# Catalogue Of Potential Magnetic Topological Insulators From Materials Database

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## Abstract:

Due to the quantum anomalous Hall effect (QAHE), magnetic topological insulators (TI) provide dissipationfree edge transport. However, there are only a small number of magnetic topological insulators that are known. This research identifies a huge number of magnetic topological insulators. Topological insulators with magnetic characteristics are found using data from two materials databases, considering the spin polarization of the unit cell at the Fermi level, magnetization per atom, and magnetization per cell. In addition to classifying, a machine learning model is employed to detect other magnetic topological materials by making use of a large variety of material attributes. The implementation of the quantum anomalous Hall effect (QAHE) at high temperatures will be made possible by a huge number of magnetic TI.

Key Word: Topological Insulator; Magnetic Materials; Machine Learning; Materials Database; Spintronics.

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## I. Introduction

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With the various prospective applications like dissipation less edge transport due to quantum anomalous Hall effect (QAHE), topological magneto-electric effect, topological magneto-optical effect, spintronic functionalities having nonlinear unidirectional magnetoresistance and high spin Hall angle, magnetic topological insulators are having growing interest of research community.<sup>1-7</sup> Topological insulators and topological semimetals are becoming new frontiers of research and can offer various functionalities similar to transistors.<sup>8-26</sup> Magnetic materials and magnetic topological insulators are receiving more and moreattention in the recent years.<sup>27–32</sup> To achieve QAHE, spontaneous magnetization is needed instead of external magnetic field. For this purpose, magnetic topological insulators (TIs) are generally made by transition metal doping. For example (Bi,Sb)<sub>2</sub>Te<sub>3</sub> (BST) is doped with Cr, V and Mn. But the Curie temperature of magnetic TIs is a few tens of Kelvins for a moderate doping level. The Curie temperature increases with the doping level, but dopants also create disorder which eventually affects QAHE. An alternative route towards the higher temperature realization of the QAHE is to incorporate the magnetic proximity effect. Generally TI is sandwiched between two ferromagnetic insulators with high Curie temperature (EuS, Cr<sub>2</sub>Ge<sub>2</sub>Te<sub>6</sub>, YIG and TIG). Besides this approach of high temperature realization of TIs, the MB<sub>2</sub>T<sub>4</sub>-family materials (M = transition-metal or rare-earth element, B =Bi or Sb, T = Te, Se, or S) have attracted significant attention due to their inherent magnetization (MnBi<sub>2</sub>Te<sub>4</sub> is an example). The monolayer is predicted to be an magnetic insulator for varying element of M (Ti, V, Mn, Ni,Eu) and their phonon dispersions are found in the literature. Considering their thickness, these materials behave as an axion insulators with Chern number, C = 0 in even number of layers and become quantum anomalous Hall (QAH) insulators with C = 1 in odd number of layers. But still the temperature at which QAHE can be achieved is only up to 4.5 K. Temperature at which QAHE can be achieved needs to be higher to use magnetic topological insulators in industry applications.

This study identifies a significant number of intrinsic magnetic topological insulators, which expands the field of materials research and paves the way for the experimental demonstration of the quantum anomalous Hall effect at higher temperatures. Recent initiatives have been made to leverage materials databases to uncover numerous topological insulators. Using magnetization data from a materials database, magnetic topological insulators are retrieved from these topological insulators. A few additional magnetic topological insulators are found using machine learning approach in addition to these two. Along with their magnetization statistics, certain recently discovered magnetic topological insulators' band structures are provided.

# II. Method

Topological Materials Database [topologicalquantumchemistry.com] provides total 4,321 topological insulator (TI) which were originally found from ICSD materials database. Total 4,321 topological insulators (TI) Topological Materials Database, found in the which may be accessed were at topologicalquantumchemistry.com. The Automatic FLOW (Aflow) [http://aflowlib.org] materials database, which contains 60,325 materials with ICSD tags, is searched for these topological insulators (Fig. 1). 455 topological insulators are discovered to have spin polarization at Fermi level that is larger than zero after paramagnetic or nonmagnetic materials were eliminated from the list of TI materials that were present in the Aflow database.



Figure 1. flowchart for determining magnetic topological insulators out of materials that are simultaneously listed in both the Topological Materials Database and the Aflow Materials Database and have the ICSD tag.



Figure 2. Flowchart for applying a machine learning algorithm to distinguish magnetic TI from materials without ICSD tags.

