

Schrödinger Equation unfit for fundamental law

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Abstract:

This paper proves that the Schrödinger equation is no longer valid to stand as the fundamental postulate of quantum mechanics. Most of the non-harmonic wave equations cannot be represented by this equation including the one's in their simplest form has been derived. As the motion of a particle is more complicated and not always harmonic or Gaussian which forms the basis of defining an alternative equation that is all encompassing, complete and accurate to correctly describe the properties and behaviour of a quantum particle.

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I. Introduction:

The Schrödinger equation is a fundamental equation in quantum mechanics that describes the behaviour of wave functions, which represent the quantum states of particles. It is named after the Austrian physicist Erwin Schrödinger, who developed it in 1925.

The time-dependent Schrödinger equation is given by:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

In this equation, \hbar is the reduced Planck's constant ($\hbar/2\pi$), ψ represents the wave function of the particle, t is time, x is position, m is the mass of the particle, and $V(x)$ is the potential energy function.

The left-hand side of the equation involves the time derivative of the wave function, which represents the rate of change of the wave function with respect to time. The right-hand side consists of two terms: the first term represents the spatial derivative of the wave function, which describes the curvature of the wave function in space, and the second term represents the potential energy of the particle.

The Schrödinger equation is a differential equation that relates the time evolution of the wave function to the energy and potential of the system. It provides a mathematical framework to describe the behaviour of quantum systems, such as atoms, molecules, and particles, and allows us to determine the probabilities of various outcomes when measuring physical quantities.

By solving the Schrödinger equation, one can obtain the wave function of a quantum system, which contains all the information about the system's quantum state. From the wave function, various observable properties of the system can be calculated, such as the energy spectrum, particle position, momentum, and more.

The Schrödinger's equation is not limited to free particles and can be used to describe particles in a wide range of scenarios, including those influenced by external potentials.

It is generally believed that the Schrödinger's equation does not assume harmonic oscillation of electrons. The equation is a general equation that describes the behaviour of quantum particles, including electrons, in a wide range of systems and potential energy landscapes. While harmonic motion can be observed in certain systems involving electrons, such as in simple harmonic oscillators or in the motion of electrons in atoms, it is not an assumption of the Schrödinger's equation itself.

As we know the electrons do not always move in harmonic motion. While harmonic motion, or oscillatory motion, is observed in some systems involving electrons, such as in atoms and molecules, electrons can also exhibit other

types of motion. For example, in a conductor, electrons can move in a random motion known as thermal motion. Additionally, in certain situations, electrons can move in a straight line or follow complex paths, depending on the forces acting on them.

It will be shown below that the above postulate generally believed by the scientific community that the Schrödinger's equation can solve for most of the wave function in a particle is not true and that there are lot of non-harmonic wave functions, conditions and assumptions that the equation is not valid.

II. Observations:

We all know that the harmonic wave functions can easily describe solutions to the Schrödinger equation. In fact, the simplest and most well-known solutions to the time-independent Schrödinger equation are harmonic wave functions.

A harmonic wave function is a mathematical representation of a harmonic oscillator, which is a system that oscillates around an equilibrium position with a restoring force proportional to the displacement.

The harmonic wave function, often denoted as $\psi(x)$, has the general form:

$$\psi(x) = A \exp(-\alpha x^2)$$

In this equation, A is the amplitude of the wave function, α is a constant related to the stiffness of the harmonic oscillator, and x represents the position of the particle.

When this wave function is plugged into the time-independent Schrödinger equation for a harmonic oscillator potential, it satisfies the equation and represents a valid stationary state. There are thousands of literature available on this both on-line and in books, research articles and the like that shows the derivation of Schrödinger equation from the harmonic wave function.

So, harmonic wave functions, which have a sinusoidal or Gaussian-like shape, can accurately describe the behaviour of particles in certain systems and are commonly used in quantum mechanics to solve the Schrödinger equation for the harmonic oscillator potential. In fact, it can also be derived from the more general wave equations, such as the wave equation for electromagnetic waves or the wave equation for sound waves.

Non-harmonic waves can exhibit a wide range of distributions, depending on their specific characteristics and mathematical forms. Here are a few examples of common non-harmonic wave distributions:

1. Gaussian Distribution: The Gaussian distribution, also known as the normal distribution, is a common example of a non-harmonic wave distribution. It is characterized by a bell-shaped curve, with a peak at the mean value and symmetrical tails extending to both sides. The Gaussian distribution is often used to describe wave packets or localized wave functions in quantum mechanics.

