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Application Of Cayley Hamilton's 3rd Order Matrix Theorem In N-Loop DC Electric Circuit Problems (2≤N≤3)

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Abstract:
The problem in the analysis of DC electrical circuits using Kirchoff I's law and Kirchoff II's law is obtained by the System of Linear Equations. In addition to using the elimination substitution method, the System of Linear Equations can also be solved using the help of matrices. Some numerical methods that can solve DC electrical circuit problems are the Gauss Elimination Method, Cramer Method, Gauss-Jordan Elimination Method, and Matrix Inversion Method. Cayley Hamilton's theorem is a theorem that can be used to determine Inverse Matrices. Every square matrix must have an Eigencharacteristic equation in the form of a polynomial $p(\lambda) = \lambda^n + C_1\lambda^{n-1} + \ldots\ldots + C_n = 0$ *and based on the Cayley-Hamilton theorem the characteristic equation also applies* $p(\mathrm{A}) = \mathrm{A}^n + p_1\mathrm{A}^{n-1} + p_2\mathrm{A}^{n-2} + \ldots\ldots + p_n\mathrm{I} = 0$. *This equation can be converted into the form* $A^{-1} = \frac{1}{P_3}\left(\mathrm{A}^2 - p_1\mathrm{A} + p_2 I\right)$. *The purpose of this study is to apply Cayley Hamilton's theorem in determining the strength of electric current in 2-loop and 3-loop DC electric circuits. The results of the application of Cayley Hamilton's theorem can be used to determine the strength of electric current in DC electric circuits 2 Loops and 3 Loops even n infinite loops because the shape of this theorem is polynomial.*

Key Word: *Electric circuits; System of linear equations; Cayley Hamilton's theorem.*

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I. Introduction

Physics is a science that studies all natural phenomena and is explained mathematically so that it is easy to understand [1]. Understanding physics usually uses two approaches, namely the empirical approach and the mathematical approach [2], [3]. According to Tzanakis in Hasyim & Ramadhan, physics statements and ways of thinking require mathematical equations, so that mathematical methods are needed to explain and solve physics problems [4]. So the relationship between physics and mathematics should not be ignored because in reality physics cannot be separated from the calculations of adding, subtracting, multiplying, differentiating, integrating, etc.

Physics and mathematics cannot be separated because many physics concepts are written using mathematical notation [5]. A good supply of mathematical knowledge is needed to deepen physical knowledge. If mathematics has been mastered then physics problems can be solved easily [6]. Therefore, mathematical abilities such as problem solving and mathematical communication must be mastered at a minimum [4]. This is supported by the results of Mardiyatmi & Abdullah's research, stating that basic mathematical abilities have a significant influence on students' physics learning competence [7].

DC electric circuits are one of the physics materials that are considered difficult [8]. Electric circuits are considered difficult because many electrical components are interconnected and the current distribution cannot be observed directly [9]. A DC electrical circuit is an electrical circuit that connects a source of voltage (V) and current (I) as well as resistance (R) arranged in series or parallel. There are two Kirchoff's laws in DC electrical circuits, with these two laws the solution to DC electrical circuit problems is obtained by a linear equation [10].

The solution to a system of linear equations can be expressed in matrix multiplication form [11]. A linear equation can have either a single or infinite solution or have no solution [12]. There are two methods for finding solutions to systems of linear equations, namely the first is the indirect method, namely using Jacobi and Gauss-Seidel literacy. The two direct methods are the Gauss-Jordan elimination method, Cramer method, LU decomposition method and inverse matrix method [13].

The Cayley-Hamilton theorem is a method that can be used as a basis for calculating matrix functions [14]. Hartini's research states that the application of the Cayley-Hamilton theorem can be used to find the inverse of a rectilinear matrix [15]. In addition, the Cayley-Hamilton theorem has been extended to n-dimensional real polynomial matrices and discrete-time linear systems with time delays [16]. The results of the extension of Cayley Hamilton's theorem can be used to solve n infinite square matrices because the form of this theorem is a polynomial. This theorem is one of the most powerful and classic matrix theories and many applications result from this theorem. The Cayley-Hamilton theorem and its generalizations can be used in control systems, electrical networks, systems with time delays, singular systems, 2-D linear systems and others [17].