	Group	Polarization	Moment per $Atom(\mu_B)$
MnRh	221 (Pm-3m)	0.375308	2.54044
B <sub>2</sub> Gd	191 (P6/mmm)	0.346653	2.47534
CeCo <sub>8</sub> Mn <sub>4</sub>	139 (I4/mmm)	0.42245	2.43938
C <sub>3</sub> Fe <sub>17</sub> Ho <sub>2</sub>	166 (R-3m)	0.321525	2.35146
C <sub>3</sub> Er <sub>2</sub> Fe <sub>17</sub>	166 (R-3m)	0.281013	2.34616
Mn <sub>2</sub> NdSi <sub>2</sub>	139 (I4/mmm)	0.476767	2.19403
GaFe <sub>2</sub> Co	225 (Fm-3m)	0.451509	2.09263
Mn <sub>2</sub> Sb <sub>2</sub> Yb	164 (P-3m1)	0.384063	1.99097
SrMn <sub>2</sub> Sb <sub>2</sub>	164 (P-3m1)	0.263674	1.98692
Fe <sub>2</sub> Ge <sub>2</sub> Nd	139 (I4/mmm)	0.456738	1.98096
BaMn <sub>2</sub> Sb <sub>2</sub>	139 (I4/mmm)	0.275589	1.95873
Fe <sub>2</sub> NdSi <sub>2</sub>	139 (I4/mmm)	0.749423	1.87554
Fe <sub>3</sub> Se <sub>4</sub>	12 (C2/m)	0.417269	1.86067
Ge <sub>2</sub> Mn <sub>2</sub> Sr	139 (I4/mmm)	0.300165	1.73069
Ge <sub>2</sub> Mn <sub>2</sub> Yb	139 (I4/mmm)	0.348794	1.71251
DyGe <sub>2</sub> Mn <sub>2</sub>	139 (I4/mmm)	0.258104	1.62617
Mn <sub>2</sub> Si <sub>2</sub> Th	139 (I4/mmm)	0.317836	1.62291
DyMn <sub>2</sub> Si <sub>2</sub>	139 (I4/mmm)	0.342696	1.59548
ErMn <sub>2</sub> Si <sub>2</sub>	139 (I4/mmm)	0.300883	1.58879
Cr <sub>4</sub> U	87 (I4/m)	0.392315	1.49104
CeFe <sub>2</sub> Ge <sub>2</sub>	139 (I4/mmm)	0.270352	1.46693
CaGe <sub>2</sub> Mn <sub>2</sub>	139 (I4/mmm)	0.531608	1.44171
AuPr	221 (Pm-3m)	0.682797	1.43658
$Mn_2S_4Sn$	65 (Cmmm)	0.653758	1.42778
Fe <sub>7</sub> Nb <sub>6</sub>	166 (R-3m)	0.647596	1.40032

 Table 1. Part-1 of identified magnetic topological insulators which have ICSD tag. The space group, the unit cell's Fermi level spin polarization, and the magnetization per atom are all displayed.



Figure 3. Unit cell structure of some identified magnetic topological materials: (a) CaFe<sub>2</sub>As<sub>2</sub>, (b) Fe<sub>3</sub>Se<sub>4</sub>, (c) MnAlAu<sub>2</sub>, (d)MnAlRh<sub>2</sub>, (e) MnRh, (f) PuGe<sub>3</sub>, (g) SrFe<sub>2</sub>As<sub>2</sub>, (h) SrMn<sub>2</sub>Sb<sub>2</sub>, (i) GaFe<sub>2</sub>Co.