2. Lorentzian Distribution: The Lorentzian distribution, also known as the Cauchy distribution, is another example of a non-harmonic wave distribution. It has a peaked shape similar to the Gaussian distribution but with heavier tails that extend infinitely. The Lorentzian distribution is often used to describe wavefunctions with long-range interactions or resonant behaviour.

3. Uniform Distribution: The uniform distribution is a simple non-harmonic wave distribution where values are evenly distributed within a specified range. It is characterized by a constant probability density function over this range. The uniform distribution is commonly used in signal processing and random number generation.

4. Exponential Distribution: The exponential distribution is a non-harmonic wave distribution that is often used to describe decay processes or the waiting times between events in stochastic systems. It has a decreasing exponential shape, with a higher probability of occurrence for smaller values and a long tail extending towards larger values.

5. Square Distribution: A square wave is a non-harmonic wave that alternates between two distinct values. It is often used in signal processing and electronics. The general formula for a square wave is:

$$\psi(x) = A, \text{ for } x \text{ within a certain range}$$

$$\psi(x) = -A, \text{ for } x \text{ outside that range}$$

In this equation, A represents the amplitude of the wave.

Shown below are some of the proofs that show that the Schrödinger equation cannot describe the particle motion for some of the most common non-harmonic wave functions with the exception of Gaussian wave function:

1. The Lagrangian distribution is defined as:

$$\psi(x, t) = A * \exp(-iEt/\hbar) * \exp(-(x-x_0)^2/2\sigma^2)$$

Where A is the normalization constant, E is the energy, t is time, x is position, x_0 is the mean position, σ is the standard deviation of the distribution, and \hbar is the reduced Planck's constant.

To determine whether this wave function satisfies the Schrödinger equation, we need to substitute it into the time-dependent Schrödinger equation:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

Let's first calculate the partial derivative of ψ with respect to time:

$$\partial\psi/\partial t = -iEA * \exp(-iEt/\hbar) * \exp(-(x-x_0)^2/2\sigma^2)$$

Now, let's calculate the second partial derivative of ψ with respect to position:

$$\partial^2\psi/\partial x^2 = A * \exp(-iEt/\hbar) * \exp(-(x-x_0)^2/2\sigma^2) * (1/\sigma^2 - (x-x_0)^2/\sigma^4)$$

Now, let's substitute these derivatives into the Schrödinger equation:

$$i\hbar * (-iEA * \exp(-iEt/\hbar) * \exp(-(x-x_0)^2/2\sigma^2)) = -\hbar^2/2m * (A * \exp(-iEt/\hbar) * \exp(-(x-x_0)^2/2\sigma^2) * (1/\sigma^2 - (x-x_0)^2/\sigma^4)) + V(x) * (A * \exp(-iEt/\hbar) * \exp(-(x-x_0)^2/2\sigma^2))$$

Simplifying this equation, we get:

$$EA = -\hbar/2m * (1/\sigma^2 - (x-x_0)^2/\sigma^4) + V(x)$$

This equation should hold for all x and t if the Lagrangian distribution wave function satisfies the Schrödinger equation. However, since the Lagrangian distribution wave function does not have a time-independent potential term V(x), it cannot satisfy the Schrödinger equation in its current form.

2. The Poisson distribution wave function is given by:

$$\psi(x, t) = A * e^{(-iEt/\hbar)} * e^{(-\lambda)} * (\lambda^k / k!)$$

Where A is the normalization constant, E is the energy, t is time, x is position, λ is the average number of events, k is the number of events, and \hbar is the reduced Planck's constant.

To determine whether this wave function satisfies the Schrödinger equation, we need to substitute it into the time-dependent Schrödinger equation:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

Let's first calculate the partial derivative of ψ with respect to time:

$$\partial\psi/\partial t = -iEA * e^{(-iEt/\hbar)} * e^{(-\lambda)} * (\lambda^k / k!)$$

Now, let's calculate the second partial derivative of ψ with respect to position:

$$\partial^2\psi/\partial x^2 = A * e^{(-iEt/\hbar)} * e^{(-\lambda)} * (\lambda^k / k!) * (-\lambda^2)$$

Now, let's substitute these derivatives into the Schrödinger equation:

$$i\hbar * (-iEA * e^{(-iEt/\hbar)} * e^{(-\lambda)} * (\lambda^k / k!)) = -\hbar^2/2m * (A * e^{(-iEt/\hbar)} * e^{(-\lambda)} * (\lambda^k / k!) * (-\lambda^2)) + V(x) * (A * e^{(-iEt/\hbar)} * e^{(-\lambda)} * (\lambda^k / k!))$$

Simplifying this equation, we get:

$$EA = -\hbar^2/2m * \lambda^2 * (\lambda^k / k!) + V(x) * (\lambda^k / k!)$$

This equation should hold for all x and t if the Poisson distribution wave function satisfies the Schrödinger equation. However, since the Poisson distribution wave function does not have a time-independent potential term V(x), it cannot satisfy the Schrödinger equation in its current form.

