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Effects of Temperature Dependent Viscosity and Magnetic Field on the Oscillations in Triply Diffusive Convection

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Abstract:*The stability of the triply diffusive convective layer of fluid is studied to analyze the effects of temperature dependent viscosity and uniform magnetic field on the growth rate of oscillations. A condition for the stability and consequently for the existence ofoscillatory motions of growing amplitude are derived, using some nontrivial integral estimates obtained from the eigenvalue equations. The upper bounds for the complex growth rate of oscillatory modes are prescribed, which yields that the oscillations can be controlled by an appropriate choice of the parameters of the problem. It is observed that both the temperature dependent viscosity and the magnetic field have the stabilizing effect on the oscillatory instability i.e. both postpone the onset of oscillatory motions in triply diffusive convection. Some consequences of the derived results are also deduced, which are also uniformly valid for the quite general nature of bounding surfaces.*

Keywords: *Triply-diffusive convection; linear stability; Growth rate; Magnetic field; Temperature dependent Viscosity, Oscillatory motions.*

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I. Introduction

A natural convection flow in a gravity field is termed as Thermohaline convection/ Double-diffusive convection, if both thermal and concentration gradients are affecting the density of the fluid. In the context of the thermohaline instability problem, two fundamental configurations have been studied by Stern [1]/ Veronis [2], wherein the temperature gradient is stabilizing/ destabilizing and the concentration gradient is destabilizing/stabilizing. Gupta *et. al.* [3] studied the stability of Veronis and Stern type's configurations in horizontal layer geometry and derived the sufficient conditions for the stability of an initially top-heavy or an initially bottom-heavy configuration by controlling the growth rate of the oscillatory motions of neutral or growing amplitude. For a broad view of the subject one may be referred to Brandt and Fernando [4].

The influence of magnetic field on an electrically conducting fluid contribute to two kinds of effects: first, by the motion of electrically conducting fluid across the magnetic lines of force, electric currents are generated and the associated magnetic field contribute to change in the existing fields and second, the fluid elements carrying currents transverse magnetic lines of force contributes to additional forces like Lorentz Force acting on the fluid elements. The effect of magnetic field on the stability of thermal convection problem has been thoroughly investigated by Chandrasekhar [5].

Straughan [6] has pointed that the viscosity of a fluid is one of the properties which are most sensitive to temperature. Torrance & Turcotte [7] observed that the viscosity of the liquids decreases with increasing temperature, while the reverse trend is observed in gases. Therefore, the type of fluid and the range of operating temperatures are very crucial parameters in the study of fluid dynamics. Authors have considered the viscosity of the fluid to be constant in most of the studies pertaining to single or double-diffusive convection. Because, additional mathematical complexities arises in convective instability problems, when the viscosity of the fluid is varying with temperature. Some authors, including Stengel *et al.* [8] and Dhiman and Kumar [9] have studied the stability of thermal convection problem by taking into account the variations in viscosity. Dhiman and Kumar [10] studied the thermohaline convection problems under the influence of temperature dependent viscosity and derived some general qualitative and quantitative results for each combination of rigid and dynamically free boundary conditions.

The problem of multi diffusive-convection (when temperature and two or more component agencies, or three different salts are present) becomes more interesting than double-diffusive convection problem. Triple-diffusive convection is of great importance because of its usefulness in describing so many natural phenomena such as acid rain effects, underground water flow, warming of stratosphere etc. Griffiths [11], Pearlstein *et. al.* [12] have studied theoretically the onset of convection in a horizontally fine layer of a triply-diffusive fluid (in which the density depends on three independently diffusing components having different diffusivities) and derived some general results. Many applications of triply-diffusive convection have been discussed by

Pearlstein *et. al.*and they predicted that the onset of convection in triply-diffusive fluid might occur via a quasi-periodic bifurcation from the motionless basic state. For further details in the discipline of multi component diffusion one may be referred to Lopez *et. al.* [13].

