Inflation and the Cosmological Constant

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Abstract: In this study, we use the Friedman – Robertson - Walker (FRW) model with \( \kappa = +1 \) for cyclic universe for understanding the physics of cosmic inflation. According to this model, kinetic energy of the universe is converted into gravitational energy during deceleration (compression) cycle. This builds up a dense assembly of photons close to the singularity of Planck dimensions at temperature \( T \sim < T_{Pl} \). Black body radiation approximation is used to analyze the distribution of the photons in the phase space using quantum statistical mechanics. It is shown that the available states in the phase space are completely filled up when the temperature reaches \( T_{Pl} \). When the temperature increases above \( T_{Pl} \), the assembly of superheated photons transforms to a non-equilibrium state. We show that an adiabatic transition from the non-equilibrium state to equilibrium state at \( T_{Pl} \) is accompanied by the inflation of space-time and the big bang.

The measurements of the red shifts of radiation from very distant galaxies and the measurement of anisotropy in Cosmic Microwave Background using Wilkinson and Planck observatories have confirmed that dark matter and dark energy (or a positive cosmological constant \( \Lambda \)) make contribution to the expansion of the universe. We discuss here the mechanism which builds up large amount of energy \( E_{inf} \) in the phase space created during the cosmic inflation. We propose that this energy is the source of the cosmological constant \( \Lambda \). Other mechanisms which transfer some of the energy released during the inflation to the observable universe are also discussed. The contribution from these sources to \( \Lambda \) is comparatively small.

Keywords: Cosmology, Big Bang, Inflation, Cosmological Constant, Dark Energy

I. Introduction

Einstein’s general theory of relativity together with the standard model of fundamental particles has been successful in providing a good understanding of the expansion of the universe, cosmic ray background and creation of light atoms, formation of large structures and many other observations in experimental cosmology [1]. Friedmann [2], Robertson [3] and Walker [4] applied Einstein’s equations for universe to an accelerating reference frame. This led to the interpretation of the Hubble expansion and to widespread use of their model. The finding of flat velocity curves of the stars in Andromeda galaxy (M31) by Rubin and Ford [5] gave support to the dark matter hypothesis. Investigations have shown that the dark matter resides in the spaces between the spiral arms of galaxies, in the intergalactic space, and around supermassive black holes. Its presence is also inferred from the gravitation lensing produced by it. The amount of dark matter far exceeds the observed quantity of the known forms of (normal) matter in the universe. Dark matter responds only to the gravitational force. Vaidya [6] has recently proposed that the ‘lowest energy state mesons made up of u and d quarks’ are the much sought after dark matter particles. Dark matter particles have not been directly observed or characterized till now; Trimble [7], de Stewart et al [8].

Another important finding was discovery by Penzias and Wilson [9] of the cosmic microwave background (CMB). This is a uniformly distributed 2.7 Kmicrowave background radiation which is relic of a glow of \( \sim 3000 \) K temperature which filled the universe \( \sim 360,000 \) years after the big bang. The measurement of rate of expansion of the universe by using the Hubble telescope and CMB lends support to the big bang hypothesis. Guth’s contribution [10] to inflationary cosmology led to many other studies using different types of potentials and ideas from the quantum field theory. He proposed that the universe grew out of a singularity of Planck dimensions \( L_{Pl} \sim 10^{-33} \) cm and (vacuum of) very high energy density \( \sim 10^{19} \) GeV. This highly unstable state then adiabatically transformed to low energy ground state accompanied by inflation - creation of space time at a rate which is not limited by the velocity of light. Guth’s hypothesis provided answers to homogeneity and flatness of the universe, and the horizon problem, namely, the inter-connectedness of its distant regions. Linde [11] proposed spontaneous formation of a vacuum state of very high energy density and its transformation to the ground state via Higg’s field or thorough tunnelling.

Einstein’s field equation for the universe [12] is

\[
R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = -8\pi G T_{\mu\nu} .
\]  

(1)
Here $R_{\mu\nu}$ is Ricci tensor, $R$ is Ricci scalar refer to the curvature of space time, $g_{\mu\nu}$ is metric tensor, $T_{\mu\nu}$ is energy - momentum tensor and $\Lambda$ is the cosmological constant. Einstein’s first paper did not contain the $\Lambda$ term. He added the term $-\Lambda g_{\mu\nu}$ (with $\Lambda > 0$) in his second paper. Weinberg [13] has given historical details pertaining to this subject. Energy associated with this term is also called dark energy. Like the first two terms in eq. (1), energy $\Lambda g_{\mu\nu}$ (arising from $\Lambda$) is stored as curvature of the space time.

