

Coherence towards De-Coherence: Through Environment Monitoring

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Abstract:

While coherence is an intrinsic characteristic of quantum systems, de-coherence describes the environmental intrusion during the measurement process, leading to transition from quantum to classical reality, hence playing a pivotal role in quantum information processing. The authors emphasize the need to understand physical aspect of various elements of the density matrix of a quantum state and how these elements control the coherence/de-coherence behaviour of quantum physics and the transition to classical domain.

Key Word: Quantum states; Density matrix; Coherence; de-coherence; measurement; Environment

Date of Submission: 14-06-2025

Date of Acceptance: 29-06-2025

I. Introduction

The density matrix corresponding to a quantum system is completely deconstructed to interpret the correlation between the intangible (the mathematical representation) and the tangible (the actual physical system) quantities pertaining to the quantum and classical events. It is elaborated with suitable mathematics that a diagonal density matrix (with all the off-diagonal terms to be zero), represents a completely mixed quantum state. Whereas, the density matrix of a pure quantum state contains all the non-zero off-diagonal terms. Due to the measurement process, the off-diagonal terms either exponentially decay towards null values (T_2 relaxation of the quantum objects), or increase towards unity (T_1 relaxation of the quantum objects). This finds applications in MRI (magnetic Resonance Imaging).

II. Quantum World vs. Classical World

Generally a physics experiment is considered 'good' if it gives the results with minimum noise from the environment, e.g., friction, resistance, Brownian motion due to finite temperature, to name a few examples. Hence till the beginning of the twentieth century, the quantum physics experiments were performed in isolated systems (i.e., closed environment), e.g., to study a single atom and proving the laws of quantum mechanics, making it of little use in classical applications. Schrodinger's cat indeed resulted in a paradigm shift to now understand the quantum to classical transition (Schlosshauer 2007). The infamous "CAT PARADOX" initiated the debate on whether the quantum "superposition state" is realizable (causal) or just a mathematical notation to simplify and justify the quantum experiment outcomes. The interference phenomenon earlier observed in the quantum domain and later viewed in the classical realm, indeed supports the interconnection between the quantum and classical worlds.

One of the many reasons as to why the classical physics and quantum physics behave differently, is the concept of "locality" or "individuality" retained by classical states. Whereas, the quantum theory in itself is a 'local' phenomenon, but the quantum states obey 'non-locality' due to the local quantum interactions (Barrera et al. 2010). This non-locality of the quantum states gives rise to the concept of 'quantum entanglement' (Griffiths 2003). Entangled quantum states capsule the quantum correlation among the entangled systems such that the composite system has characteristics entirely different from the sum of its constituent parts. Any 'local' interaction between a quantum system and its environment leads to entanglement between the interacting systems, thereby changing the characteristics of the entangled whole. In fact the environment coupling defines the observable parameters of the quantum system. Another important effect of environment coupling (measurement process) is the *irreversible* de-localization of the 'quantum-coherence', which is an estimate of the quantum-ness of any system. This de-coherent process (i.e., the measurement process) of any quantum system leads to the emergence of the classical world from the underlying quantum platform. De-coherence is an inherent property of quantum

theory which cannot be freely controlled. Its consequences must be taken into account in order to reach to the classical realm of any quantum event (Gerry and Knight 2023).

III. Mathematical Description

There are two different but equivalent formulations to describe Quantum mechanics: The matrix mechanics given by Heisenberg and the wave mechanics given by Schrodinger. In this paper, we shall be dealing with the matrix mechanics description. If $|\psi\rangle$ (a column vector) denotes a pure quantum state (which can be a superposition of n –qubit computational basis), then the corresponding density matrix ρ is defined as the outer product of $|\psi\rangle$ with its conjugate (Blum 2012):

$$\rho = |\psi\rangle \langle\psi| \tag{1}$$

The physical interpretation of density matrix is that its diagonal terms correspond to the probability density of the state $|\psi\rangle$ in its constituent computational basis. Hence the sum of all the diagonal terms tracing to unity (Trace=1 for the density matrix).

