

Role of Elliptic flow in Quark Gluon Plasma

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Abstract

This paper is a short review about the elliptic flow. Due to pressure gradients, collective flows occur in the ultra relativistic heavy ion collisions and this flow can be calculated from anisotropic momentum distributions of particles relative to the reaction plane which is given by the second Fourier coefficient, V_2 . Here first section contains brief introduction and the motivation behind its calculation. Second section contains terms needed to calculate elliptic flow. Third and fourth section tells physics behind the elliptic flow and the progresses in these years. Its measurement provides many informations related to Quark Gluon Plasma (QGP) and expansion of early universe.

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I. Introduction and Motivation

Quark-Gluon-Plasma (QGP), a new state was produced at extreme density and temperature in the first few microseconds after the big bang explosion. QGP is a state of matter where the quarks and the gluons are in deconfined state. At Relativistic Heavy Ion Collider (RHIC) in Brookhaven National Laboratory (BNL) and Large Hadron Collider (LHC), CERN, many collision experiments of heavy ions have been performed to find out and analyse their properties. Angular correlations between the produced particles help to study the properties of the QGP during the collisions of heavy ions.

When a system of two nuclei with large mass number collide at high energy and extreme density, a flow pattern develops slowly during the expansion of the system^{1,2,3}. Flow can be described in terms of hydrodynamics and it relates the fundamental properties of the fluid with conservation laws (mass, momentum, energy).

The relation between pressure with density and temperature builds an equation of state (EoS) for a system of nuclear matter and those EoS are important for astrophysical study like the big bang, explosions of supernovae etc. Depending upon the density and temperature, nuclear matter experiences two phase transitions⁴. At low density and temperatures below 20 MeV, the liquid-to-vapour phase transition occurs. At high density and temperatures above 150 MeV, another phase transition of hadrons to QGP occurs. The EoS sometimes also contains the incompressibility parameter K, which calculates the resistance versus compression and affects the flow phenomena of QGP directly.

Collisions among nucleons and nuclei are described by fluid dynamics model given by Belenkij and Landau in 1955⁵. When a high-energy particle moving faster than the speed of sound of nucleus passes through another nucleus, shockwaves are formed and Glassgold, Heckrotte, and Watson⁶ have studied these shockwaves propagating in the longitudinal direction in 1959 using hydrodynamics model^{7,8,9}.

Depending on the size of impact parameter collisions are of two types. Those are “central” collisions having small impact parameter and “peripheral” or “non-central collision” having large impact parameter. When two nuclei collide and subsequently expand, three types of transverse flows are considered: radial transverse flow, directed flow and elliptic flow. Radial transverse flow is allowed for azimuthally isotropic central collisions and for non-central collisions, anisotropic flows i.e. directed and elliptic flow is allowed. A plane called as reaction plane, can be determined to describe those events which are not isotropic azimuthally and with respect to this plane, anisotropy of the particles are calculated for directed and elliptic flow. The azimuthal distribution of the particles with respect to this plane can be calculated from a Fourier expansion and the amplitude of the first harmonic gives the directed flow which was found out at the Bevalac¹⁰.

Elliptic flow measures the non-uniform flow of particles during the subsequent expansion of the system in the collision and this confirms the existence of QGP. The amplitude of second harmonic coefficient V_2 of the Fourier expansion gives the elliptic flow. This flow restated the anisotropic space of the overlap region of the nucleus in the transverse plane to the momentum distribution of known particles. Elliptic flow is important in the early stages of system evolution as anisotropy of space is largest at the beginning of the evolution. Its measurement provides thermalization time scale as well as informations about the initial stages of a relativistic collision.

Scheid, Müller and Greiner have used ideal hydrodynamics model and shown the importance of transverse expansion for the first time⁸. For beam energies 12.5A MeV, the transverse flow was more than the longitudinal flow in the first 15 fm/c after penetration revealing that the products are pushed outwards in the perpendicular direction with respect to the relative motion of the two nuclei.