A major breakthrough in this field occurred after 1998 with accurate measurements of red shift of very distant galaxies and Type1A supernovae as standard candle for distance measurements using Hubble telescope (Perlmutter et al. [14], Schmidt et al. [15], Reiss et al. [16], and with the measurements of anisotropy in CMB using space borne Wilkinson probe [17] and Planck Coll. [18]. These studies together showed that the dark matter and dark energy make contribution to the expansion of the universe. The red shift measurements showed that the energy density associated with dark energy or the cosmological constant $\Lambda$, is $\rho_\Lambda = (\Lambda c^2/4\pi G) = 5.96 \times 10^{-32}$ g/cc $\sim 10^{-9}$ erg/cc. Dark energy makes large contribution to the expansion because it is spread out throughout the universe and is needed to form the observationally flat universe.

There are good reviews on theoretical models for $\Lambda$, Narlikar and Padmanabhan [19], Carroll [20], Piatella [21]. It has been speculated that the zero-point energy of the vacuum, regularized due to the existence of a suitable ultraviolet cut-off scale, could be the source of the non-vanishing cosmological constant. However, Mahajan, Sarkar and Padmanabhan [22] have shown that the presence of such a cut-off can significantly alter the results for the Casimir force between parallel conducting plates and even lead to repulsive Casimir force when the plate separation is smaller than the cut-off scale length. Using the experimental data they ruled out the possibility that the observed cosmological constant arises from the zero-point energy which is made finite by a suitable cut-off. Explanations based on quantum field theory have been widely explored. According to the quantum field theory, empty space is a collection of these fields and their zero point energy make contribution to $\rho_\Lambda$.

When calculations are performed, the total zero point energy of the quantum fields exceeds estimated value of $\Lambda$ by some 120 orders of magnitude, a discrepancy that has been called the worst theoretical prediction in the history of physics [13], [19].

In this article we take up two topics, namely, ‘what caused big bang?’ and ‘the source of energy associated with the cosmological constant $\Lambda$’. In Sec. II, we summarize main results of Friedman-Robertson-Walker (FRW) model and its extension called $\Lambda$CDM model. In Sec. III, we discuss the formation of a non-equilibrium state of a dense assembly of photons in singularity region of dimension $l < \ell_{Pl} \sim 10^{-33}$ cm, which leads to the big bang. In Sec. IV, we discuss the origin of $\Lambda$. Narlikar and Padmanabhan [19]have pointed out that the solution to the cosmological problem should be compatible with the inflationary scenario. Our proposal about the cosmological constant meets this requirement.

II. FRIEDMAN – ROBERTSON - WALKER (FRW) MODEL

For a spherically symmetric space, proper time in terms of metric $g_{\mu\nu}$ is

$$\begin{align*}
\mathrm{d}t^2 &= -g_{\mu\nu} \mathrm{d}x^\mu \cdot \mathrm{d}x^\nu = -g_\theta \mathrm{d}t^2 - g_\rho \mathrm{d}r^2 - g_{\theta\rho} \mathrm{d}\theta^2 - g_{\phi\phi} \mathrm{d}\phi^2 \quad (2)
\end{align*}$$

The metric of the FRW model of the universe is based on the assumption that the universe is homogeneous and isotropic, and is an ideal fluid. FRW metric is [2-4]

$$\begin{align*}
\mathrm{d}t^2 &= -R^2(t) \left( \mathrm{d}r^2/(1-\kappa r^2) + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \right) \quad (3)
\end{align*}$$

Here $R(t)$ is the radius of the universe at time $t$ in four dimensional space-time. $R_0 = R(t_0)$ is the radius at the present time $t_0$. A comparison of eqs. (2) and (3) gives [1]

$$\begin{align*}
g_\theta = -1, & \quad g_\rho = R(t)^2/(1-\kappa r^2), \quad g_{\theta\rho} = rR(t)^2, \\
g_{\theta\theta} = r^2 \sin^2\theta, & \quad g_{\phi\phi} = 0 \quad \text{for} \mu \neq \nu. \quad (4)
\end{align*}$$