3. The wave function for a **Uniform distribution** is given by:

$$\psi(x, t) = A * e^{(-iEt/\hbar)}$$

where A is the normalization constant, E is the energy, t is time, x is position, and \hbar is the reduced Planck's constant.

To determine whether this wave function satisfies the Schrödinger equation, we need to substitute it into the time-dependent Schrödinger equation:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

Let's first calculate the partial derivative of ψ with respect to time:

$$\partial\psi/\partial t = -iEA * e^{(-iEt/\hbar)}$$

Now, let's calculate the second partial derivative of ψ with respect to position:

$$\partial^2\psi/\partial x^2 = 0$$

Now, let's substitute these derivatives into the Schrödinger equation:

$$i\hbar * (-iEA * e^{(-iEt/\hbar)}) = -\hbar^2/2m * (0) + V(x) * (A * e^{(-iEt/\hbar)})$$

Simplifying this equation, we get:

$$EA = V(x)A * e^{(-iEt/\hbar)}$$

This equation should hold for all x and t if the uniform distribution wave function satisfies the Schrödinger equation. However, this equation implies that $V(x) = 1$, which means the potential is constant throughout space. This is not generally true for physical systems, as the potential usually depends on position. Therefore, the uniform distribution wave function does not satisfy the Schrödinger equation in its current form.

4. The **Lorentzian distribution** wave function is given by:

$$\psi(x, t) = A / (1 + ((x - x_0)/\sigma)^2)$$

where A is the normalization constant, x_0 is the mean position, σ is the standard deviation of the distribution, and x is position.

To determine whether this wave function satisfies the Schrödinger equation, we need to substitute it into the time-dependent Schrödinger equation:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

Let's first calculate the partial derivative of ψ with respect to time:

$$\partial\psi/\partial t = 0$$

Now, let's calculate the second partial derivative of ψ with respect to position:

$$\partial^2\psi/\partial x^2 = A * (2\sigma^2 / (1 + ((x - x_0)/\sigma)^2)^3)$$

Now, let's substitute these derivatives into the Schrödinger equation:

$$i\hbar * (0) = -\hbar^2/2m * (A * (2\sigma^2 / (1 + ((x - x_0)/\sigma)^2)^3)) + V(x) * (A / (1 + ((x - x_0)/\sigma)^2))$$

Simplifying this equation, we get:

$$0 = -\hbar^2/2m * (A * (2\sigma^2 / (1 + ((x - x_0)/\sigma)^2)^3)) + V(x) * (A / (1 + ((x - x_0)/\sigma)^2))$$

This equation should hold for all x and t if the Lorentzian distribution wave function satisfies the Schrödinger equation. However, this equation is not generally true since the left-hand side is zero, and it is unlikely that the right-hand side will be zero for all x and t unless $V(x) = 0$. Therefore, the Lorentzian distribution wave function does not satisfy the Schrödinger equation in its current form.

5. The **Exponential distribution** wave function is given by:

$$\psi(x, t) = A * e^{(-iEt/\hbar)} * e^{(-\lambda|x|)}$$

where A is the normalization constant, E is the energy, t is time, x is position, λ is the rate parameter, and \hbar is the reduced Planck's constant.

To determine whether this wave function satisfies the Schrödinger equation, we need to substitute it into the time-dependent Schrödinger equation:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

Let's first calculate the partial derivative of ψ with respect to time:

$$\partial\psi/\partial t = -iEA * e^{(-iEt/\hbar)} * e^{(-\lambda|x|)}$$

Now, let's calculate the second partial derivative of ψ with respect to position:

$$\partial^2\psi/\partial x^2 = A * e^{(-iEt/\hbar)} * (-\lambda^2) * e^{(-\lambda|x|)}$$

Now, let's substitute these derivatives into the Schrödinger equation:

$$i\hbar * (-iEA * e^{(-iEt/\hbar)} * e^{(-\lambda|x|)}) = -\hbar^2/2m * (A * e^{(-iEt/\hbar)} * (-\lambda^2) * e^{(-\lambda|x|)}) + V(x) * (A * e^{(-iEt/\hbar)} * e^{(-\lambda|x|)})$$

Simplifying this equation, we get:

$$EA = -\hbar^2/2m * \lambda^2 * e^{(-\lambda|x|)} + V(x) * e^{(-\lambda|x|)}$$

This equation should hold for all x and t if the exponential distribution wave function satisfies the Schrödinger equation. However, this equation implies that $V(x) = \lambda^2$, which means the potential is proportional to the square of the rate parameter λ . This is not generally true for physical systems, as the potential usually depends on position in a more complex manner. Therefore, the exponential distribution wave function does not satisfy the Schrödinger equation in its current form.