	Group	Polarization	Moment per Atom( $\mu_B$ )
CaFe <sub>2</sub> As <sub>2</sub>	139 (I4/mmm)	0.393105	1.38434
CdCu <sub>2</sub> Nd <sub>2</sub>	65 (Cmmm)	0.540909	1.38215
Al <sub>8</sub> Mn <sub>4</sub> U	139 (I4/mmm)	0.454842	1.37541
B <sub>6</sub> Fe <sub>3</sub> Pr <sub>4</sub>	166 (R-3m)	0.523744	1.35104
AgPr	221 (Pm-3m)	0.355468	1.34174
GdMn <sub>2</sub> Si <sub>2</sub>	139 (I4/mmm)	0.426179	1.33711
ErFe <sub>6</sub> Ga <sub>6</sub>	71 (Immm)	0.290458	1.32759
Fe <sub>2</sub> Si <sub>2</sub> Yb	139 (I4/mmm)	0.493325	1.24471
Al <sub>8</sub> Cr <sub>4</sub> Gd	139 (I4/mmm)	0.272686	1.22723
MnAlRh <sub>2</sub>	225 (Fm-3m)	0.302948	1.22407
PuGe <sub>3</sub>	221 (Pm-3m)	0.520987	1.18466
Al <sub>8</sub> CaMn <sub>4</sub>	139 (I4/mmm)	0.438069	1.16135
Fe <sub>2</sub> Si <sub>2</sub> Zr	139 (I4/mmm)	0.314701	1.15057
GdSn <sub>4</sub> Ti <sub>6</sub>	166 (R-3m)	0.735934	1.14793
DyFe <sub>2</sub> Si <sub>2</sub>	139 (I4/mmm)	0.503306	1.14548
As <sub>2</sub> Co <sub>2</sub> Nd	139 (I4/mmm)	0.730453	1.14179
Al <sub>8</sub> Mn <sub>4</sub> Th	139 (I4/mmm)	0.362908	1.12451
SrAs <sub>2</sub> Fe <sub>2</sub>	139 (I4/mmm)	0.308873	1.12139
MnAlAu <sub>2</sub>	225 (Fm-3m)	0.250696	1.11648
Cu <sub>2</sub> InMn	225 (Fm-3m)	0.254598	1.1052
Mn <sub>2</sub> Si <sub>2</sub> Yb	139 (I4/mmm)	0.393555	1.08173
Ga <sub>2</sub> Gd	191 (P6/mmm)	0.392402	1.07775
AlCu <sub>2</sub> Mn	225 (Fm-3m)	0.250066	1.06962
Al <sub>8</sub> Fe <sub>4</sub> Nd	139 (I4/mmm)	0.39693	1.06959
PrSb	225 (Fm-3m)	0.280469	1.05381

 Table 2. Part-2 identified magnetic topological insulators which have ICSD tag. Space group, spin polarization of unit cellat Fermi level and magnetization per atom are shown.

	Group	Moment per Atom $(\mu_B)$	Moment per $Cell(\mu_B)$
AgAlCr	216 - F43m	1.12052	3.36155
BiV	65 - Cmmm	1.04088	2.08177
AgAlMn	216 - F43m	1.73379	6.93518
Bi <sub>2</sub> V	164 - P3m1	1.02988	3.08965

 Table 3. Identified magnetic topological insulators using machine learning algorithm from the materials which do not have ICSD tag.

We choose a polarization range of 0.25 to 0.75 to look for prospective magnetic TIs because most common ferromagnetic materials (Fe or Co) have spin polarization at Fermi levels of around 0.5 while ferromagnetic half metals have spin polarization of one (the greatest conceivable value). A total of 50 materials were discovered to be magnetic topological insulators after only those materials are chosen that have magnetism per atom of greater than one Bohr magneton. Using the QUANTUM-ESPRESSO software, first-principle calculations are used to determine the band structure of various magnetic TI materials. The findings section displays the spin polarized band structure together with spin orbit coupling (SOC).

A further machine learning method is then used to find more magnetic Tis (Fig. 2). To do this, the AFLOW materials database's first 21 ICSD material attributes that can be found are extracted. They include topological materials that are mentioned in the Topological Quantum Chemistry database. The machine learning model is trained using these data using the extreme gradient boosting (XGBOOST) technique. A trained model is used to forecast topological insulators from materials in the AFLOW database that are missing the ICSD tag. Utilizing the magnetic moment data of all these projected topological insulators is necessary to identify magnetic topological insulators with magnetization per atom higher than one Bohr magneton.

# III. Results

For 50 identified magnetic topological materials with ICSD tags, space group, spin polarization of the unit cell at Fermi level, and atom-level magnetization are displayed in Tables 1 and 2. Table 3 provides details of detected magnetic topological insulators from non-ICSD materials using machine learning algorithms. In both situations, only topological insulators with magnetic moments per atom greater than one Bohr magneton are taken into consideration.



**Figure 4.** Calculated band structure of some identified magnetic topological materials: (a) CaFe<sub>2</sub>As<sub>2</sub>, (b) Fe<sub>3</sub>Se<sub>4</sub>, (c)MnAlAu<sub>2</sub>, (d) MnAlRh<sub>2</sub>. Blue color denotes up spin and red color denotes down spin.