This paper is aimed precisely in the direction to include the effects of viscosity variation and magnetic field (acting opposite to the gravity) in triply-diffusive convection problem. Following linear stability theory, a sufficient condition for the stability of oscillatory motions for this general problem is derived, using some integral estimates obtained from the non-dimensional linearized perturbations equations governing the problem and hence the bounds for the complex growth rate for arbitrary neutral or unstable oscillatory modes (when it exist) are prescribed. The obtained results are uniformly valid for all cases of boundary conditions. Some consequences for the triply-diffusive convection problem and double-diffusive convection problems are also worked out.

II. Physical Configuration and Basic Equations

A horizontal layer of viscous incompressible fluid of infinite horizontal extension and finite vertical depth is statically confined between two horizontal boundaries $z = 0$ and $z = d$, which are respectively maintained at uniform temperatures$T_0$and $T_1 (< T_0)$ and uniform solute concentrationsS_{a0} , $S_{b0}$and$S_{a1} (< S_{a0})$, $S_{b1} (< S_{b0})$. This layer of fluid is kept under the influence of uniform vertical magnetic field. Let the viscosity of the fluid depend upon temperature. Let the origin be taken on the lower boundary $z = 0$ with z-axis perpendicular to it. It is further assumed that cross diffusion effects are neglected.

Following the usual steps of linear stability theory, the non-dimensional linearized perturbation equations for triply-diffusive convection problem with magnetic field and temperature dependent viscosity can be put in the following form (*c.f.* Gupta *et. al.* [3] and Dhiman and Kumar [10]);

$$f(D^2 - a^2)^2 w - \frac{p}{\sigma}(D^2 - a^2)w + 2(Df)D(D^2 - a^2)w + D^2 f(D^2 + a^2)w$$

$$= R_T a^2 \theta - R_S a^2 \phi_1 - R_S' a^2 \phi_2 - QD(D^2 - a^2)h_z \tag{1}$$

$$(D^2 - a^2 - p)\theta = -w \tag{2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_1}\right)\phi_1 = -\frac{w}{\tau_1} \tag{3}$$

$$\left(D^2 - a^2 - \frac{p}{\tau_2}\right)\phi_2 = -\frac{w}{\tau_2} \tag{4}$$

$$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)h_z = -Dw \tag{5}$$

together with following boundary conditions

$$w = 0 = \theta = \phi_1 = \phi_2 = h_z \ \ at\ z = 0\ and\ z = 1; \tag{6}$$

and the bounding surface is dynamically free or rigid according as $D^2 w = 0\ or\ Dw = 0$ on the lower/upper boundary *i.e.*at $z = 0$and $z = 1$.

In the foregoing equations, $z$is the real independent variable, $D = d/dz$ is the differentiation with respect to z, $a^2$is the square of the wave number, $\sigma$is the thermal Prandtl number, $\sigma_1$is the magnetic Prandtl number,$\tau_1, \tau_2$are the Lewis numbers for the two concentrationsS_a and $S_b$respectively, $R_T$is the thermal Rayleigh number, $R_S$and $R_S'$are the concentration Rayleigh numbers for the two concentration components, Q is the Chandrasekhar number, $p(= p_r + ip_i)$ is the complex growth rate, w, θ, h_z are the perturbations in the vertical velocity, temperature, magnetic field respectively, ϕ_1, $\phi_2$are the perturbations in the respective concentrations of the two components and $\mu[= \mu_0 f(T)]$is the viscosity of the fluid; μ_0 is the viscosity at the lower boundary and $f(T)$ is an arbitrary function of vertical coordinate z.

We have also followed the assumptions of Stengel*et. al.*[8] (regarding the small ratio of the viscosities at the top to the bottom boundaries), in deriving the above equations.

System of equations (1) – (5) together with either of the boundary conditions (6) governing triple diffusive convection problem with temperature dependent viscosity in the presence of magnetic field constitutes an eigen value problem for$R_T$for given values of the other parameters. Further, a given state of system is stable, neutral or unstableaccording as p_r is negative, zero or positive. Further if $p_r = 0$ implies $p_i = 0$ for every wave number$a$ then the *principle of the exchange of stabilities* (PES) is valid, otherwise we shall have *overstability* at least when instability sets in as certain modes.

