# Investigation Of Point Spread Function Of Four-Zone Apertures With Defocus And Aberrations 

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#### Abstract

The point spread function (PSF) produced by a coherent optical system under the influence of defocus, primary spherical aberration (PSA) and astigmatism is examined in this work. This paper discusses the use of amplitude apodization to mitigate the impact of monochromatic aberrations on the diffracted Point Spread Function (PSF). It effectively conveys the main idea of the paper's focus on using amplitude apodization to address the influence of monochromatic aberrations on the diffracted Point Spread Function (PSF). We computed the central peak intensity and Full Width at Half Maximum (FWHM) and then analyzed the results. The resolution of a diffraction-limited optical imaging system can be improved by using a symmetric optical filter to minimize the impact of defocus.


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## I. Introduction

Point spread function (PSF), which involves modulating the light distribution in the focal region of an optical system to enhance the performance of optical systems used in various applications. [1-18]. The mechanism of using phase or amplitude filter masks in the Fourier plane of an optical system to achieve changes in the focal spot size and the intensity of its surrounding side lobes. [1-4]. The presence of optical aberrations typically leads to a degradation in the performance of an optical system. [5, 6]. In general, the detrimental effects of aberrations can be reduced by inserting an optical element with a suitable optical transmittance function [7-23], the proposed technique suggests that the possibility for pupil filters design and can be used to develop the off-axis imaging systems in fields such as of microscopy, telescope and spectroscopy. Apodization is the process for removal of secondary side-lobes or side-bands in the diffraction field, which is known as the point spread function (PSF), for any optical system to function effectively as a super resolver. And this can be acquired by appropriately selecting the transmittance of the pupil function of the optical s The positive impact of using a Hanning amplitude filter in optical systems that are highly defocused and suffering from aberrations includes improving the quality of the resulting image by reducing side-lobes in the Point Spread Function (PSF) and enhancing the central lobe's intensity. ystem, the intensity around the focused fields can be completely eliminated or significantly reduced, all without the need to increase the dimensions of the pupil. The effects of defocusing or optical aberrations on the response of an optical system to a point defect. It indicates that such conditions can lead to significant distortion of the image and cause the intensity in the central lobe to scatter into the optical side lobes.

The results of a point spread function with non-zero minima which leads to an undesirable image characterized by an increase in the intensity within the ring structure of the diffraction pattern. The positive impact of using a Hanning amplitude filter in optical systems that are highly defocused and suffering from aberrations includes improving the quality of the resulting image by reducing side-lobes in the Point Spread Function (PSF) and enhancing the central lobe's intensity. It highlights how the filter can significantly improve the intensity in the central lobe of the point spread function while redistributing the intensity and achieving zero first minima, which is considered a desirable characteristic of the point spread function.

In the current study, the imaging characteristics of the diffracted field of rotationally symmetric optical systems with Shaded, Connes, Triangular and Hanning amplitude filters have been investigated. This leads to the reduction of secondary side-lobes by modifying a four-zone aperture with varying degrees of amplitude apodization $\beta$. This modification is done in the context of dealing with defocus, primary spherical aberrations, and astigmatism. It highlights the significance of understanding the Point Spread Function (PSF) when studying the imaging properties of optical systems and how it plays a crucial role in the design of optical imaging systems.

The present study provides a significant contribution to the resolution studies. We know that by employing suitable apodization filters can be used to modify the point spread function in the maximum out-offocus image plane to match specific axial shape requirements. By using suitable shading aperture can be very
helpful in correcting the Seidel aberration effect in the image plane of an optical system. Based on the investigations done in the four-zone apodization process can be done by arranging the Shaded amplitude filter in the first zone, Connes amplitude filter in the second zone, Triangular amplitude filter in the third zone and Hanning amplitude filter in the outer zone could be the solution for modifying the point spread function of the optical system under the strong combined influence of defect-of-focus, primary spherical aberration and astigmatism. In this study, we examined the use of a four-zone aperture to modify the light distribution in the focal plane of aberration-induced optical systems.