**Figure 5.** Calculated band structure of some identified magnetic topological materials: (a) MnRh, (b) PuGe<sub>3</sub>, (c) SrFe<sub>2</sub>As<sub>2</sub>, (d) SrMn<sub>2</sub>Sb<sub>2</sub>, (e) GaFe<sub>2</sub>Co. Blue color denotes up spin and red color denotes down spin.

Some of the magnetic TIs' unit cell structures are depicted in Figure 3 together with their band structures in Figures 4 and 5. CaFe<sub>2</sub>As<sub>2</sub> has topological indices of  $Z_2 = 1$ ,  $Z_4 = 1$ ,  $Z_8 = 5$ , and magnetic moment per unit cell of 6.92 Bohr magneton. Fe<sub>3</sub>Se<sub>4</sub> has topological indices of  $Z_4 = 1$ , and magnetic moment per unit cell of

13.02 Bohr magneton. MnAlAu<sub>2</sub> has topological indices of  $Z_2 = 1$ ,  $Z_4 = 1$ ,  $Z_8 = 5$ , and magnetic moment per unit cell of 4.47 Bohr magneton. MnAlRh<sub>2</sub> has topological indices of  $Z_4 = 2$ ,  $Z_8 = 2$ , and magnetic moment per unit cell of 4.9 Bohr magneton. MnRh has topological indices of  $Z_{4\pi} = 2$ ,  $Z_8 = 4$ , and magnetic moment per unit cell of 5.08 Bohr magneton. PuGe<sub>3</sub> has topological indices of  $Z_2 = 1$ ,  $Z_{2w,3} = 1$ ,  $Z_4 = 3$ ,  $Z_{4\pi} = 1$ ,  $Z_{2w,1} = 1$ ,  $Z_{2w,2} = 1$ ,  $Z_8 = 7$ , and magnetic moment per unit cell of 4.74 Bohr magneton. SrFe<sub>2</sub>As<sub>2</sub> has topological indices of  $Z_2 = 1$ ,  $Z_{2w,3} = 1$ ,  $Z_4 = 3$ ,  $Z_{2w,1} = 1$ ,  $Z_{2w,2} = 1$ ,  $Z_8 = 7$ , and magnetic moment per unit cell of 4.74 Bohr magneton. SrFe<sub>2</sub>As<sub>2</sub> has topological indices of  $Z_2 = 1$ ,  $Z_{2w,3} = 1$ ,  $Z_4 = 3$ ,  $Z_{2w,1} = 1$ ,  $Z_{2w,2} = 1$ ,  $Z_8 = 7$ , and magnetic moment per unit cell of 5.6 Bohr magneton. SrMn<sub>2</sub>Sb<sub>2</sub> has topological indices of  $Z_{2} = 1$ ,  $Z_{4} = 1$ , and magnetic moment per unit cell of 9.93 Bohr magneton. GaFe<sub>2</sub>Co has topological indices of  $Z_2 = 1$ ,  $Z_4 = 1$ ,  $Z_8 = 5$ , and magnetic moment per unit cell of 8.37 Bohr magneton.

As the results are from two materials database and the Topological Materials Database do not have MnBi<sub>2</sub>Te<sub>4</sub> as a topological insulator, it is not shown in the result. But MnBi<sub>2</sub>Te<sub>4</sub> is mentioned in the introduction as it has been gaining research interests for the last few years. Some of the identified materials are antiferromagnetic topological insulator e.g., GdMn<sub>2</sub>Si<sub>2</sub>, ErMn<sub>2</sub>Si<sub>2</sub>, SrAs<sub>2</sub>Fe<sub>2</sub>. GdMn<sub>2</sub>Si<sub>2</sub> can be made as a layered antiferromagnet and can be of future research interest to explore thickness dependent property. Not all materials share the same characteristic. However, when a machine learning model is trained using known topological insulators, unit cell valence electron quantity and number of atoms in unit cell are discovered to be the most prominent features. Two materials databases were used to find the results, one of which solely included topological materials. Once more, several of the predicted materials have previously been demonstrated to be topological insulators, demonstrating the veracity of the findings.

#### **IV. Conclusions**

Using two materials databases, many magnetic topological insulators are found. To distinguish magnetic TI from non-ICSD materials, machine learning is also applied. These many different materials will make it easier for experimenters to investigate QAHE at greater temperatures because of the possibility of dissipation-less edge transport caused by QAHE.

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