6. To show how the **Gaussian distribution** wave function satisfies the Schrödinger equation, let's start with the Gaussian wave function:

$$\psi(x, t) = A * e^{(-iEt/\hbar)} * e^{-(x - x_0)^2 / (4\sigma^2)}$$

where A is the normalization constant, E is the energy, t is time, x is position, x_0 is the mean position, σ is the standard deviation of the distribution, and \hbar is the reduced Planck's constant.

We need to substitute this wave function into the time-dependent Schrödinger equation:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

First, let's calculate the partial derivative of ψ with respect to time:

$$\partial\psi/\partial t = -iEA * e^{(-iEt/\hbar)} * e^{-(x - x_0)^2 / (4\sigma^2)}$$

Next, let's calculate the second partial derivative of ψ with respect to position:

$$\partial^2\psi/\partial x^2 = A * e^{(-iEt/\hbar)} * (-2\sigma^2 - 2(x - x_0)^2 / (4\sigma^4)) * e^{-(x - x_0)^2 / (4\sigma^2)}$$

Now, let's substitute these derivatives into the Schrödinger equation:

$$i\hbar * (-iEA * e^{(-iEt/\hbar)} * e^{-(x - x_0)^2 / (4\sigma^2)}) = -\hbar^2/2m * (A * e^{(-iEt/\hbar)} * (-2\sigma^2 - 2(x - x_0)^2 / (4\sigma^4)) * e^{-(x - x_0)^2 / (4\sigma^2)}) + V(x) * (A * e^{(-iEt/\hbar)} * e^{-(x - x_0)^2 / (4\sigma^2)})$$

Simplifying this equation, we get:

$$E = -\hbar^2/2m * (-2\sigma^2 - 2(x - x_0)^2 / (4\sigma^4)) + V(x)$$

Now, let's simplify the right-hand side further:

$$E = (\hbar^2/2m) * (2\sigma^2 - 2(x - x_0)^2 / (2m\sigma^4)) + V(x)$$

$$E = (\hbar^2/4m\sigma^2) + [(\hbar^2(x - x_0)^2) / (2m\sigma^4)] + V(x)$$

$$E = (\hbar^2/4m\sigma^2) + [(\hbar^2(x - x_0)^2) / (2m\sigma^4)] + V(x)$$

$$E = (\hbar^2/4m\sigma^2) + [(\hbar^2(x - x_0)^2) / (2m\sigma^4)] + V(x)$$

$$E = (\hbar^2/4m\sigma^2) + (\hbar^2(x - x_0)^2) / (2m\sigma^4) + V(x)$$

This equation shows that the right-hand side is equal to the energy E, which is a constant. Therefore, the Gaussian distribution wave function satisfies the Schrödinger equation in its current form.

7. The **Square distribution** wave function is given by:

$$\psi(x, t) = A * e^{(-iEt/\hbar)} * \text{sgn}(x)$$

where A is the normalization constant, E is the energy, t is time, x is position, $\text{sgn}(x)$ is the sign function, and \hbar is the reduced Planck's constant.

To determine whether this wave function satisfies the Schrödinger equation, we need to substitute it into the time-dependent Schrödinger equation:

$$i\hbar \partial\psi/\partial t = -\hbar^2/2m \partial^2\psi/\partial x^2 + V(x)\psi$$

Let's first calculate the partial derivative of ψ with respect to time:

$$\partial\psi/\partial t = -iEA * e^{(-iEt/\hbar)} * \text{sgn}(x)$$

Now, let's calculate the second partial derivative of ψ with respect to position:

$$\partial^2\psi/\partial x^2 = A * e^{(-iEt/\hbar)} * 2\delta(x)$$

where $\delta(x)$ is the Dirac delta function.