Mathematical methods that have succeeded in solving electric circuits include Batarius stating that the Gauss-Jordan method can be used to determine the electric current in a unidirectional circuit [18]. The Cramer method can also be used as a solution for electrical circuit analysis carried out by Anam and Arnas [19].

Research by Ghozanfar & Ramdan uses Norton's Thevenin theorem to determine the electric current in a DC circuit. Taufik & Ramdan state that the Superposition theorem can be used to determine the electric current in a DC circuit [20]. The Mesh Theorem resulting from previous research can also be used to determine the current strength in a DC circuit [21]. Apart from the several mathematical models above, there is one form of mathematical model that is quite important, namely Cayley Hamilton's theorem.

Cayley Hamilton's theorem is a mathematical method that can solve systems of linear equations and the results of the analysis of two-loop closed electrical circuit problems are also systems of linear equations, so researchers use Cayley Hamilton's theorem to solve two-loop and three-loop closed electric circuit problems.

II. Material And Methods

An electric circuit is a collection of electrical components that are connected to each other and have at least one closed path [22]. Electric current is a charge (q) that moves or flows in a conductor within a certain time interval [23]. George Simon Ohm explained that the amount of current flowing is directly proportional to the potential difference (V), but the electric charge is inversely proportional to the resistance (R). There are two Kirchoff's laws in closed electrical circuits, namely Kirchoff's first law and Kirchoff's second law

Kirchoff's First Law states that the total amount of current entering a branch point must be the same as the total amount of current leaving the branch point [23].

$$\sum \boldsymbol{I_{input}} = \sum \boldsymbol{I_{output}} \tag{2.1}$$

Meanwhile, Kirchoff's Second Law states that the algebraic sum of the GGL in each circuit *loop* is equal to the algebraic sum of the products (IR) in the same *loop* circuit [12]. Mathematically it can be written as follows.

$$\sum \mathcal{E} = \sum IR \tag{2.2}$$

By using Kirchoff's two laws in DC electrical circuits, the solution to DC electrical circuit problems is obtained by a system of linear equations [24].

$A_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$	
$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$	
$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$	(2.3)

The system of linear equations in equation (2.3) can be converted into matrix multiplication form to obtain a matrix A or a square matrix of order n x n.

$$A\ X = B$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_n \end{bmatrix} \tag{2.4}$$

Solving equation (2.4) can use several methods, namely: Crammer's rule, Gauss Elimination, Matrix Inverse, and Matrix Decomposition [12].

Every square matrix always has a scalar quantity called the determinant [25]. The method for calculating the determinant of a matrix can use several methods, namely the Cofactor Method, Sarrus Method and Schur's Complement [26]. A square will have an inverse if the determinant value of a matrix is not equal to zero and if the determinant value of a matrix is equal to zero then the matrix does not have an inverse [25]. Determining the inverse of a matrix depends on the size of the matrix. If the size of the matrix is large then the level of difficulty in determining the inverse of a matrix becomes higher

If A is a matrix $n \times n$ then the nonzero vector $\boldsymbol{x}$ in R^n is called an eigenvector of A if $A\boldsymbol{x}$ is a scalar multiple of $\boldsymbol{x}$

$$A\,x = \lambda\,x \tag{2.4a}$$

Equation (2.4a) is changed to the following equation

$$(A - \lambda\,I)x = 0 \tag{2.4b}$$

The vector x is called the eigenvector corresponding to λ [27]. A scalar is an eigenvalue of A if and only if equation (2.4b) has a non-trivial solution. Equation (2.4b) will have a non-trivial solution if and only if

$$det(A - \lambda I) = 0 \quad (2.5)$$

Equation (2.5) is called the characteristic equation for matrix A. If the determinant in equation (2.5) is expanded, we will obtain a polynomial degree to–*n* in the variable λ expressed with

$$p(\lambda) = \det(A - \lambda I) = 0$$

$$p(\lambda) = \lambda^n + C_1\lambda^{n-1} + \ldots.. + C_n = 0 \quad (2.6)$$

Equation (2.6) is called the characteristic polynomial. The roots of this characteristic polynomial are the eigenvalues of the matrix A [28]. If we calculate the roots according to their multiples, then the characteristic polynomial will have n roots.