III. Stability of the oscillatory modes

We shall investigate the character of the oscillatory modes. Let us suppose that$p_i \neq 0$, *i.e.* PES is not valid and instability is through oscillations.

Multiplying equation (1) by w^*(the complex conjugate of w) and integrating the resulting equation by parts a suitable number of times and using the relevant boundary conditions (6) and equations (2)-(5), we have

$$\int_0^1 f(|D^2 w|^2 + 2a^2|Dw|^2 + a^4|w|^2)dz + \frac{p}{\sigma}\int_0^1 (|Dw|^2 + a^2|w|^2)dz + \int_0^1 (D^2 f)|w|^2 dz$$

$$= R_T a^2 \int_0^1 (|D\theta|^2 + a^2|\theta|^2)dz - R_S a^2 \tau_1 \int_0^1 (|D\phi_1|^2 + a^2|\phi_1|^2)dz$$

$$-R_S' a^2 \tau_2 \int_0^1 (|D\phi_2|^2 + a^2|\phi_2|^2)dz - Q \int_0^1 (|D^2 h_z|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2)dz$$

$$+a^2 p^* \left[R_T \int_0^1 |\theta|^2 dz - R_S \int_0^1 |\phi_1|^2 dz - R_S' \int_0^1 |\phi_2|^2 dz - \frac{Q\sigma_1}{\sigma} \int_0^1 (|Dh_z|^2 + a^2|h_z|^2)dz\right] \quad (7)$$

Subsequently, for convenience, we will omit the limits of integration from the integral sign and dz from the integrand.Now, equating the real and imaginary parts of both sides of equation (7) and cancelling $p_i(\neq 0)$ throughout from the imaginary part, we have

$$\int f(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2) + \frac{p_r}{\sigma}\int (|Dw|^2 + a^2|w|^2) + a^2 \int (D^2 f)|w|^2 - R_T a^2 \int (|D\theta|^2 + a^2|\theta|^2)$$

$$-R_S a^2 \tau_1 \int (|D\phi_1|^2 + a^2|\phi_1|^2) - R_S' a^2 \tau_2 \int (|D\phi_2|^2 + a^2|\phi_2|^2) + Q \int (|D^2 h_z|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2)$$

$$= a^2 p_r \left[R_T \int |\theta|^2 - R_S \int |\phi_1|^2 - R_S' \int |\phi_2|^2 - \frac{Q\sigma_1}{\sigma} \int (|Dh_z|^2 + a^2|h_z|^2)\right] \quad (8)$$

$$\frac{1}{\sigma}\int (|Dw|^2 + a^2|w|^2) + R_T a^2 \int |\theta|^2 - R_S a^2 \int |\phi_1|^2 - R_S' a^2 \int |\phi_2|^2$$

$$-\frac{Q\sigma_1}{\sigma} a^2 \int (|Dh_z|^2 + a^2|h_z|^2) = 0 \quad (9)$$

From equation (9), it follows that

$$R_S a^2 \int |\phi_1|^2 > \frac{1}{\sigma}\int (|Dw|^2 + a^2|w|^2) - R_S' a^2 \int |\phi_2|^2 - \frac{Q\sigma_1}{\sigma} a^2 \int (|Dh_z|^2 + a^2|h_z|^2) \quad (10)$$

Since$w = 0 = \theta = \phi_1 = \phi_2 = h_z \ at\ z = 0$ and $z = 1$; therefore, Rayleigh-Ritz inequality [14] yields that

$$\int |D\psi|^2 \geq \pi^2 \int |\psi|^2 \quad (11)$$

where $\psi = \{w, \theta, \ \phi_1, \ \phi_2, \ h_z\}$.