An interacting system attains thermal equilibrium locally after the initial collisions between two nuclei. The gradients of the pressure increases and becomes sharp along the impact parameter which gives an anisotropy momentum distribution of the particles, defined as elliptic flow. This was presented in ref.^{11,12} along with directed flow for the first time for Pb+Pb collisions at energy 158 GeV/nucleon. It has been found that directed flow exhibits opposite results for Protons and Pions at large rapidities. The elliptic flow did not depend on rapidity for Pions. But for the protons, this elliptic flow depends on rapidity and becomes sharp near mid-rapidity region. Pressure gradients lead to collective flows, whereas matter can also expand due to its internal pressure which reduces in the transition region and causes the directed flow in the reaction plane^{13, 14}. The squeeze out process has been studied at different energies. The above results motivate further study of elliptic flow to know more about initial pressure distribution in the high density region in the non-central collisions. 4π detectors give first experimental evidence about sideward flow^{15, 16, 17}. In these detectors, events could be characterized by the momenta of the particles which were emitted after the collision. These results confirmed many theoretical analysis forecasted by fluid dynamics. There must exist a reaction plane to describe sideward flow.

By taking a hydrodynamic model, Ollitrault^{18, 19} has shown that the elliptic flow depends on the anisotropy of the overlapping space and is sensitive to the evolution of the system. This anisotropy is presented by the number of collisions of the nucleons in the beam direction²⁰. This space anisotropy is elliptic in nature and is denoted as ϵ . For a Woods-Saxon distribution of space, it did not depend on the nucleon-nucleon cross section²¹. From the graph V_2/ϵ verses centrality^{22, 23}, properties of the nuclear matter can be found out. Thus study of elliptic flow is important for QGP.

II. Terms related to elliptic flow

In the heavy ion collision, momenta of the charged hadrons are necessary to calculate the azimuthal anisotropy. Identification of product particles is also very useful for the future analysis.

2.1 Event Evolution

The system created for extended objects like heavy ions are different for head-on and peripheral collisions and can be categorized by centrality which is defined by the impact parameter b . However impact parameter cannot be observed directly.

QGP can be produced in laboratories like RHIC and LHC by bombarding two nuclei having large mass number. At RHIC, QGP can be formed by bombarding gold ions at 99.999% the speed of light(c). At that speed, the energy per nucleon pair of the reaction in center of mass frame $\sqrt{s_{NN}}$ is 200 GeV. A gold nucleus has a diameter of about 15 fm, but when accelerated to 99.9999% c ; it is Lorentz contracted along the direction of travel to a 0.13 fm thick disk. This contraction causes the ions to only overlap during the collision for 4.38×10^{-25} second (0.13 fm/c). Not all collisions have complete overlap of the nuclei. Particles that interact in the collision are called participants and particles that fly by the collision are named spectators. A large overlap produces a large number of participants, while small overlap has a large number of spectators. Outgoing particles with high transverse to the beam line momentum (>5 GeV/c) are called jets. They get their momentum from hard scattering processes of the parent nuclei's quarks or gluons. After the collision, the QGP expands and cools which causes it to transition back to hadrons. This phase transition is called hadronization. At this point in the event, the quark flavors inside the hadrons are fluctuating due to inelastic scattering of the hadrons. After further expansion and cooling, chemical freeze out occurs, ceasing inelastic scattering and this causes the flavors of the quarks to stop changing. Thermal freeze out occurs after more expansion when the outgoing hadrons and leptons are no longer interacting. They can decay if it is energetically favorable. Once all of this occurs, the outgoing particle's momenta and type are identified by detectors. This entire process is called an event.