The Cayley-Hamilton theorem states that every matrix *nxn* fulfills its own characteristic equation, where the characteristic equation of matrix A is like equation (2.5). If equation (2.6) is a characteristic polynomial of A, a square matrix of order n, then according to Cayley Hamilton's theorem, $p(A) = A^n + p_1A^{n-1} + p_2A^{n-2} + \ldots.. + p_nI = 0$ with I is an identity matrix of order n [15]. Because p(a)=0 then the function.

$$p(\lambda) = \lambda^n + p_1\lambda^{n-1} + \ldots.. + p_n \quad (2.7)$$

is a characteristic polynomial of A and is valid

$$p(A) = A^n + p_1A^{n-1} + p_2A^{n-2} + \ldots.. + p_nI = 0 \quad (2.8)$$

The steps that must be taken to solve 2 *loop* and 3 *loop* electrical circuit problems using Cayley Hamilton's theorem are as follows:

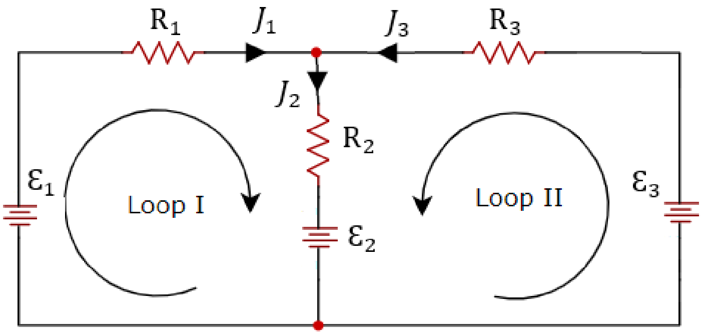

Figure 1. Two loop DC electrical circuit

a. Analyze the electrical circuit above using Kirchhof's law I and Kirchhof's law II. The results of the analysis will be a system of linear equations. Form a linear equation for a 2-*loop* electrical circuit

$$\boldsymbol{R_1x + R_2y + 0z} = -\sum\varepsilon \quad (2.9a)$$

$$\boldsymbol{0x + R_2y + R_3z} = -\sum\varepsilon \quad (2.9b)$$

$$\boldsymbol{x - y + z = 0} \quad (2.9c)$$

b. Changing the form of a system of linear equations to a square matrix of order 3x3. The following is the matrix for a 2 loop electrical circuit

A X = B

$$\begin{bmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sum\varepsilon \\ -\sum\varepsilon \\ 0 \end{bmatrix} \quad (2.10)$$

c. Looking for the eigen characteristic equation of matrix A

$$p(\lambda) = \det(A - \lambda I) = 0$$

$$p(\lambda) = \lambda^n + C_1\lambda^{n-1} + \ldots.. + C_n = 0 \quad (2.11)$$

d. Using Cayley Hamilton's theorem to determine the inverse matrix. The Cayley-Hamilton theorem states that every matrix *nxn* fulfills its own characteristic equation

$$p(\lambda) = -\lambda^3 + p_1\lambda^2 - p_2\lambda + p_3 = 0$$

$$p(A) = -A^3 + p_1A^2 - p_2A + p_3 = 0 \quad (2.12)$$

Next, both sides are multiplied by A^{-1} and the equation is obtained

$$A^{-1} = \frac{1}{p_3}\left(A^2 - p_1A + p_2I\right) \quad (2.13)$$

e. Use the inverse matrix A to find the value of X or current strength

$$X = A^{-1}B$$

III. Result and Discussion

The following is an illustration of a 2-*loop* electrical circuit problem which is solved using Cayley Hamilton's Theorem.