Cosmological principle is used in this derivation. Accordingly, $g_{\mu\nu}$ now the metric in co-moving coordinates in maximally symmetric space. The Einstein field equation is a tensor equation relating a set of symmetric $4 \times 4$ tensors. Each tensor has 10 independent components. The four Bianchi identities reduce the number of independent equations from 10 to 6. Using $g_{\mu\nu}$ from Eq. (4) in Einstein’s equations and solving them gives [1]

$$\begin{align*}
3R &= -4\pi G (\rho + 3p) \quad (5) \\
R^2 &= -\kappa + (R^2/3) (8\pi G \rho) \quad (6)
\end{align*}$$

And the energy conservation equation

$$\frac{\mathrm{d}pR^3}{\mathrm{d}R} = -3pR^2. \quad (7)$$

In these equations, dot over $R$ represents time derivative, $\rho$ is the energy density, $p$ is pressure and Newton’s gravitational constant $G = (6.67)\times 10^{-8}$ dyne. cm$^2$. gm$^{-2}$. The spatial curvature is $\kappa /R_0^2$; $\kappa = +1$, 0 and -1 depends on whether the present energy density $\rho_0$ is $> , =$ or $< \rho_c$. The values of $\kappa = +1$, 0 and -1 lead to closed, steady-state and open universe, respectively. In terms of Hubble parameter $H_0 = (R/\rho_0)$, critical energy density

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FRW model becomes applicable when photons and all forms of matter in the universe exchange energy with one another and are in equilibrium. The early universe, following the big bang, has relativistic particles in equilibrium. Hence \( \rho = 3p^2 \) and eq. (7) gives \( \rho_0 \propto R^{-4} \). We have used here suffix \( \nu \) for photons. Since energy density follows the law of black body radiation and \( \rho_0 \propto T^4 \), one gets \( R \propto T^{-1} \). In matter dominated universe \( \rho = 0 \) and from eq. (7) one gets \( \rho_0 R^2 = \) constant, or \( \rho \propto R^{-3} \).

Contributions to energy density from matter and cosmological constant can be included in the FRW model eq. (3) by replacing \( \rho \) by a weighted sum of energy densities \( \rho = \Sigma \rho_i \) with \( \rho_{DM} = \Omega_{DM} \cdot \rho_0 \), \( \rho_M = \Omega_M \cdot \rho_0 \), \( \rho_\Lambda = \Omega_\Lambda \cdot \rho_0 \) and \( \rho_\gamma = \Omega_\gamma \cdot \rho_0 \). It is seen that \( \rho = \Sigma \rho_i = \rho_{tot} \cdot \rho_0 \) with \( \rho_{tot} = (\Omega_{DM} + \Omega_M + \Omega_\Lambda + \Omega_\gamma) \). We can use equation of state \( p_i = w_i \rho_i \) with \( w_i = 0 \) for non-relativistic matter and \( w_i = 1/3 \) for relativistic matter. In \( \Lambda \)CDM model FRW equation takes the form

\[
\frac{\dot{R}^2}{R^2} = \Sigma \Omega_i \left( \frac{R_0}{R} \right)^{3(1+w_i)} - \left( \kappa / R^2 \right). \tag{8}
\]

Values of parameters \( H = (R / R) \) and \( \Omega_s \) are obtained from \( \Lambda \)CDM eq. (10) by fitting a CMB power spectrum or by fitting the line shift data.

Schmidt et al. [15] found that \( \Omega_M \sim 0.4 \) and \( \Omega_\Lambda \sim 0.6 \), and Reiss et al. [16] found that \( \Omega > 0 \). The Wilkinson Microwave Anisotropy Probe (WMAP) [17] spacecraft seven-year analysis estimated that the universe is made up of 72.8\% dark energy, 22.7\% dark matter, and 4.5\% ordinary matter. Planck Collaboration [18] measurements of the CMB give estimates 68.3\% dark energy, 26.8\% dark matter, and 4.9\% ordinary matter.