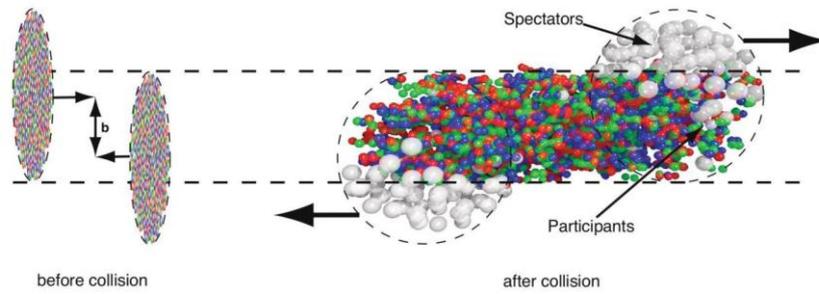


Figure 1: Left: The two heavy-ions in a non-central collision. Right: The spectators continue to remain unaffected and production of particle take place in the participant zone ¹⁴.

2.2 Centrality

Since each event is unique, there is a set of variables used to classify them. The most descriptive of these is the distance between the centers of the colliding nuclei in a plane transverse to the beam axis of the collision, called as impact parameter, b , which gives information about the overlap volume of the two nuclei. This overlap volume determines counts number of participant nucleons (N_{part}), number of spectator nucleons (N_{spec}) and the number of binary collisions (N_{coll}). For small b , the overlap between the nuclei is large and the number of participants is large. For large b , the overlap is small and there are a small number of participants. Unfortunately, impact parameters cannot be measured directly in experiments. Instead, a variable called centrality is used. From simulation of the experimental observables, impact parameter and hence the centrality of the event can be calculated. The centrality is expressed in terms of charged particle multiplicity. Multiplicity defines the number of particles created in a collision. Events with large multiplicity are named central, while small multiplicity events are called peripheral. The dependence of elliptic flow on centrality is discussed in ^{24, 25, 26}.

2.3 Coordinate Systems

A few coordinate systems can be placed about the nominal collision point in order to map out where the outgoing particles travel. Mostly used coordinate system is the Cartesian coordinate system. The $+z$ -axis is taken as one of the beam lines and the $+y$ -axis as straight up is the most convenient. Spherical coordinate system can also be used. Here θ is the angle above the beam line. Here $\theta = 0$ means $+z$ -axis. ϕ is the angle around the beam line, with $\phi = 0$ along the $+x$ -axis.

There are two useful variables, rapidity(y) and pseudorapidity (η) that describe forward and backward angles with respect to the nominal collision point and beam line. Rapidity is defined as

$$Y = \frac{1}{2} \ln \frac{1+\beta \cos \theta}{1-\beta \cos \theta} = \frac{1}{2} \ln \frac{1+\beta_z}{1-\beta_z} \quad (1)$$

here $\beta = v/c$ and v represents particle's velocity. It conveys a particle's velocity along the beam line, its velocity in the $+z$ direction, $\beta_z = V_z/c$. The problem with rapidity is that it requires particle's velocity. The better quantity is pseudorapidity, only relies on the particles angle. It is defined as

$$\eta = \frac{1}{2} \ln \frac{1+\cos \theta}{1-\cos \theta} \quad (2)$$

Notice that when particle's velocity approaches the speed of light, η goes to one and rapidity and pseudorapidity are equal except at 0° and 180° .

2.4 The Reaction Plane

Reaction plane is represented by the impact parameter b and the beam line. The angle of this plane (Ψ_{RP}) is measured over the beam line by determining the azimuthal asymmetry of the outgoing particles. The determination of the reaction-plane counts on the particle's number in the event and on the flow's strength. Measurement of transverse momentum with high precision of the heaviest fragment of the reactions is the second way to choose the reaction plane.

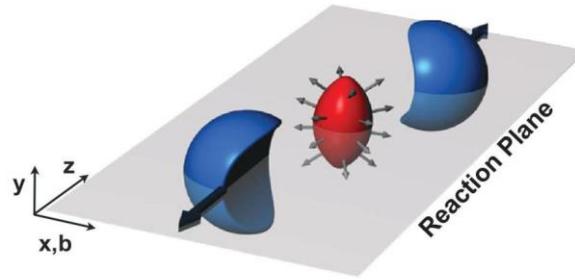


Figure 2: Almond shaped interaction volume after a non-central collision of two nuclei. The spatial anisotropy with respect to the x-z plane (reaction plane) translates into a momentum anisotropy of the produced particles (anisotropic flow)¹⁴.