Now, using *Schwartz* inequality, we can have

$$\int |D\theta|^2 = |-\int \theta^* D^2\theta| \leq \int |\theta^* D^2 \theta| = \int |\theta||D^2\theta| \leq (\int |\theta|^2)^{\frac{1}{2}} (\int |D^2\theta|^2)^{\frac{1}{2}} \quad (12)$$

Which upon utilizing inequality (11)$(with\ \psi = \theta)$ gives

$$\int |D^2\theta|^2 \geq \pi^4 \int |\theta|^2 \quad (13)$$

Now, combining inequality (13)and inequality (11)$(with\ \psi = \theta)$, we have

$$\int (|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2) \geq (\pi^2 + a^2)^2 \int |\theta|^2 \quad (14)$$

Proceeding analogously as in the derivation of the inequality (14), we have following inequalities

$$\int f(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2) \geq f_{min.}(\pi^2 + a^2)^2 \int |w|^2 \quad (15)$$

$$\int (|D^2h_z|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2) \geq (\pi^2 + a^2) \int (|Dh_z|^2 + a^2|h_z|^2) \quad (16)$$

Here, $f_{min.}$is the minimum value of $f(z)$ in closed interval$[0, 1]$.

Again, using *Schwartz* inequality, we can have

$$\int (|D\theta|^2 + a^2|\theta|^2) = |-\int \theta^*(D^2 - a^2)\theta| \leq \int |\theta^*(D^2 - a^2)\theta| = \int |\theta||(D^2 - a^2)\theta|$$

$$\leq (\int |\theta|^2)^{\frac{1}{2}} (\int |(D^2 - a^2)\theta|^2)^{\frac{1}{2}} \quad (17)$$

Further, multiplying equation (2) by its complex conjugate and integrating the various terms on left hand side of the resulting equation by parts for an appropriate number of times and making use of boundary condition (6), we obtain

$$\int (|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2) + 2p_r \int (|D\theta|^2 + a^2|\theta|^2) + |p|^2 \int |\theta|^2 = \int |w|^2 \quad (18)$$

If permissible, let$p_r \geq 0$. In view of this equation (18) implies that

$$\int |(D^2 - a^2)\theta|^2 = \int (|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2) < \int |w|^2 \quad (19)$$

Inequality (17) together with inequalities (14) and (19), yields

$$\int (|D\theta|^2 + a^2|\theta|^2) < \frac{1}{\pi^2 + a^2}\int |w|^2 \quad (20)$$

Now, inequality (10) upon using inequality (11)$(with\ \psi = w)$, yields either of the inequalities

$$R_S a^2 \int |\phi_1|^2 > \frac{\pi^2 + a^2}{\sigma}\int |w|^2 - R_S' a^2 \int |\phi_2|^2 - \frac{Q\sigma_1}{\sigma} a^2 \int (|Dh_z|^2 + a^2|h_z|^2) \quad (21)$$

$$R_S' a^2 \int |\phi_2|^2 > \frac{\pi^2 + a^2}{\sigma}\int |w|^2 - R_S a^2 \int |\phi_1|^2 - \frac{Q\sigma_1}{\sigma} a^2 \int (|Dh_z|^2 + a^2|h_z|^2) \quad (22)$$

$$Q \int (|Dh_z|^2 + a^2|h_z|^2) > \frac{\pi^2 + a^2}{\sigma_1}\int |w|^2 - \frac{R_S a^2 \sigma}{\sigma_1}\int |\phi_1|^2 - \frac{R_S' a^2 \sigma}{\sigma_1}\int |\phi_2|^2 \quad (23)$$

Now, multiplying equation (9) by p_r and adding the resulting equation to equation (8), we have

$$\int f(|D^2w|^2 + 2a^2|Dw|^2 + a^4|w|^2) + \frac{2p_r}{\sigma}\int (|Dw|^2 + a^2|w|^2) + a^2 \int (D^2 f)|w|^2$$

$$-R_T a^2 \int (|D\theta|^2 + a^2|\theta|^2) - R_S a^2 \tau_1 \int (|D\phi_1|^2 + a^2|\phi_1|^2) - R_S' a^2 \tau_2 \int (|D\phi_2|^2 + a^2|\phi_2|^2)$$

$$-Q \int (|D^2h_z|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2) = 0 \quad (24)$$