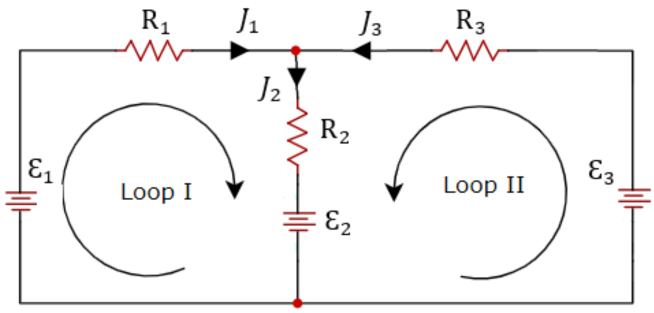

Figure 2. Illustration of a 2-*loop* electrical circuit problem

Using Kirchoff's law I and Kirchoff's law II, the solution to the problem of a closed DC 2 *loop* electric circuit is to obtain a linear equation, namely:

Loop 1: $\sum \varepsilon + \sum IR = 0$

$$R_1 I_1 + R_2 I_2 = \varepsilon_1 - \varepsilon_2 \quad (3.1)$$

Loop 2: $\sum \varepsilon + \sum IR = 0$

$$R_2 I_2 + R_3 I_3 = \varepsilon_3 - \varepsilon_2 \quad (3.2)$$

Hukum I Kirchoff:

$$I_1 - I_2 + I_3 = 0 \quad (3.3)$$

If viewed from the form of the system of linear equations in equations (3.1, 3.2, and 3.3), it can also be expressed in matrix multiplication form as follows:

A X = B

$$\begin{bmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \varepsilon_1 - \varepsilon_2 \\ \varepsilon_3 - \varepsilon_2 \\ 0 \end{bmatrix} \quad (3.4)$$

Equation 4.4 above shows that there is a square matrix of order 3 x 3 or matrix A.

$$A = \begin{bmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \\ 1 & -1 & 1 \end{bmatrix}$$

The Cayley-Hamilton theorem states that every matrix *nxn* fulfills its own characteristic equation, where the characteristic equation of matrix A can be written as follows:

$$\det(A - \lambda \mathrm{I}) = 0$$

$$\det(A - \lambda \mathrm{I}) = \begin{bmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \\ 1 & -1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\det(A - \lambda \mathrm{I}) = \begin{bmatrix} R_1 - \lambda & R_2 & 0 \\ 0 & R_2 - \lambda & R_3 \\ 1 & -1 & 1 - \lambda \end{bmatrix} = 0$$

Based on the matrix above, the determinant size of the matrix $(A - \lambda \mathrm{I})$ is

$$\det(A - \lambda \mathrm{I}) = (R_1 - \lambda)(R_2 - \lambda)(1 - \lambda) + R_2 R_3 1 + (0)(0)(-1) - (0(R_2 - \lambda)1) - (R_1 - \lambda)R_3(-1) - R_2(0)(1 - \lambda)$$

$$= (R_1 - \lambda)(R_2 - \lambda)(1 - \lambda) + R_2 R_3 + 0 - 0 + (R_1 - \lambda)R_3 - 0$$

$$= (R_1 - \lambda)\left(R_2 - (R_2 + 1)\lambda + \lambda^2\right) + R_2 R_3 + R_1 R_3 - R_3 \lambda$$

$$= R_1 R_2 - (R_1 R_2 + R_1)\lambda + R_1 \lambda^2 - R_2 \lambda + (R_2 + 1)\lambda^2 - \lambda^3 - R_3 \lambda + R_2 R_3 + R_1 R_3$$

$$= -\lambda^3 + (R_1 + R_2 + 1)\lambda^2 - (R_1 R_2 + R_1 + R_2 + R_3)\lambda + R_1 R_2 + R_2 R_3 + R_1 R_3$$

$$\boldsymbol{p_1} = (R_1 + R_2 + 1)$$

$$\boldsymbol{p_2} = (R_1 R_2 + R_1 + R_2 + R_3)$$

$$\boldsymbol{p_3} = (R_1 R_2 + R_2 R_3 + R_1 R_3)$$

Thus obtained

$$p(\lambda) = -\lambda^3 + p_1\lambda^2 - p_2\lambda + p_3 = 0$$
$$p(A) = -A^3 + p_1A^2 - p_2A + p_3 = 0$$