## II. Theory and Formulation

Figure1 shows the general schematic representation of a symmetric optical imaging system with a circular aperture


Point Source


Figure1. General schematic representation of optical imaging system.
It describes the amplitude of diffracted light associated with a rotationally symmetric pupil using the given formula. This appears to be part of a technical or scientific discussion.

$$
\begin{equation*}
A(Z)=2 \int_{0}^{1} f(r) J_{0}(Z r) r d r \tag{1}
\end{equation*}
$$

The general expression for diffraction field of four-zone amplitude filters is given by:
$A(z)=\left\{2\left(\int_{0}^{a} f_{1}(r) J_{0}(Z r) r d r+\int_{a}^{b} f_{2}(r) J_{0}(Z r) r d r+\int_{b}^{c} f_{3}(r) J_{0}(Z r) r d r+\int_{c}^{1} f_{4}(r) J_{0}(Z r) r d r\right)\right\}$
Where $\boldsymbol{f 1}(\boldsymbol{r})$ is Shaded amplitude pupil function, $\boldsymbol{f Z}(\boldsymbol{r})$ is Connes amplitude pupil function, $\boldsymbol{f 3}(\boldsymbol{r})$ is Triangular amplitude pupil function, $\boldsymbol{f 4}(\boldsymbol{r})$ is Hanning amplitude pupil function of the optical system; $\mathbf{Z}$ is the dimension less variable which forms the distance of the point of investigation from the centre of the diffraction field; and $\boldsymbol{J O} \mathbf{( Z r})$ is the zero order Bessel function of the first kind; ' $\boldsymbol{r}$ ' is the reduced radial coordinate on the exit-pupil of aberrations influenced optical system. The generalized expression for the amplitude impulse response of the pupil function in the presence of higher degree of primary spherical aberration, astigmatism and defocusing with phase can be written as:

$$
\begin{gather*}
A(z)=\left\{2 \left(\int_{0}^{a} f_{1}(r) e^{-i\left(\phi_{d} \frac{r^{2}}{2}+\phi_{s} \frac{r^{4}}{4}+\phi_{a} \frac{r^{2}}{4}(1+\cos 2 \theta)\right)} J_{0}(Z r) r d r+\right.\right. \\
\int_{a}^{b} f_{2}(r) e^{-i\left(\phi_{d} \frac{r^{2}}{2}+\phi_{s} \frac{r^{4}}{4}+\phi_{a} \frac{r^{2}}{4}(1+\cos 2 \theta)\right)} J_{0}(Z r) r d r+(\text { Phase }) \int_{b}^{c} f_{3}(r) e^{-i\left(\phi_{d} \frac{r^{2}}{2}+\phi_{s} \frac{r^{4}}{4}+\phi_{a} \frac{r^{2}}{4}(1+\cos 2 \theta)\right)} J_{0}(Z r) r d r+ \\
\left.\left.\int_{c}^{1} f_{4}(r) e^{-i\left(\phi_{d} \frac{r^{2}}{2}+\phi_{s} \frac{r^{4}}{4}+\phi_{a} \frac{r^{2}}{4}(1+\cos 2 \theta)\right)} J_{0}(Z r) r d r\right)\right\} \tag{3}
\end{gather*}
$$

Where $\phi_{d}, \phi_{s}$ and $\phi_{a}$, are the defect-of-focus, primary spherical aberration and astigmatism parameters respectively. In current study, the four pupil functions we have considered to study variable anodization Shaded, Connes, Triangular and Hanning filter in the first, second, third and outer zone respectively with induced phase is $(-\pi / 7)$ which can be represented by:
$f(r)=1-\beta r^{2}$
Shaded amplitude filter
$f(r)=\left(1-\beta^{2} r^{2}\right)^{2}$ Connes amplitude filter
$f(r)=1-\beta r$
Triangular amplitude filter (6)
$f(r)=\cos (\pi \beta r) \quad$ Hanning amplitude filter

Where ' $\beta$ ' is the amplitude apodization parameter controlling the non-uniform transmission of the pupil function. The intensity PSF $B(Z)$ which is the measurable quantity can be obtained by taking the squared modulus of $A(Z)$. Thus,

$$
\begin{equation*}
A(Z)=|B(Z)|^{2} \tag{8}
\end{equation*}
$$



Figure2. Four-zone Apertures

## III. Results and Discussions

The investigations on the effect four-zone aperture shading on the images of point object formed by coherent optical system apodized by the Shaded, Connes, Triangular and Hanning amplitude filters in the presence of defocus and higher order spherical aberrations have been evaluated using the expression (8) by employing MATLAB simulation.