For $D^2 f \geq 0$ (which is true for the most of the temperature dependent viscosity variation laws), and $p_r \geq 0$, equation (24) yields:

$$\int f(|D^2 w|^2 + 2a^2|Dw|^2 + a^4|w|^2) + R_S a^2 \tau_1 \int (|D\phi_1|^2 + a^2|\phi_1|^2)$$

$$+R_S' a^2 \tau_2 \int (|D\phi_2|^2 + a^2|\phi_2|^2) + Q \int (|D^2 h_z|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2)$$
$$< R_T a^2 \int (|D\theta|^2 + a^2|\theta|^2) \quad (25)$$

Now, using inequality (11)($with\ \psi = \phi_1\ and \phi_2$), inequalities (15), (16), (20) and using inequalities (21) – (23) successivelyin inequality (25), we haveeither of the following inequalities

$$\frac{(\pi^2+a^2)^3}{a^2\sigma}(\sigma f_{min.} + \tau_1) \int |w|^2 + (\pi^2 + a^2)^2 R_S' a^2 (\tau_2 - \tau_1) \int |\phi_2|^2$$
$$+\frac{(\pi^2+a^2)^2}{a^2\sigma} Q\sigma_1 \left(\frac{\sigma}{\sigma_1} - \tau_1\right) \int (|Dh_z|^2 + a^2|h_z|^2) < R_T \int |w|^2 \quad (26)$$

$$\frac{(\pi^2+a^2)^3}{a^2\sigma}(\sigma f_{min.} + \tau_2) \int |w|^2 + (\pi^2 + a^2)^2 R_S a^2 (\tau_1 - \tau_2) \int |\phi_1|^2$$
$$+\frac{(\pi^2+a^2)^2}{a^2\sigma} Q\sigma_1 \left(\frac{\sigma}{\sigma_1} - \tau_2\right) \int (|Dh_z|^2 + a^2|h_z|^2) < R_T \int |w|^2 \quad (27)$$

$$\frac{(\pi^2+a^2)^3}{a^2\sigma}\left(\sigma f_{min.} + \frac{\sigma}{\sigma_1}\right) \int |w|^2 + (\pi^2 + a^2)^2 R_S a^2 \left(\tau_1 - \frac{\sigma}{\sigma_1}\right) \int |\phi_1|^2$$
$$+(\pi^2 + a^2)^2 R_S' a^2 \left(\tau_2 - \frac{\sigma}{\sigma_1}\right) \int |\phi_2|^2 < R_T \int |w|^2 \quad (28)$$

Taking $\Gamma = min.\{\tau_1\ ,\ \tau_2\ ,\ \sigma/\sigma_1\}$, inequalities (26) - (28) can be written as follows

$$\frac{(\pi^2+a^2)^3}{a^2\sigma}(\sigma f_{min.} + \Gamma) \int |w|^2 < R_T \int |w|^2. \quad (29)$$

Since, minimum value of $(\pi^2 + a^2)^3/a^2$ with respect to a^2 is $27\pi^4/4$, therefore (29) yields

$$\left[\frac{27\pi^4}{4\sigma}(\sigma f_{min.} + \Gamma) - R_T\right] \int |w|^2 < 0.$$

From the above inequality, it is clear that if $R_T < 27\pi^4(\sigma f_{min.} + \Gamma)/4\sigma$, then we have a contradiction. Hence, we must have $p_r < 0$.

Thus, when $D^2 f \geq 0$ and $R_T < 27\pi^4(\sigma f_{min.} + \Gamma)/4\sigma,\ \Gamma = min.\{\tau_1\ ,\ \tau_2\ ,\ \sigma/\sigma_1\}$, the oscillatory modes of the system are stable, *i.e.*the oscillatory modes of growing amplitude are not allowed for triply-diffusiveconvection problem with magnetic field and temperature dependent viscosity.