Next, both sides are multiplied by and the equation is obtained

$$A^{-1} = \frac{1}{P_3}\left(A^2 - p_1A + p_2I\right)$$

If matrix A in equation 4.4 results in a system of linear equations in electrical circuit 2*loop* inserted into the eigen characteristic equation in Cayley Hamilton's theorem then the solution to equation 3.4 in the electrical circuit 2*loop* can be written as follows:

$$X=A^{-1}B$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{P_3}\left(\begin{bmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \\ 1 & -1 & 1 \end{bmatrix}^2 - P_1\begin{bmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \\ 1 & -1 & 1 \end{bmatrix} + p_2I\right)\begin{bmatrix} \Sigma\varepsilon_1 \\ \Sigma\varepsilon_2 \\ 0 \end{bmatrix}$$

The following is an example of a 2-*loop* electrical circuit problem that is solved using Cayley Hamilton's theorem

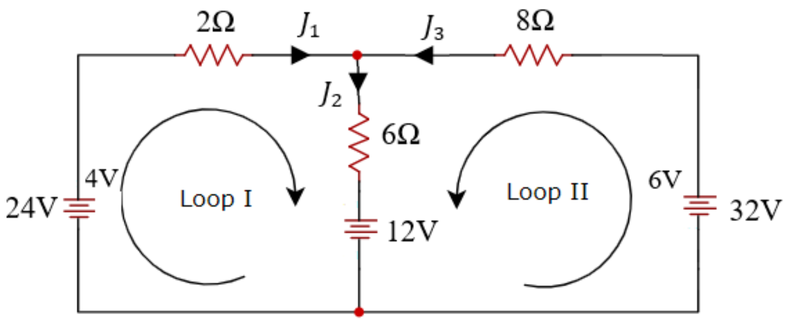

Figure 3. two-loop electrical circuit problem

Analysis of Kirchoff's first law and Kirchoff's second law

Loop 1

$$2I_1+6I_2 = 12$$

Loop 2

$$6I_2 + 8I_3 = 20$$

Kirchoff's first law

$$I_1 - I_2 + I_3 = 0$$

The results of the analysis of Kirchoff's law obtained a system of linear equations

$$2x + 6y + 0z = 12$$
$$0x + 6y + 8z = 20$$
$$x - y + z = 0$$

The system of linear equations above is converted to matrix multiplication

$$\begin{bmatrix} 2 & 6 & 0 \\ 0 & 6 & 8 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 0 \end{bmatrix}$$

Looking for eigen characteristic equations in matrix A

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 2 & 6 & 0 \\ 0 & 6 & 8 \\ 1 & -1 & 1 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\det(A - \lambda I) = -\lambda^3 + 9\lambda^2 - 28\lambda + 76$$

Substitute A to Replace to Obtain

$$det\ A = -A^3 + 9A^2 - 28A + 76$$

Then divide by A to obtain

$$0 = A^2 - 7A + 16I - 20A^{-1}$$
$$76A^{-1} = A^2 - 9A + 28I$$

$$A^{-1} = \frac{1}{76}(A^2 - 9A + 28I)$$

$$A^2 = \begin{bmatrix} 4 & 48 & 48 \\ 8 & 28 & 56 \\ 3 & -1 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{76}\left(\begin{bmatrix} 4 & 48 & 48 \\ 8 & 28 & 56 \\ 3 & -1 & -7 \end{bmatrix} - 9\begin{bmatrix} 2 & 6 & 0 \\ 0 & 6 & 8 \\ 1 & -1 & 1 \end{bmatrix} + 28\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$A^{-1} = \begin{bmatrix} \frac{7}{38} & -\frac{3}{38} & \frac{12}{19} \\ \frac{2}{19} & \frac{1}{38} & -\frac{4}{19} \\ -\frac{3}{38} & \frac{2}{19} & \frac{3}{19} \end{bmatrix}$$

$$X = A^{-1} B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{7}{38} & -\frac{3}{38} & \frac{12}{19} \\ \frac{2}{19} & \frac{1}{38} & -\frac{4}{19} \\ -\frac{3}{38} & \frac{2}{19} & \frac{3}{19} \end{bmatrix}\begin{bmatrix} 12 \\ 20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0{,}63152 \\ 1{,}78952 \\ 1{,}1578 \end{bmatrix}$$

So the magnitude of the current flowing through each resistor in the circuit problem above is I_1 = 0.63152 A, I_2 = 1.78952 A, and I_3 = 1.1578 A.