The quantum state $|\psi\rangle$ can be expanded in terms of the superposition of the n –qubit computational basis as follows:

$$|\psi(t)\rangle = \sum_{i=0}^N C_i(t) |N\rangle \tag{2}$$

Where $N = 2^n - 1$, and $C_i(t)$ is a time-dependent complex number corresponding to i^{th} -computational basis, given by a real amplitude and a phasor part:

$$C_i(t) = C_i \exp(j\phi_i(t)) \tag{3}$$

Where $j = \sqrt{-1}$, Substituting Eqn. (2) in Eqn. (1), we get:

$$\rho = |\psi(t)\rangle \langle\psi(t)| = \sum_{i=0}^N C_i |i\rangle \sum_{k=0}^M C_k^* \langle k| \tag{4}$$

Tracing over the diagonal elements of the density matrix, we get

$$Tr(\rho) = \sum_{i=0}^N C_i \exp(j\phi_i(t)) C_i^* \exp(-j\phi_i(t)) |i\rangle \langle i| \tag{5}$$

$$Tr(\rho) = \sum_{i=0}^N |C_i|^2 \tag{6}$$

The diagonal elements give the probability of occupancy of a quantum state $|i\rangle$ residing at the diagonal location $|i\rangle \langle i|$ of the $N \times N$ square matrix. For this reason, the diagonal elements are also cited as the populations of the respective computational states. Hence the name density matrix is justified.

Regarding the off-diagonal terms,

$$\rho_{ik} = C_i(t)C_k^*(t) = C_iC_k^* \exp j(\phi_i(t) - \phi_k(t)) \tag{7}$$

$$\text{Let } j(\phi_i(t) - \phi_k(t)) = -j\omega_{ki}t \tag{8}$$

In Eqn. (8), ω_{ki} is the Rabi frequency (Ashhab et al. 2006). This equation describes the coherent oscillatory behaviour corresponding to the coherent superposition states in the quantum system, hence the off-diagonal elements are designated as the coherences. Any measurement or environment monitoring results in the de-coherence process, *i.e.*, dephasing of the off-diagonal terms towards zero (the corresponding dephasing time for the off-diagonal terms to exponentially decay towards zero is coined as Phase relaxation time or the T_2 relaxation time). This process yields $\rho_{\text{pure}} \xrightarrow{\text{decoherence}} \rho_{\text{mixed}}$, implying that dephasing results in de-coherence of the pure quantum states into mixed states.

IV. Interpretation of the T_2 and the T_1 relaxation Times

The T_2 value is generally smaller than the T_1 value, meaning that the T_2 relaxation is much faster than the T_1 relaxation. The T_2 relaxation describes the decay of Transverse magnetization in the xy – plane after a RF pulse is applied along the z – axis. T_2 is spin-spin relaxation because the loss of the phase coherence arises from interactions between spins.

Similarly, if an environment event results in the off-diagonal terms to increase from 0 to 1, this is coined as the energy relaxation time (T_1 relaxation time). This is spin-lattice relaxation because energy transfer occurs between spins and the lattice (phonon interaction). A large value of T_1 implies slow recovery. The T_1 measures how quickly the spins return to their equilibrium longitudinal magnetization.

V. Discussion

The authors have critically analyzed the density matrix and its various elements, giving emphasis on the physical interpretation of the diagonal and off-diagonal terms with adequate mathematics. The coherence and de-coherence of the quantum states is discussed along with the environment intrusion and its effects on the dephasing of the off-diagonal terms. The relaxation times T_1 and T_2 are described in detail. These relaxation times find applications in MRI (Magnetic Resonance Imaging), the tissues with shorter T_1 values appear brighter. This helps to diagnose various medical issues and hence initiate medical treatments on time.

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