Using \( \rho_\gamma = 1.1 \times 10^{-29} \text{ g/cm}^3 \), this gives \( \Omega_\Lambda = \rho_\gamma / \rho_0 \). \( \Omega_\Lambda = (1.1 \times 10^{-29} \text{ g/cc}) (0.683) = (0.75) \times 10^{-29} \text{ g/cc} \).

III. INFLATION

The inflation models of Guth [10] and Linde [11] postulate spontaneous formation of vacuum state of very high energy density and do not describe the physical process which creates such a state. We propose a different scenario. We use FRW model for \( \kappa = +1 \). In this case, the universe initially expands and then it collapses into a singularity. We show that formation of singularity of high energy density is admissible within the framework of this model.

Mostly photons are present in the singularity region which has volume \( l_0^3 \), where \( l_0 > l_{pl} \sim 10^{-33} \text{ cm} \), and \( T > T_{pl} \sim 10^{-12} \text{ K} \). We use quantum statistical mechanics for photons to calculate the number of states or cells per unit volume \( \zeta \) occupied by photons in singularity region. \( \zeta \) is a product of number of states per unit volume between \( k \) and \( k + dk \) in the phase space, and its probability of occurrence of \( e^{h \nu/kT} - 1 \) integrated over \( k \). Here \( k \) is the wave vector and \( \nu = (c/2\pi k) \) is the frequency of the photon. From quantum statistical mechanics [23] it follows that

\[
\zeta = \int_0^\infty [2(4\pi k^2 dk)/(2\pi)^3] e^{h \nu/kT} - 1 \] \tag{9}

Since \( k = (2\pi \nu/c) \), it follows that

\[
\zeta = \int_0^\infty [(8\pi /c^3) \nu^2 dv] e^{h \nu/kT} - 1 \] \tag{10}

which simplifies to

\[
\zeta = 8\pi (2.4) (kT/\hbar)^3 \] \tag{11}

We use \( h = 6.67 \times 10^{-27} \text{ ergs} \cdot \text{s} \) for Planck constant, \( k = 1.38 \times 10^{-16} \text{ erg/K} \) for Boltzmann constant and \( c = 3 \times 10^{10} \text{ cm/sec} \). Since each state occupies volume \( \hbar^3 \) in the phase space, volume occupied by \( \zeta \) states per unit volume is

\[
V_{ps}(T) = \zeta \hbar^3 = (19.785) T^{-3} \cdot \hbar^3 \] \tag{12}

In the equilibrium state, volume fraction \( V_{ps}(T) \) is always less than or equal to 1. Thermodynamic equilibrium is maintained up to temperature \( T = T_E \) defined by condition

\[
\frac{V_{ps}}{\zeta} \hbar^3 = 1 \quad \text{or} \quad \zeta = \frac{\hbar^3}{(19.785) T_E^3} \] \tag{13}

Eqs. (11) and (12) give

\[
T_E = (60.288) c^2 / (c/\kappa) = (5.52) \times 10^{25} \text{ K} \] \tag{14}

The above analysis shows that the assembly of photons transforms to a non-equilibrium state above \( T_E \). Using expression \( S = k \ln \zeta \) for entropy [23] and eq. (11) one gets

\[
S = 3k (1 + \ln T) \] \tag{15}

and the entropy at \( T_E \) as

\[
S_E = k \ln \zeta = 3k (1 + \ln T_E) = k (180.8) \] \tag{16}

Since \( V_{ps} \) cannot exceed 1 at \( T = T_E \), \( \zeta \) and \( S \) remain constant at temperature \( T > T_E \). Mean energy density of this dense assembly of photons from Stefan Boltzmann law is

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E\text{E}M close to the vertical axis Ox because its slope is greater than the velocity of light, c. As

\begin{align*}
\tau = \frac{7.56 \times 10^{19}}{T} \text{sec, by } e^{15} \\
\end{align*}

This question is not answered by many models on inflation. As mentioned earlier, the inflation ends at B when the temperature of the expanding universe decreases to T_E and the contents of the universe attain thermal equilibrium. Thermal equilibrium is attained at T_E because the cells of the phase space of volume

\begin{align*}
\frac{1}{2} m c^2 = \frac{1}{2} m c^2
\end{align*}

is the enormous difference in the energy densities

\begin{align*}
U_M = U_E = \frac{1}{2} m c^2 \text{ cm}
\end{align*}

We shall state the main results of our model of inflation based on instability of an assembly of photons

\begin{align*}
\tau = \frac{7.56 \times 10^{19}}{T} \text{sec, by } e^{15}
\end{align*}

