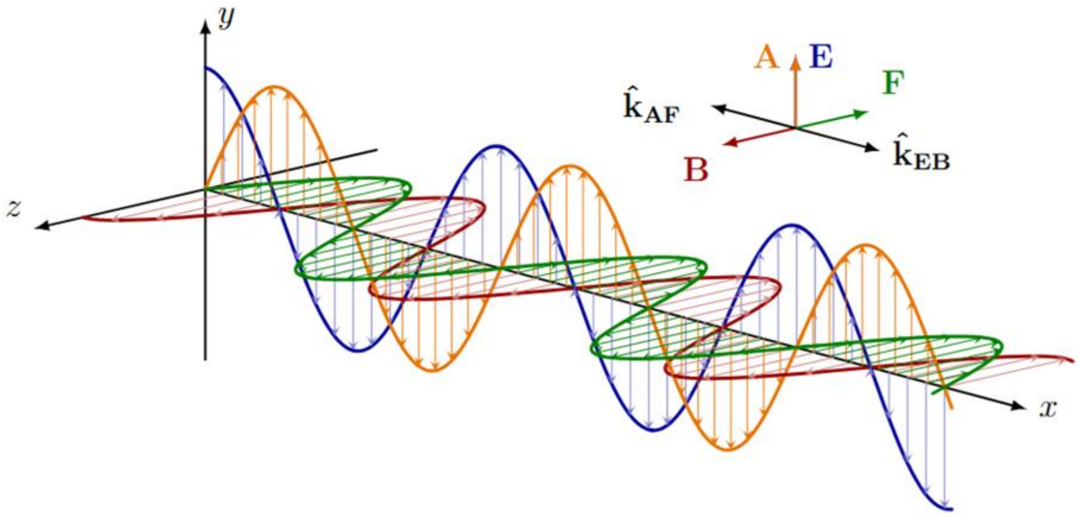

FIGURE 3. A snapshot of the quaternary electromagnetic waves at $t = 0$.

In correlation with the mechanical waves, (60) states that PE electric field, **E** and NE electric field, **A** form one wave parameter pair and NE magnetic field, **F** and PE magnetic field, **B** form the other. Using the wave equations of **E** and **B** in (9) and (10), wave equations of **A** and **F** can be obtained, using (60), as follows,

$$\boldsymbol{A} = \frac{E_m}{\omega} \sin\,(kx - \omega t)\mathbf{a_y} \quad (61)$$

$$\mathbf{F} = -\omega B_m \sin\,(kx - \omega t)\mathbf{a_z} \quad (62)$$

The Poynting vector in terms of **A** and **F** is then obtained as follows,

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{A}\text{x}\mathbf{F} = -\frac{1}{\mu_0} E_m B_m sin^2(kx - \omega t)\mathbf{a_x} \quad (63)$$

This is a vector travelling in +x direction with a negative energy. Note that the direction and absolute max value of the energy is the same as those of the energy associated with **E** and **B** fields, given in (15). Note also that this forward travelling negative energy can alternatively be interpreted as absolute negative energy propagating in backward (-x) direction. Figure 3 shows a snapshot of the quaternary form of the electromagnetic waves for backward propagation interpretation. If this negative energy is interpreted as a new form of energy (rather than having a mathematically negative sign), then comparing (63) and (15) it can be seen that the quaternary wave form of the electromagnetic fields has the same behaviour as the mechanical waves, where energy oscillates between two forms, positive and negative energy during propagation in vacuum.

V.Conclusions And Future Considerations

Propagation of electromagnetic fields in vacuum has been an observed phenomenon without a satisfactory explanation for decades. In this article, we tried to make an explanation using the concept of negative energy. In this context, a conditional vector calculus identity enabled the expression of electromagnetic energy flow in an alternative form where two new fields were defined. Energy flow analysis in this quaternary form of electromagnetic fields has shown that during the propagation in vacuum, while the conventional electric and magnetic fields carry a positive alternating energy the newly defined fields carry a negative energy in a complementary manner as in mechanical waves. The emerging negative energy concept immediately brings to mind the negative energy theories of Hawking and Dirac. In a manner in accordance with Hawking's hypothesis, it then can be interpreted that the negative energy carried by the new fields corresponds to the negative energy part of the vacuum and therefore the vacuum can be considered as the required medium for the electromagnetic waves where they propagate by exchanging the total energy between positive and negative energies. And also, the direction of the negative energy flow can be interpreted as consistent with the new interpretations of Dirac's equation. It is clear that the quaternary wave approach requires experimental verification to qualify as a theorem. In this context, the phenomenon of energy propagation in vacuum by electromagnetic waves can be considered a strong evidence for the theory, since it seems to be the only explanation.

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