It is remarkable to note that, when we consider the compliment of the above sufficient condition for the stability of the oscillatory motions, *i.e.* when $R_T \geq 27\pi^4(\sigma f_{min.} + \Gamma)/4\sigma$, the oscillatory modes of growing amplitude may exist. Hence it becomes important to prescribe the bounds for the growth rate of these motions.

IV. Bounds for complex growth rate

In following analysis, we have derived the bounds, which arrest the complex growth rate of the arbitrary neutral or unstable ($p_r \geq 0$) oscillatory motions ($p_i \neq 0$)for thepresent problem.

We now, prove the following theorem.

Theorem 1. If $(p,\ w, \theta, \phi_1, \phi_2, h_z), p = p_r + ip_i\ , p_i \neq 0, p_r \geq 0,\ D^2 f \geq 0$ is a solution of equations (1)-(5) together with either of boundary conditions (6) and $R_T > 0, R_S > 0, R_S' > 0, R_T > 27\pi^4(\sigma f_{min.} + \Gamma)/4\sigma$, then $|p| < \sigma R_T \sqrt{\Omega^2 - 1}/27\pi^4(\sigma f_{min.} + \Gamma)$, where $\Omega = 4\sigma R_T/27\pi^4(\sigma f_{min.} + \Gamma)$ and $\Gamma = min.\{\tau_1\ , \tau_2\ , \sigma/\sigma_1\}$.

Proof. Taking $p_r \geq 0$ in the equation (18), we have

$$\int (|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2) + |p|^2 \int |\theta|^2 < \int |w|^2$$

Now, using inequality (14), above inequality yields

$$\int |\theta|^2 < \frac{1}{[(\pi^2+a^2)^2+|p|^2]} \int |w|^2. \quad (30)$$

Now, using inequalities (19) and (30) in inequality (17), we have

$$\int (|D\theta|^2 + a^2|\theta|^2) < \frac{1}{(\pi^2+a^2)\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{1/2}} \int |w|^2. \quad (31)$$

Now, using inequality (11)($with\ \psi = \phi_1\ and \phi_2$), inequalities (15), (16), (31) and using inequalities (21) – (23) successivelyin inequality (25), we haveeither of the following inequalities

$$\frac{(\pi^2 + a^2)^3}{a^2\sigma}(\sigma f_{min.} + \tau_1) \int |w|^2 + \frac{(\pi^2 + a^2)^2}{a^2\sigma} Q\sigma_1 \left(\frac{\sigma}{\sigma_1} - \tau_1\right) \int (|Dh_z|^2 + a^2|h_z|^2)$$
$$+(\pi^2 + a^2)^2 R_S' a^2 (\tau_2 - \tau_1) \int |\phi_2|^2 < R_T \left[1 + \frac{|p|^2}{(\pi^2+a^2)^2}\right]^{-1/2} \int |w|^2. \quad (32)$$

$$\frac{(\pi^2 + a^2)^3}{a^2\sigma}(\sigma f_{min.} + \tau_2) \int |w|^2 + \frac{(\pi^2 + a^2)^2}{a^2\sigma} Q\sigma_1 \left(\frac{\sigma}{\sigma_1} - \tau_2\right) \int (|Dh_z|^2 + a^2|h_z|^2)$$

$+(\pi^2+a^2)^2 R_S a^2(\tau_1-\tau_2)\int|\phi_1|^2 < R_T\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{-1/2}\int|w|^2.$ (33)

$$\frac{(\pi^2+a^2)^3}{a^2\sigma}\left(\sigma f_{min.}+\frac{\sigma}{\sigma_1}\right)\int|w|^2+(\pi^2+a^2)^2 R_S a^2\left(\tau_1-\frac{\sigma}{\sigma_1}\right)\int|\phi_1|^2$$