2.5 Elliptic Flow and Eccentricity

If momentum distribution of particles with respect to the reaction plane is not isotropic then it represents an anisotropic collective flow. The production of particles that are not isotropic azimuthally can be described by a Fourier coefficient over the reaction plane angle (Ψ_{RP}) and is given by

$$\frac{1}{N} \frac{dN}{d\phi} = \frac{1}{2\pi} [1 + \sum_n 2 V_n \cos(n(\phi - \Psi_{RP}))] \quad (3)$$

The sine terms correspond to those particles which are symmetric, are dropped from this expansion. Elliptic flow is given by the second Fourier coefficient, V_2 and quantitatively this flow is calculated by the eccentricity

$$\epsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \quad (4)$$

here x coordinate represents along the reaction plane angle and y coordinate represents perpendicular to the reaction plane angle and their averages are calculated with respect to the positions of nucleons which are participating in the collision. As early stage eccentricity cannot be determined directly, different models can be made for its description. The Glauber model is one of them and is used to calculate initial geometric quantities like impact parameter.

In this model, the positions of the participating nucleons in the collision have been determined from the description of the colliding nuclei from Hydrodynamic calculations and it has been shown that the elliptic flow depends on the system's viscosity and event-by-event fluctuations with some uncertainties. "Optical" Glauber calculations are Monte Carlo based models with Fermi distribution function and here collision properties are calculated by averaging over multiple events. Through reducing elliptic flow fluctuations and uncertainties in eccentricity calculations, thermodynamic properties of the medium can be found out precisely both theoretically and experimentally.

III. The Physics of Elliptic Flow

In 1992, by considering ideal hydrodynamic calculations, Ollitrault predicted "Anisotropies in transverse momentum distributions"¹⁸ in ultrarelativistic nucleus-nucleus collisions for different initial conditions and equations of state. Later in 1994, Voloshin proposed Fourier analysis to study transverse flow effects²¹. He named second term in the Fourier expansion as "elliptic flow" and since then this transverse flow has been measured at the AGS^{27, 28, 29}, SPS³⁰ and RHIC^{31, 32, 33} for different heavy ion collisions. The reason behind this flow was the rescattering of produced particles during the collisions; hence this flow should be proportional to the particle density. The elliptic flow is zero for a symmetric system and for anisotropic system it is proportional to eccentricity which was shown by Ollitrault even for large ϵ ¹⁸. Based on Ollitrault's predictions, Voloshin and Poskanzer have studied elliptic flow by plotting a graph between V_2/ϵ and particle density^{22, 25}. Their results match with the predicted hydrodynamic limit for higher central collisions and satisfy the hydrodynamic calculation assumptions of early thermalization and zero mean-free path interactions limit. Another study at RHIC has shown that the elliptic flow parameter is directly proportional to the characteristic mean free path of the matter formed in the central rapidity region of the collision³⁴.

IV. Progress of Elliptic Flow

The target of current heavy-ion colliders is to create QGP with ultrarelativistic energies and to study its different properties. QGP contains deconfined quarks and gluons. But the deconfined quanta of a QGP are not directly observed. The hadronic and leptonic residues like Λ, Y, π etc are directly observed and calculated. They carry informations about the electromagnetic current fluctuations, quark flavor chemistry etc of the QGP state.

Theory suggests that the study of decay channels such as $\rho \rightarrow e^+ e^-$ provides information about the hadronization and Chiral symmetry breaking.

Due to pressure gradients, collective flows occur in a reaction and its measurement provides much information about the pressure in those reactions³⁵. The natural matter can expand due to its internal pressure also but this expansion becomes slow in the transition region. The directed flow and the squeeze out have been studied largely at available energies around 1 AGeV at AGS and CERN in early 90's^{27,36}. Matter mostly escapes in a direction orthogonal to the reaction plane at this low beam energy. The squeeze-out effect is the blocking of the path of the participant hadrons by the spectator nucleons. E877 group have found nonzero first and second moment of the Fourier expansion of particle multiplicity and transverse energy. The dominant flow is parallel to the impact parameter. Later NA49 group also found transverse energy flow for semi-peripheral Pb + Pb collision with beam energy 158AGeV per nucleon¹¹.