Figure.3. Variation in the axial shape of the point spread function of four-zone apertures for different degrees of defocus, primary spherical aberration and astigmatism with Phase ( $-\pi / 7$ )

$-\beta=0.0-\beta=0.2-\beta=0.4-\beta=0.6-\beta=0.8-\beta=1.0$

Figure. 3 depicts the intensity distribution curves for various amounts of apodization parameter for fourzone circular aperture with central zone parameter $\mathrm{a}=0.2, \mathrm{~b}=0.6$ and $\mathrm{c}=0.8$, with Shaded filter in the first zone and Connes filter in the second zone, Triangular filter in the third zone and Hanning filter in the outer zone in
the presence of higher degrees of defocus, primary spherical aberration and astigmatism. Figure 3 (a) shows the effect of the apodization parameter $\beta$ on the intensity distribution of PSF in the absenceof defocus, primary spherical aberration and astigmatism with increase in the apodization, there is a decrease in the intensity of the central lobe. As the apodization parameter $\beta$ is increased from $\beta=0$ to $\beta=0.4$, the optical side lobes are completely suppressed. When employing higher orders of apodization ( $\beta=1$ ), it is observed that the radius of the first dark ring in the diffraction pattern diminishes compared to the Airy case. In Figures 3(b) and (c), the impact of the apodization parameter $\beta$ on the intensity distribution of the Point Spread Function (PSF) is demonstrated with defocus, primary spherical aberration, and astigmatism ( $\phi_{\mathrm{d}}=\pi / 2, \phi_{\mathrm{s}}=\pi / 2$ and $\phi_{\mathrm{a}}=\pi / 2$ ). As the apodization increases, there is a concurrent reduction in the intensity of the central lobe. When the apodization parameter $\beta$ is increased from $\beta=0$ to $\beta=1.0$ under conditions where $\phi_{\mathrm{d}}=\pi, \phi_{\mathrm{s}}=\pi$ and $\phi_{\mathrm{a}}=\pi$, there is an observable escalation in the intensity of the central lobe. And, this increase in the apodization parameter does lead to enhanced intensity, but it does not result in the suppression of side lobes. In Figure 3(d), the impact of the apodization parameter $\beta$ on the intensity distribution of the Point Spread Function (PSF) is illustrated under conditions of defocus, primary spherical aberration, and astigmatism $\phi_{\mathrm{d}}=2 \pi, \phi_{\mathrm{s}}=2 \pi$ and $\phi_{\mathrm{a}}=2 \pi$. Figure. 3 (d) presents the effect of the apodization parameter $\beta$ on the intensity distribution of PSF defocus, primary spherical aberration and astigmatism when the value of $\phi_{d}=2 \pi, \phi_{s}=2 \pi$ and $\phi_{\mathrm{a}}=2 \pi$. From the profile of the intensity distribution curves it is evident that for $\beta=0.6$, i.e., for partial apodization there appears to be a total elimination of the optical sidelobes and for higher values of apodization with $\beta=1$. The intensity in the central lobe shapes to the desired profile resulting in an increase in the intensity of the central maximum and reduction in the radius of the first dark ring. In other words the energy in the central maximum is increased with reduced size of the spot rendering it to be super-resolved.

Figure. 4. 3D Intensity distribution for four-zone aperture at $a=0.2, b=0.6, c=0.8$ under highest degree of defocus, primary spherical aberration and astigmatism.


Table. 1 FWHM for apodization parameter ( $\beta$ )

| $\boldsymbol{\beta}$ | FWHM |
| :---: | :---: |
| 0.00 | 10.8823 |
| 0.20 | 10.5289 |
| 0.40 | 7.6553 |
| 0.60 | 4.8327 |
| 0.80 | 3.5889 |
| 1.00 | 3.1838 |

It is observed that, FWHM decreased for higher values of $\beta$ when an induced phase $(-\pi / 7)$, and it attains minimum for $\beta=1$. There is an increased intensity in the central maxima, when $\phi_{d}=2 \pi, \phi_{s}=2 \pi \& \phi_{a}=2 \pi$. It is more clearly shown from the Table1.

## IV. Conclusions

From the present work, it is found that employing combination of Shaded amplitude filter (from 0 to 0.2 ), Connes amplitude filter (from 0.2 to 0.6 ), Triangular amplitude filter (from 0.6 to 0.8 ) and Hanning amplitude filter (from 0.8 to 1.0) is effective in achieving a super-resolved PSF for higher values of amplitude apodization ( $\beta=1$ ), even in the presence of high degree of defocus $\left(\phi_{\mathrm{d}}=2 \pi\right.$ ), primary spherical aberration ( $\phi_{\mathrm{s}}=$ $2 \pi)$ and astigmatism $\left(\phi_{\mathrm{a}}=2 \pi\right)$. The process of apodizing the optical system, suppresses the optical side-lobes. For $\beta=0.6$, the side-lobes are almost eliminated. For $\beta=1$, the axial shape and the lateral resolution of the PSF is modified into the required module of maximum intensity and suppressed side lobes. On the whole it is emphasized that, the four-zone aperture with the combined Shaded, Connes, Triangular and Hanning amplitude apodization filters has got good results in terms of the intensity of PSF under the combined influence of defocusing effect, the primary spherical aberration and astigmatism.

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