$+(\pi^2+a^2)^2 R_S' a^2\left(\tau_2-\frac{\sigma}{\sigma_1}\right)\int|\phi_1|^2 < R_T\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{-1/2}\int|w|^2.$ (34)

Taking$\Gamma = min.\{\tau_1\ ,\ \tau_2\ ,\ \sigma/\sigma_1\}$, inequalities (32) - (34) can be written as follows

$\frac{(\pi^2+a^2)^3}{a^2\sigma}(\sigma f_{min.}+\Gamma)\int|w|^2 < R_T\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{-1/2}\int|w|^2.$ (35)

Since, minimum value of$(\pi^2+a^2)^3/a^2$ with respect to a^2 is $27\pi^4/4$, therefore (35) yields

$\left[\frac{27\pi^4}{4\sigma}(\sigma f_{min.}+\Gamma)-R_T\left\{1+\frac{|p|^2}{(\pi^2+a^2)^2}\right\}^{-1/2}\right]\int|w|^2 < 0.$ (36)

Inequality (36), clearly implies that

$|p| < (\pi^2+a^2)\sqrt{\Omega^2-1}$ (37)

Where$\Omega = 4\sigma R_T/27\pi^4(\sigma f_{min.}+\Gamma)$.

Also, from inequality (29), we have

$\frac{(\pi^2+a^2)^3}{a^2\sigma}(\sigma f_{min.}+\Gamma) < R_T.$ (38)

Since, minimum value of$(\pi^2+a^2)^2/a^2$ with respect to a^2 is $4\pi^2$, therefore (38) yields

$\pi^2+a^2 < \frac{\sigma R_T}{4\pi^2(\sigma f_{min.}+\Gamma)}.$ (39)

Now, using inequality (39) in inequality (37), we get

$|p| < \sigma R_T\sqrt{\Omega^2-1}/4\pi^2(\sigma f_{min.}+\Gamma)$.

Theorem 2. If $(p,\ w,\theta,\phi_1,\phi_2,h_z), p=\ p_r+ip_i\ , p_i\neq 0, p_r\geq 0,\ D^2 f\geq 0$ is a solution of equations (1)-(5) together with either of boundary conditions (6)-(7) and $R_T>0, R_S>0, R_S'>0, R_T>\frac{27\pi^4\left(f_{min.}+\frac{\tau_1+\tau_2}{\sigma}\right)}{4}$, then$|p|<\frac{\sigma R_T\sqrt{M^2-1}}{27\pi^4[\sigma f_{min.}+(\tau_1+\tau_2)]}$, where,$M = 4\sigma R_T/27\pi^4[\sigma f_{min.}+(\tau_1+\tau_2)]$.

Proof. Proceeding as in the derivation of the above Theorem, considering the other forms of inequality (10) derived from equation (9) and making use of these inequalities together with inequality (11)$(with\ \psi =$ *$\phi 1$ and$\phi 2$*, inequalities (15), (16), (20), in inequality (25) and following the analogous steps used there in the derivation of the Theorem 1, we can easily prove the theorem.

V. Results and Discussion

In terminology of hydrodynamic stability theory, the above result can be stated as, '*the complex growth rate $p(=p_r+ip_i)$ of an arbitrary oscillatory$(p_i\neq 0)$ perturbation of growing amplitude $(p_r\geq 0)$, in triply-diffusive convection problem with magnetic field and temperature dependent viscosity, when, $D^2 f\geq 0$ and$\Omega>1$, lies inside a semicircle in the right half of the $p_i p_r$ −plane, whose center is at the origin and radius is*$\sigma R_T\sqrt{\Omega^2-1}/27\pi^4(\sigma f_{min.}+\Gamma)$'. From the analysis, we observed that the radius of growth rate of oscillations for an arbitrary oscillatory perturbation decreases under the effects of diffusion ratios (Lewis numbers for the two concentrations), the thermal Prandtl number, the magnetic Prandtl number and the viscosity variation parameter.