Another transport model like relativistic quantum molecular dynamics (RQMD) has shown a larger flow of participants in the reaction plane for RHIC energy. This model is a Monte-Carlo simulation based model which uses interactions of string and excitations of resonances for collective flow calculations. RQMD contains pressure changing options which help finding observables of final-state. In ref. [35], RQMD model is employed to calculate the azimuthal asymmetries for Au (11.7AGeV) on Au non-central collisions whose results signify the dependency of the azimuthal asymmetry on the pressure difference.

The first data on collective flow for Pions and protons for Pb + Pb collisions at beam energy 158 GeV/nucleon is presented at Bevalac and at higher rapidity, the Pions and the protons show opposite directed flow¹¹. It was found that Pion's elliptic flow did not dependent on rapidity whereas for the protons, it was peaking near mid-rapidity.

Subsequently, Lattice quantum chromo dynamics (QCD) calculations noticed a phase transition near QGP formation region and in this region the pressure rises more slowly with temperature than the energy density. Since gradients of pressure provide the driving force for collective flow, its measurements could be useful for the pressure development mechanism in heavy ion collisions and for QGP formation. This signal also strongly depends on transport properties and their uncertainty increases with beam energy. But the transverse elliptic flow dependencies are weaker on these transport properties. At the initial stages of the collision, elliptic flow is negative due to the squeeze-out of nuclear matter whereas this flow becomes positive at the later stages due to the exposure of larger reaction plane surface area which favors in-plane emission. E_{tr} is the beam energy where transition of elliptic flow occurs from negative to positive value. This transition beam energy depends on the parameters of the EoS. Basically the pressure during compression stage and the passage time decide types of elliptic flow. The time required to cross the shadowing area between the projectile and target spectators is known as the passage time and is given by $\sim 2R = (Y_0 V_0)$, where V_0 is spectator velocity, R is nuclear radius. From the squeeze-out contribution, the ratio $\frac{C_s}{Y_0 V_0}$ can be calculated. Elliptic flow measurements are also used to calculate speed of sound C_s and their excitation functions from QGP Phase transition can give the EoS. Theoretical calculations using hydrodynamical models focus on qualitative nature of collective flow whereas experiments describe various dynamical effects quantitatively.

Later many experiments were conducted to calculate elliptic flow for different particles by bombarding different heavy nuclei with more beam energies in RHIC and LHC laboratories. For Ni+Ni collisions, the elliptic flow signal is calculated for η and π^0 mesons. It was negative for η mesons whereas positive for π^0 mesons which indicates more emission in the plane which is perpendicular to the plane of reaction for η mesons. In peripheral collisions this effect should be minimum whereas for π^0 mesons, this flow is observed only in peripheral collisions. With increasing pion transverse momentum, elliptic flow changes from positive to negative sign for π^0 mesons³⁷.

Directed flow is positive when baryons are emitted in the plane of projectile or target nucleons, whereas for baryons emitted out-of-plane, elliptic flow becomes negative. Later SIS (GSI) had found negative elliptic flow for neutral and charged pions from experiments by taking Gold (Au) nuclei at beam energy around 1 AGeV. Unlike positive directed flow for baryons, this flow was positive for charged pions here and predicted an increase in the elliptic flow magnitude for pions with the increasing size of spectator matter. Later a small variation of its magnitude with the change of the number of spectator nucleons was found for charged π mesons in Bi+Bi collisions at 0.4-1.0 AGeV. So, a doubt arises about Pion's anisotropic flow.

Theoretical studies of EoS for QGP phase transition have a “softest point” where densities of quarks and antiquarks can be comparable to the matter densities ρ_0 in the ground-state. At energies between $1 \lesssim E_{\text{Beam}} \lesssim 11$ AGeV, matter densities of collision-zone are expected to be $\rho \sim 6-8\rho_0$ which favour EoS softening. Elliptic flow calculations in this energy range may provides new things about the parameters of the EoS and about softening point.