Now, let's substitute these derivatives into the Schrödinger equation:

$$i\hbar * (-iEA * e^{(-iEt/\hbar)} * \text{sgn}(x)) = -\hbar^2/2m * (A * e^{(-iEt/\hbar)} * 2\delta(x)) + V(x) * (A * e^{(-iEt/\hbar)} * \text{sgn}(x))$$

Simplifying this equation, we get:

$$EA = -\hbar^2/m * \delta(x) + V(x) * \text{sgn}(x)$$

This equation should hold for all x and t if the square distribution wave function satisfies the Schrödinger equation. However, this equation implies that the potential $V(x)$ is proportional to the Dirac delta function $\delta(x)$ and the sign function $\text{sgn}(x)$. In general, the potential in the Schrödinger equation does not have this specific form. Therefore, the square distribution wave function does not satisfy the Schrödinger equation in its current form.

However, it is important to note that the Schrödinger equation has additional limitations and may not be applicable for certain scenarios as per below:

1. Non-Quantum Systems: The Schrödinger equation is specifically formulated for quantum systems, where particles exhibit wave-like properties. For classical systems or macroscopic objects, the Schrödinger equation is not applicable. Classical waves, such as electromagnetic waves or waves in a fluid, are described by different equations, such as Maxwell's equations or the wave equation.

2. Strongly Interacting Systems: In systems with strong interactions, such as those involving strong nuclear forces or high-energy particle collisions, the Schrödinger equation may not provide an accurate description. In these cases, more advanced theoretical frameworks, such as quantum field theory, are needed to account for the complex interactions and dynamics.

3. Relativistic Effects: The Schrödinger equation is based on non-relativistic quantum mechanics, which neglects relativistic effects, such as time dilation and length contraction. For particles moving at speeds close to the speed of light or in systems involving strong gravitational fields, relativistic quantum mechanics, such as the Dirac equation, is required.

Furthermore, the Schrödinger equation assumes weak interactions, where the potential energy terms are relatively small compared to the kinetic energy of the particle. In systems involving strong nuclear forces or high-energy particles, the interactions can be highly complex and cannot be accurately described by the simplified potential energy terms used in the Schrödinger equation.

However, when dealing with particles moving at speeds close to the speed of light or systems with strong gravitational fields, relativistic effects become significant. In these cases, relativistic quantum mechanics, such as the Dirac equation or quantum field theory, is required to provide an accurate description.

In summary, when the potential energy becomes negligible, the Schrödinger equation simplifies to the free-particle Schrödinger equation. When the kinetic energy is high, relativistic effects become important and the non-relativistic Schrödinger equation is no longer sufficient. In cases where the potential energy dominates the system's behaviour, and the Schrödinger equation provides a means to analyse and predict the particle's behaviour in this regime.

Furthermore, the Schrödinger equation is a linear equation, and it does not account for certain quantum phenomena that arise in strongly interacting systems or at very small length scales, such as particle creation and annihilation. In these cases, more advanced theoretical frameworks, such as quantum field theory, are required.

III. Discussions:

The Schrödinger equation is considered the most fundamental postulate of quantum mechanics because it provides a mathematical framework for describing the behaviour of quantum systems. It is the cornerstone of quantum mechanics and is used to calculate and predict the properties and behaviour of particles and systems at the microscopic level.

It can be easily seen from the observations section that the Schrödinger equation cannot predict most non-harmonic waves including the simplest form of square waves that are used in electronics and signal processing worldwide. All these common wave functions were not found to be the solution to the equation when substituted into it, including the time and spatial derivatives. In addition the equation is no longer applicable for systems with strong relativistic effects and the presence of an ill-defined potential energy function that includes strong nuclear forces, high-energy particles, particle creation and annihilation (not to mention the non-quantum systems).

We all know that a typical wave function of an electron is very complicated and is not always simple harmonic, harmonic or Gaussian. How can an equation be called the fundamental postulate of a branch of study if it cannot explain the path of an electron or a particle, in general when it is only valid to model a few ideal or close to ideal cases of its wave trajectory? This is the biggest anti-axiom of quantum mechanics where an equation is being postulated which is not sufficient to describe all the scenarios there is? How can this postulate be true to building the building of further theory, if the foundation is hollow and is standing on just a few pillars? Why are we using alternative methods and equations, such as density functional theory or quantum field theory, to build these additional pillars which should be one solid foundation block to rest the whole quantum mechanics building?

IV. Conclusion:

It can be concluded from the above facts that the Schrödinger equation can no longer be considered the fundamental postulate of quantum mechanics to define, calculate and predict the properties and behaviour of particles and systems at the microscopic level. This is based on the true definition of a postulate itself that provides the building block and lays the foundation for a study area rather than provide just a few pillars and depend on the rest of the other pillars of varying strength to support the entire quantum physical system. An alternative equation needs to be developed that is all encompassing, complete and accurate enough to define the wave equation to the dot.

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