The obtained result is uniformly valid for all combinations of rigid and dynamically free boundary conditions.Also, Theorem 1 implies that when $R_T < 27\pi^4(\sigma f_{min.}+\Gamma)/4\sigma$, the oscillatory motions of growing amplitude are not allowed.

Further, from the expression for the growth rate given in Theorem 1, we can see that for Triply-diffusive convection problem with magnetic field and temperature dependent viscosity, one can control the radius of arbitrary oscillatory perturbation by appropriate choice of diffusion ratio's(Lewis numbers for the two concentrations)$\tau_1\ ,\ \tau_2$,thermal Prandtl number (σ), magnetic Prandtl number(σ_1) and viscosity of the fluid$(f_{min.})$ and hence by modulating the viscosity through temperature, one can control the oscillations of the growth rate.In particular, when the viscosity is a linear, quadratic or exponential function of the temperature, the condition on viscosity *i.e.* $D^2 f\geq 0$is automatically satisfied and$f_{min.}=1$.

From the point of view of hydrodynamic stability theory, following consequences arededuced form Theorem 1.

The complex growth rate $p(=p_r+ip_i)$ of an arbitrary oscillatory$(p_i\neq 0)$ perturbation of growing amplitude$(p_r\geq 0)$lies inside a semicircle in the right half of the $p_i p_r$ −plane, whose center is at the origin and radius is as given below, for the following convection problems;

1. For*HydromagneticTriply-diffusive convection problem*with*constant viscosity* $\left(f_{\min.}=1\right)$, the radius is given by $\sigma R_T\sqrt{N^2-1}/4\pi^2(\sigma+\Gamma)$, where$N=\frac{4\sigma R_T}{27\pi^4(\sigma+\Gamma)}and\Gamma=min.\{\tau_1,\ \tau_2,\ \sigma/\sigma_1\}$.
2. For*Triply-diffusive convection problem*with *constant viscosity*$(Q=0, f_{min.}=1)$,the radius is given by $\sigma R_T\sqrt{P^2-1}/4\pi^2(\sigma+\Gamma_1)$, where$P=4\sigma R_T/27\pi^4(\sigma+\Gamma_1)$ and $\Gamma_1=min.\{\tau_1,\ \tau_2\}$.
3. For*Hydromagnetic Double-diffusive convection* with *temperature dependent viscosity*$(\tau_1=\tau, \tau_2=0)$, the radius is given by $\sigma R_T\sqrt{L^2-1}/4\pi^2(\sigma f_{min.}+\Gamma_2)$,where$L=4\sigma R_T/27\pi^4(\sigma f_{min.}+\Gamma_2)and\Gamma_2=min.\{\tau,\ \sigma/\sigma_1\}$.
4. For*Hydromagnetic Double-diffusive convection* with *constant viscosity*$(\tau_1=\tau, \tau_2=0,\ f_{min.}=1)$, the radius is given by $\sigma R_T\sqrt{S^2-1}/4\pi^2(\sigma+\Gamma_3)$, where $S=4\sigma R_T/27\pi^4(\sigma+\Gamma_3)$ and $\Gamma_3=min.\{\tau,\ \sigma/\sigma_1\}$.
5. For*Double-diffusive convection* with *temperature dependent viscosity*$(\tau_1=\tau, \tau_2=0,\ Q=0)$, the radius is given by $\sigma R_T\sqrt{X^2-1}/4\pi^2(\sigma f_{min.}+\tau)$, where$X=4\sigma R_T/27\pi^4(\sigma f_{min.}+\tau)$.This is the same result as obtained by Dhiman and Kumar [18].
6. For*Double-diffusive convection* with *constant viscosity*$(\tau_1=\tau, \tau_2=0,\ Q=0, f_{min.}=1)$, the radius is given by $\sigma R_T\sqrt{Y^2-1}/4\pi^2(\sigma+\tau)$, where $Y=4\sigma R_T/27\pi^4(\sigma+\tau)$.This is the same result as obtained by Gupta *et. al.* [3].

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