In 1999, the proton elliptic flow shows a transition from negative to positive value at a beam energy, $E_{\text{tr}} \sim 4$ AGeV for the Au + Au collision having 2 - 8 AGeV energy range³⁸. A comparison with relativistic Boltzmann-equation suggests a softening of the EoS with baryon density $\rho \sim 4\rho_0$. Proton distribution function is given by

$$\frac{1}{N} \frac{dN}{d\phi} \sim [1 + 2V_1 \cos(\phi) + 2V_2 \cos(2\phi)] \quad (5)$$

Here ϕ is the azimuthal angle. For zero, positive, and negative elliptic flow, the Fourier coefficients are $\langle \cos 2\phi \rangle = 0, > 0$, and < 0 respectively.

An isospin-dependent transport model was also used to study the elliptic flow of proton in $^{48}\text{Ca}+^{48}\text{Ca}$ collisions which showed a transition from positive value to negative value with increasing incident energy. It was shown that the magnitude of this flow depends on the nuclear equation of state (EoS) as well as on the nucleon-nucleon scattering cross section³⁹.

Poskanzer and group have also studied the system with Pb(158AGeV)+Pb collisions by employing RQMD and have shown that the azimuthal asymmetry depend on the centrality of collisions^{24,25}. (3+1)-dimensional hydrodynamics model is used to study about the collective flow and its dependency on angular distributions of emitted particles, on centrality and initial energy^{40,42,43}. During the initial stages of the expansion, elliptic flow is generated whereas radial flow continues to grow until freeze-out.

Recently ALICE Collaboration have studied the hadronisation of charm quarks in p+p and Pb+Pb collisions with an improved statistical precision and a strong magnetic fields during the collision which enhances the number of nuclei which participate in collective motion⁴⁴. They have shown that recombination is necessary for hadronisation in Pb+Pb collisions.

The behavior of their localized solutions is analogous to the processes of formation of azimuthal anisotropic elliptical flows of quark-gluon plasma and hadron jets. In contrast to the hydrodynamic models of an ideal fluid, the elliptic flow in this model has a significant negative acceleration⁴⁵. The presence of the negative acceleration has proposed a hypothesis about the direct elliptic flow of photons as braking radiation QGP.

Recently the MPD detector system with Monte Carlo simulations using Au+Au and Bi+Bi ions collisions also calculated the directed V_1 and elliptic V_2 flow for charged pions, protons, K_0^S and Λ particles⁴⁶ and found elliptic flows are affected by fluctuations⁴⁷.

V. Summary and Conclusion

The production and study of the flow pattern motivates to perform experiments. Flow can be described in terms of hydrodynamics through conservation laws. Different types of collective flow have been observed at different energies. The relation between pressure with density and temperature builds an equation of state (EoS) for a system of nuclear matter and those EoS are important for astrophysical study also. Angular correlations between the produced particles help to study the properties of the QGP during the collisions of heavy ion. The azimuthal distribution of the particles can be calculated from a Fourier expansion and the amplitude gives the directed flow and elliptic flow. At fixed transition beam energy, Elliptic flow can change its sign. Elliptic flow measurements are used to calculate speed of sound C_s and their excitation functions. Detailed study of the experimental observables with improved simulation techniques is expected to provide better results about the collective flow, which is important for QGP study.