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\begin{align*}
\frac{1}{2} m c^2 = \frac{1}{2} m c^2
\end{align*}

The non-equilibrium state is retained by the inward force of the collapsing universe which opposes

\begin{align*}
\frac{1}{2} m c^2 = \frac{1}{2} m c^2
\end{align*}

Mostly photons are present in the singularity region which has volume l_s^3, where l_s is greater than the Planck length l_PL \approx 10^{-33} \text{ cm}, and temperature which is smaller than the Planck temperature T_PL \approx 10^{32} \text{ K}. Let us suppose that universe reaches maximum temperature T_U = 10^{30} \text{ K} when the confining pressure becomes zero. At this stage, the unconfined photons cause inflation of the universe – an expansion at a rate which can exceed the speed of light. Temperatures in this range have been discussed in literature. The energy density that drives the transformation from non-equilibrium state to the equilibrium state is the enormous difference in the energy densities

\begin{align*}
U_M - U_E = \frac{1}{2} m c^2 \text{ cm}
\end{align*}

This can also be seen in another way. At temperature T_M = 10^{15} \text{ K}, the U_M is \sim 10^{19} times the value U_E at T_E. This far-from-equilibrium superheated state of photons has energy density U_M \sim 7 \times 10^{45} \text{ erg cc}^{-1} and pressure p_M \approx (U_M/3) \sim 10^{28} \text{ bar}. When the confining pressure is dissipated and becomes zero, the release of energy of this superheated state causes rapid inflation of the phase space or space-time. It is seen from eq. (14) that this is an adiabatic transformation because the entropy remains constant in the temperature range T_M to T_E.

In eq. (6) which gives the rate of expansion of the universe, \kappa is very small in comparison to the second term on the right hand side and so it makes no difference in the discussion of early universe whether \kappa = +1, 0 or -1; Weinberg [1, page 530]. We can therefore take \kappa = 0 and write eq. (6) as

\begin{align*}
\dot{R}^2 = (R^2/3) (8\pi G \rho) R^2.
\end{align*}

We have assumed that photons dominate the early universe and so \rho = \rho_v = U_M, the energy density at T_M the temperature at the start of inflation. After reinserting c, and \rho = U_M the above expression becomes

\begin{align*}
\dot{R}^2 = (8\pi G U_M / 3c^2) R^2.
\end{align*}

The assembly of photons in the singularity region is in a non-equilibrium state while eq. (17) and (18) are applicable to a system in equilibrium. This should be kept in mind while using these equations for the calculation of \dot{R} during the inflation. On taking square root of eq. (18) and integration one gets

\begin{align*}
R_v = R_0 e^{\tau} \text{ with } \tau = (3c^2/8\pi G U_M)^{1/2}.
\end{align*}

Here \tau is the time constant of the exponential expansion. Using G = (6.67) \times 10^{-8} \text{ cm}^3 \text{ gm}^{-1} \text{ s}^{-2} for Newton’s gravitational constant we get

\begin{align*}
\tau = (4.01) 10^{13} / U_M^{1/2} \text{ sec} = (4.61) 10^{40} / T^2
\end{align*}

For T = 10^{30} \text{ K}, eq. (18) gives \tau = 4.61 \times 10^{40} \text{ sec}. It follows from eq. (19) that R_0 increases by 10^4 after (4.68) \times 10^{39} \text{ sec}, by 10^9 after 10^{-38} \text{ sec, and by } e^{100} after (4.6) \times 10^{-38} \text{ sec, which is an inflation scenario. The entropy remains constant during the inflation. The inflation ends when the temperature falls to T_E. When the inflation ends, and } R \text{ of the photon dominated universe decreases as } T^{-1}. \text{ Following this, there is transition to matter dominated universe below Higg’s temperature } \sim 110 \text{ GeV. In the present epoch, we are in the matter dominated universe where energy density } \rho \propto R^{-3}.