The results of calculating the current strength using Cayley Hamilton's theorem produce the same current strength as the results of Nurullaeli's research, 2020 entitled "Media Analysis of Electrical Circuits Using the Gauss-Jordan, Gauss-Seidel, and Cramer Numerical Approach" with the same example of an electric circuit problem, which produces the same current strength. namely I_1 = 0.6316 A, I_2 = 1.7895 A, and I_3 = 1.1579 A. The following is an image of the calculation results using Gauss-Jordan [24].

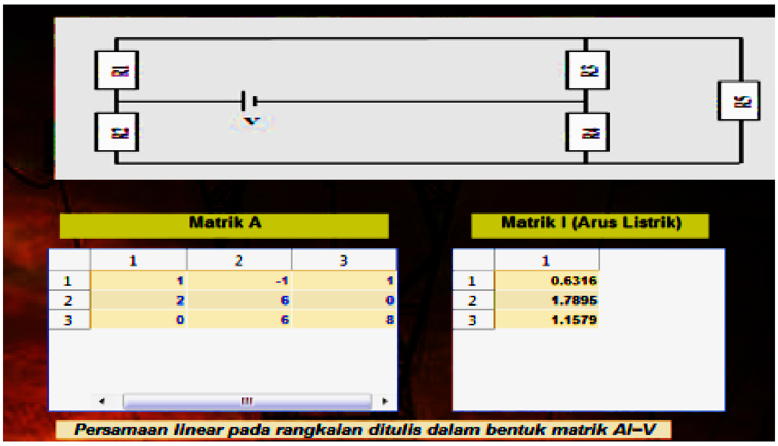

Figure 4. Results of calculating electric current strength using the Gauss-Jordan method.

The results of the same current strength between problem calculations using Cayley Hamilton's theorem and calculation results using Gauss-Jordan show that Cayley Hamilton's theorem can be used to solve two-*loop* electrical circuit problems. Apart from solving 2 *loop* electrical circuit problems, Cayley Hamilton's theorem can also be used to solve 3 *loop* electrical circuit problems where the matrix used is also of the order 3 x 3.

The following is an example of a 3-*loop* electrical circuit problem that is solved using Cayley Hamilton's theorem

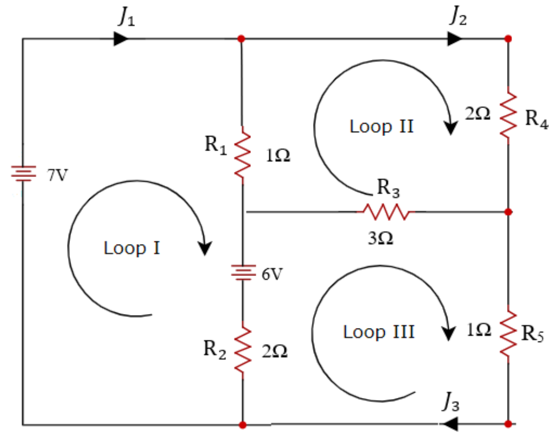

Figure 5. tree-loop electric circuit problem

Analysis of Kirchoff's law I and Kirchoff's law II where for the 3 loop electric circuit problem, Kirchoff's law I is substituted into Kirchoff's law II so that the results of the analysis of the electric circuit have 3 systems of linear equations which are converted into matrix multiplication form and still produce a 3rd order matrix like a circuit. electric 2 *loop*.

$$\text{Loop 1: } \sum \varepsilon + \sum IR = 0$$
$$\mathcal{E}_2 - \mathcal{E}_1 + R_1(I_1 - I_2) + R_2(I_1 - I_3) = 0$$
$$3I_1 - 1I_2 - 2I_3 = 1$$
$$\text{Loop 2: } \sum \varepsilon + \sum IR = 0$$
$$R_1(I_2 - I_1) + R_3(I_2 - I_3) + R_4 I_2 = 0$$
$$-I_1 + 6I_2 - 3I_3 = 0$$
$$\text{Loop 3: } \sum \varepsilon + \sum IR = 0$$
$$-\mathcal{E}_2 + R_2(I_3 - I_1) + R_3(I_3 - I_2) + R_5 I_3 = 0$$
$$-2I_1 - 3I_2 + 6I_3 = 6$$