References

- [1]. Landau LD, Lifshitz EM. Fluid Mechanics. London: Pergamon Press (1959).
- [2]. Stöcker H, Greiner W. Phys. Rep. 137:277 (1986).
- [3]. Clare RB, Strottman D. Phys. Rep. 141: 177 (1986).
- [4]. Peter F. Kolb, Josef Sollfrank, and Ulrich Heinz, Anisotropic transverse flow and the quark-hadron phase transition, Phys.Rev.C62:054909,2000,hep-ph/0006129
- [5]. Belenkij SZ, Landau LD. Usp. Fiz. Nauk 56:309 (1955); Nuovo Cimento Suppl. 3:15(1956).
- [6]. Glassgold A E, Heckrotte W, Watson KM. Ann. Phys. (NY) 6:1 (1959).
- [7]. Chapline GF, et al. Phys. Rev. D 8:4302 (1973).
- [8]. Scheid W, Müller H, Greiner W. Phys. Rev. Lett. 32:741 (1974).
- [9]. Wong CY, Welton TA. Phys. Lett. B 34: 243 (1974).
- [10]. Gustafsson H A°, et al., Phys. Rev. Lett. 52:1590(1984).
- [11]. NA49 Collaboration: H. Appelshaeuser et al, Directed and Elliptic Flow in 158 GeV/Nucleon Pb + Pb Collisions, Phys.Rev.Lett.80:4136- 4140,1998, nucl-ex/9711001