IV. Summary of our model

We shall state the main results of our model of inflation based on instability of an assembly of photons at the singularity with l_s \approx 10^{-33} \text{ cm}, and T < T_PL \sim 10^{32} \text{ K}. We have shown in Sec. III that inward compression of dense assembly of photons due to gravity in the collapsing universe creates singularity of Planck dimensions and drives it to a non-equilibrium state. The release of immense energy from this small region of singularity at temperature T_M leads to inflation. This is schematically shown in Fig. 1. This diagram is not drawn to scale. The curve OAB represents sudden expansion of the phase space (space-time) on inflation. This curve is nearly a steep close to the vertical axis Ox because its slope is greater than the velocity of light, c. As explained by Guth [10], the inflation model resolves the horizon, flatness and magnetic monopoles problems, and is consistent with the observations on the expansion of the universe.

IVA. When does the inflation end?

This question is not answered by many models on inflation. As mentioned earlier, the inflation ends at B when the temperature of the expanding universe decreases to T_E and the contents of the universe attain thermal equilibrium.
IVB. Cosmological constant

In the present work we have used FRW model with $\kappa = +1$ which leads to a cyclic universe. The photon driven expansion of the universe, inflation at $T_M$ kick starts the conversion of gravitational energy of the universe into kinetic energy during its expansion cycle. The phase space created by inflation is shown as region 1 in Fig. 1. In this region the energy density $\rho_{\text{inflation}}$ which depends on $(\dot{R}/R)^2_{\text{inflation}}$, eq. (17), is very large because of large value of $\dot{R}$. Here we are using eq. (17) for the universe and $\rho$ is the energy density of the universe. Accordingly, a very large amount of energy builds up in the region 1(Fig. 1). As mentioned above, this energy is comes from the (stored) gravitational energy of the universe. The energy $E_{\text{ps}1}$ becomes a part of the phase space, the matrix of space time. Energy density $\rho_{\text{inflation}}$ and $\Lambda$ depend on the temperature at the start of the inflation $T_M$. The change in the curvature of the space time at $T < T_M$ transforms the gravitational energy into kinetic energy and drives the FRW expansion. The same mechanism transforms energy $E_{\text{ps}1}$, through change in the curvature of space-time tensor, into kinetic energy and provides additional expansion. The two energies add up and contribute energy density $\rho_{\text{FRW}} = (\rho_v + \rho_{\Lambda})$ to the expansion of the universe. $\rho_{\Lambda}$ is the energy density that comes from $E_{\text{ps}1}$, and is associated with the cosmological constant. Since every big bang has a distinctive $T_M$ and $\rho_{\Lambda}$ and $\Lambda$ depend on $T_M$, it also has distinctive $\rho_{\Lambda}$ and $\Lambda$. We have brought out for the first time the close relationship between the inflation and $\Lambda$, which is expected from a theoretical model [19].

V. Other models of inflation

In quantum field theories (QFTs), the notion of empty space is replaced by with vacuum state defined as the lowest energy state (ground state) of the relevant quantum field. Phase transitions in the early universe are connected with the symmetry breaking of the field which leaves the vacuum state of lower symmetry than before. Nearly all the models make use of a symmetry breaking transition of the standard model for driving the inflation. In standard model, the sequence of symmetry breaking transitions and their energies are – grand unified theory (GUT) $\sim 10^{14}$ GeV, electroweak $\sim 10^2$ GeV, and quantum chromo-dynamics (QCD) $\sim 10^{-1}$ GeV respectively. The GUT transition removes supersymmetry.

In Guth’s work, the inflation is attributed to the transition at the GUT field. Guth [10] assumed that universe near singularity contains relativistic photons and electrons, and derives the rate of expansion of the universe at the inflation using eqs. (18-20). However, he presented a complicated mechanism for the inflation. As the energy is lowered, the system goes through an hierarchy of spontaneous symmetry driven transformations such as SU(5) to SU(3) X SU(2) X U(1) at energy $10^{14}$ GeV. He proposed that inflation occurs at a such phase transitions at a critical temperature $T_c$ between $10^{13}$ to $10^{15}$ GeV ($\sim 10^{25}$ to $10^{27}$ K). He also postulated that the universe somehow supercools by 28 orders of magnitude temperature below $T_c$ which is...
smaller than the singularity temperature $10^{28}$ K. This assumption was made to ensure that the inflation is an adiabatic transition. The reasoning might be that the entropy would not change when the temperature of the universe suddenly increases by 28 orders of magnitude or more from the super-cooled state during inflation because, by the Nernst theorem, the entropy of any system is zero at 0 K and at negative temperatures. We have shown that such an assumption is not required to show that the inflation is an adiabatic transition.