The results of Kirchoff's law analysis in the form of a system of linear equations are converted into matrix form

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

After obtaining matrix A, then look for the eigencharacteristic equations in matrix A

$$\det(A - \lambda I) = 0$$
$$\det\left(\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$
$$\det(A - \lambda I) = -\lambda^3 + 15\lambda^2 - 58\lambda + 39$$

Substitute A to Replace λ to Obtain

$$\det A = -A^3 + 15A^2 - 58A + 39$$

Then divide by A to obtain

$$0 = -\lambda^2 + 15\lambda - 58 + 51A^{-1}$$
$$39A^{-1} = A^2 - 15A + 58$$
$$A^{-1} = \frac{1}{39}(A^2 - 15A + 58I)$$
$$A^2 = \begin{bmatrix} 14 & -3 & -15 \\ -3 & 46 & -34 \\ -15 & -34 & 49 \end{bmatrix}$$

$$A^{-1}=\frac{1}{39}\left(\begin{bmatrix}14 & -3 & -15\\ -3 & 46 & -34\\ -15 & -34 & 49\end{bmatrix}-15\begin{bmatrix}3 & -1 & -2\\ -1 & 6 & -3\\ -2 & -3 & 6\end{bmatrix}+58\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)$$

$$A^{-1}=\begin{bmatrix}\frac{9}{13} & \frac{4}{13} & \frac{5}{13}\\ \frac{4}{13} & \frac{14}{39} & \frac{11}{39}\\ \frac{5}{13} & \frac{11}{39} & \frac{17}{39}\end{bmatrix}$$

$$X\ = A^{-1}\ \ B$$

$$\begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix}=\begin{bmatrix}\frac{9}{13} & \frac{4}{13} & \frac{5}{13}\\ \frac{4}{13} & \frac{14}{39} & \frac{11}{39}\\ \frac{5}{13} & \frac{11}{39} & \frac{17}{39}\end{bmatrix}\begin{bmatrix}1\\ 0\\ 6\end{bmatrix}$$

$$\begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix}=\begin{bmatrix}3\\ 2\\ 3\end{bmatrix}$$

So the magnitude of the current flowing through each resistor in the circuit problem above is $I_1 = 3$ A, $I_2 = 2$ A, and $I_3 = 3$ A.

The results of calculating the current strength using Cayley Hamilton's theorem produce the same current strength as the results of Muhajir et.al, 2023 research entitled "Implementation of Numerical Methods in Electric Circuits Using Octave Software" with the same example of an electric circuit problem obtained by calculating the electric current for a three-*loop* circuit using the software. Octave gets the same amount of current, namely $I_1 = 3$ A, $I_2 = 2$ A, and $I_3 = 3$ A. The following is an image of the calculation results with Octave Software [29].

Figure 6. Results of calculating electric current strength using the Software Octave

IV. Conclusion

Based on the discussion of two-*loop* and three-*loop* electrical circuit problems that has been carried out, it shows that the application of Cayley Hamilton's theorem and the third-order eigenmatrix characteristic problem can be used to solve the magnitude of the current flowing in a two-*loop* electrical circuit where the general equation is as follows:

$$X=A^{-1}\,B$$

$$\begin{bmatrix}I_1\\ I_2\\ I_3\end{bmatrix}=\frac{1}{P_3}\left(\begin{bmatrix}R_1 & R_2 & 0\\ 0 & R_2 & R_3\\ 1 & -1 & 1\end{bmatrix}^2-p_1\begin{bmatrix}R_1 & R_2 & 0\\ 0 & R_2 & R_3\\ 1 & -1 & 1\end{bmatrix}+p_2 I\right)\begin{bmatrix}\mathcal{E}_1-\mathcal{E}_2\\ \mathcal{E}_2-\mathcal{E}_3\\ 0\end{bmatrix}$$

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