- [12]. WA98 Collaboration, H. Schlagheck, Directed and Elliptic Flow in 158A GeV Pb+Pb Collisions, Nucl.Phys. A661 (1999) 337-340, nucl-ex/9907005
- [13]. W. Reisdorf, H. G. Ritter, COLLECTIVE FLOW IN HEAVY-ION COLLISIONS, Annu. Rev. Nucl. Part. sci.1997. 47:663-709
- [14]. Raimond Snellings, Elliptic flow: a brief review, New Journal of Physics 13 (2011) 055008 (18pp)
- [15]. Renfordt RE, et al., Phys. Rev. Lett. 53:763(198)
- [16]. Ströbele H, et al., Phys. Rev. C 27:1349 (1983).
- [17]. Baden A, et al., Nucl. Instr. Meth. 203:189 (1982).
- [18]. Ollitrault JY. Phys. Rev. D 46:229 (1992); Phys. Rev. D 48:1132 (1993).
- [19]. R. S. Bhalerao, N. Borghini, J. -Y. Ollitrault, Analysis of anisotropic flow with Lee-Yang zeroes, Nucl.Phys. A727 (2003) 373-426, nucl-th/0310016
- [20]. Peter F. Kolb, Ulrich Heinz, Hydrodynamic description of ultrarelativistic heavy-ion collisions, nucl-th/0305084
- [21]. S. Voloshin and Y. Zhang, Flow study in relativistic nuclear collisions by Fourier expansion of Azimuthal particle distributions. Z. Phys., C70:665-672, 1996. [arXiv:hep-ph/9407282].
- [22]. S. A. Voloshin, A. M. Poskanzer, The physics of the centrality dependence of elliptic flow, Phys.Lett. B474 (2000) 27-32, nucl-th/9906075
- [23]. Sergei A. Voloshin, Arthur M. Poskanzer, and Raimond Snellings, Collective phenomena in non-central nuclear collisions, 2008. [arXiv: 0809.2949].
- [24]. H. Sorge, Highly Sensitive Centrality Dependence of Elliptic Flow {A Novel Signature of the Phase Transition in QCD, Phys.Rev.Lett. 82 (1999) 2048-2051, nucl-th/9812057
- [25]. A.M. Poskanzer and S.A. Voloshin, Centrality Dependence of Directed and Elliptic Flow at the SPS, Nucl.Phys.A661:341-344, 1999, nucl-ex/9906013
- [26]. P. F. Kolb, J. Sollfrank, P. V. Ruuskanen, U. Heinz, Hydrodynamic simulation of elliptic flow, Nucl.Phys.A661:349-352, 1999, nucl-th/9907025
- [27]. J. Barrette et al. Observation of anisotropic event shapes and transverse flow in Au + Au collisions at AGS energy, Phys. Rev. Lett., 73:2532-2535, 1994. [arXiv:hep-ex/9405003].
- [28]. J. Barrette et al, Energy and charged particle flow in 10.8-A-GeV/c Au + Au collisions. Phys. Rev., C55:1420-1430, 1997. [arXiv:nucl-ex/9610006].
- [29]. S. A. Voloshin, Anisotropic flow from AGS to RHIC, nucl-th/0202072
- [30]. C. Alt et al. Directed and elliptic flow of charged pions and protons in Pb + Pb collisions at 40-A-GeV and 158-A-GeV, Phys. Rev., 68:034903, 2003. [arXiv:nucl-ex/0303001].
- [31]. STAR Collaboration: K.H. Ackermann, et al, Elliptic flow in Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV, Phys.Rev.Lett.86:402-407, 2001, nucl-ex/0009011
- [32]. K. Filimonov, Azimuthal Anisotropy of Charged and Identified High P_T Hadrons in Au+Au Collisions at RHIC, Nucl.Phys.A715:737-740, 2003, nucl-ex/0210027
- [33]. Peter F. Kolb, Hydrodynamic flow at RHIC, Heavy Ion Phys. 15 (2002) 279-289, nucl-th/0104089
- [34]. Marcus Bleicher, Horst Stöcker, Anisotropic flow in ultra-relativistic heavy ion collisions, Phys.Lett.B526:309-314, 2002, hep-ph/0006147
- [35]. H. Sorge, Elliptical flow: a signature for early pressure in ultra relativistic nucleus-nucleus collisions, Phys.Rev.Lett.80:4136-4140, 1998, nucl-th/9610026
- [36]. S A Bass, M Gyulassy, H Stöcker, W Greiner, Signatures of Quark-Gluon-Plasma formation in high energy heavy-ion collisions: A critical review, J. Phys. G: Nucl. Part. Phys., hep-ph/9810281v2
- [37]. A. Taranenko, A. Kugler, et. al. Azimuthal Anisotropy of η and π^0 Mesons in Heavy-Ion Collisions at 2 A GeV, Phys. Rev. Lett, nucl-ex/9903008v1
- [38]. C. Pinkenburg et. al. Elliptic Flow: Transition from out-of-plane to in-plane Emission in Au + Au Collisions, Phys.Rev.Lett.83:1295-1298, 1999, nucl-ex/9903010v1
- [39]. Yu-Ming Zheng, C. M. Ko, Bao-An Li and Bin Zhang, Elliptic flow in heavy ion collisions near the balance energy, Phys.Rev.Lett. 83 (1999) 2534-2536, nucl-th/9906076
- [40]. V. K. Magas, L. P. Csernai, D. Strottman, The source of elliptic flow and initial conditions for hydrodynamical calculations, nucl-th/0009049
- [41]. P. F. Kolb, P. Huovinen, U. Heinz, H. Heiselberg, Elliptic flow at SPS and RHIC: from kinetic transport to hydrodynamics, Phys.Lett.B500:232-240, 2001, hep-ph/0012137
- [42]. P. Huovinen, P. F. Kolb, U. Heinz, P. V. Ruuskanen, S. Voloshin, Radial and elliptic flow at RHIC: further predictions, Phys.Lett.B503:58-64, 2001, hep-ph/0101136
- [43]. P. Huovinen, P. F. Kolb, U. Heinz, Is there elliptic flow without transverse flow?, Nucl.Phys.A698:475- 478, 2002, nucl-th/0104020
- [44]. Luuk Vermunt on behalf of the ALICE Collaboration, Open heavy- flavour production from small to large collision systems with ALICE at the LHC, arXiv:2011.06291v1 [nucl-ex]
- [45]. R.K. Salimov, T.R. Salimov, On the soliton model of azimuthal - anisotropic elliptical flows of quark-gluon plasma and hadron jets, arXiv: 2011.11474 [hep-th]
- [46]. P. Parfenov, A. Taranenko for the MPD Collaboration, The comparison of methods for anisotropic flow measurements with the MPD Experiment at NICA, arXiv:2012.06763v1 [hep-ex]
- [47]. J.-Mohs, M. Ege, H. Elfner and M. Mayer, Collective flow at SIS energies within a hadronic transport approach: Influence of light nuclei formation and equation of state, arXiv: 2012.11454v1 [nucl-th]