Albrecht and Steinhardt [24] have proposed that models in which the GUT symmetry is broken radiatively can lead to inflation. Gravity mediated supersymmetry breaking mechanism can also be considered because gravitational field is very high at these energies. There is so far no evidence for supersymmetry because no pairs of particles related by supersymmetry have been discovered [25].

Khoury et al. [26] and Steinhardt and Turok [27] have proposed that the cosmic inflation is a result of collision of two branes in the structure of the universe, a subject which falls under the string theory. It is a cyclic model of the universe, which avoids the problems of entropy associated with the oscillatory universe.

VI. Other models of cosmological constant

Sahni and Starobinsky [28] have reviewed the generation of $\Lambda$ in models with spontaneous symmetry breaking and through quantum vacuum polarization effects, and the mechanisms which give rise to a large value of $\Lambda$. They also discuss the attempts to generate a small cosmological constant at the present epoch using either field theoretic techniques, or by modelling a dynamical $\Lambda$-term by scalar fields.

During the cosmic inflation, the space – time expands at velocity greater than c. This phenomenon lasts for a very short time (< 1 microsecond) after the beginning of the inflation. Quantum fluctuations dominate the region 1 (Fig. 1) because of very high energy density. Occasionally these fluctuations enter the observable universe region 2 and create bubbles of very high energy density. A strong gravitational field in the bubbles leads to the formation of primordial black holes (PBH) -- Khlopov [29], Sakai et. al [30]. Misra and Sahni [31] have shown that PBH in the mass range $10^{15}$ M$_{\odot}$ to 100 M$_{\odot}$ are formed from single field inflation in presence of small bump-like or dip-like feature in the inflaton potential. The quantum fluctuations also create bubbles of high energy density having repulsive gravity. The concept of spontaneous bubble inflation has been widely researched and extrapolated to energy scale ~ $10^{18}$ GeV as in Linde’s work [11]. We have mentioned earlier that the formation of such spontaneous bubble has been proposed as the cause of the cosmic inflation. The approach uses a suitable inflaton potential energy function. In one such formulation, a slowly moving scalar field with high but slowly varying potential energy density leads to negative pressure or inflation; Bauman and Pieris [32]. The formation bubbles of primordial black holes and primordial inflation, according to these models, bring energy $E_{\text{BH}}$ and $E_{\text{PI}}$ to the observable universe, region 2 (Figure 1).

Niikura et al. [33] have performed a dense-cadence, 7 hour-long observation of the Andromeda galaxy (M31) with the Subaru Hyper Suprime-Cam. They used microlensing of stars in M31 by PBHs lying in the halo regions of the Milky Way (MW) and M31. They found that the number of PBHs is very small and below the expected number. Such observations suggest the contribution PBHs to mass density of the universe $\rho_{\text{PBH}}$ is very small compared to the contributions from other sources. Since primordial black holes and primordial inflatons (white holes) are likely to be formed with nearly equal probability, the energy contribution of primordial inflatons, to region 2 in Figure 1 is also likely to be very small in comparison to the energy $E_{\text{inflation}}$.

A new source of cosmic energy quintessence has been proposed by Peebles and Ratra [34] and by Caldwell et al. [35] as an alternative to the cosmological constant. Quintessence is a dynamical, evolving, spatially inhomogeneous component with negative pressure, it is spatially inhomogeneous. Its negative pressure is considered to be sufficient to drive the accelerating expansion. It is not like the cosmological constant which has a very specific form of energy.

VII. Conclusion

We have proposed a new mechanism based on application of quantum statistical mechanics to an assembly of photons near a singularity of Planck dimensions at temperatures close to $T_{\text{Pl}}$ to explain the cosmic inflation. We have used, the FRW equations to identify the source of energy that drives $\Lambda$. We have also briefly discussed prominent contributions made by various groups in both the fields.

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References


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