

Evaluation Of Varimax And Promax Factorial Rotation In Interpreting The Fluid Geochemistry Of Hot Springs In India's Peninsular And Extra-Peninsular Regions

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Abstract

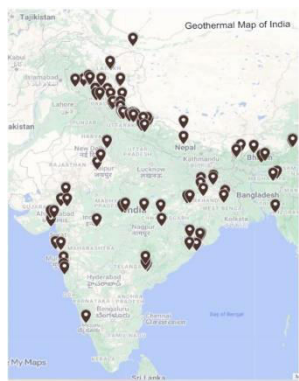
Exploratory factor analysis is performed on the same two data sets gathered from hot springs in India's Extra-Peninsular and Peninsular areas using Promax rotation, as was done before for PCA and Varimax analysis. In this fashion, different approaches or processes, such as rotated, unrotated, orthogonal, and oblique factorial rotations, are utilized in exploratory factor analysis to achieve the same basic goal: component structure simplification, often known as "dimension reduction." Rotation methods are characterized as orthogonal (Varimax) or oblique (Promax) based on whether the angle between the X and Y axes is 90° or less. This illustrates that the factors can correlate when utilizing oblique methods, but when employing varimax rotation, the two axes of the two factors become uncorrelated. The exploratory factor analysis provides a basic structure that is divided into two parts: converting variables into a few significant factors to reduce the size of the data set, and then converting factors back to variables for meaningful interpretation. A detailed examination of the successive phases of the EFA—the PCT, Varimax, and Promax matrices—and their comparison shows a gradual maximization of the range of factor loadings (in each factor column) to improve the multivariate data set. However, overmaximization appears to be less successful for spatial analysis in the current study, resulting in some redundancy and complexity in the components and making it more difficult to compare the results across different locations and samples.

Keywords: PCA analysis, Varimax Rotation, Promax Rotation, multivariate fluid geochemical data

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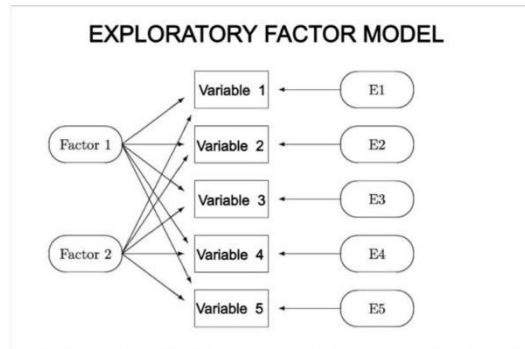
I. Introduction



The increasing demand for alternative renewable energy resources, such as geothermal, has led to a high interest in its exploration and exploitation. India has about 340 hot springs spread across the peninsular and extra-peninsular regions. The government of India constituted a 'Hot Spring Committee' in 1968 to examine the possibility of developing geothermal plants for power generation. The Central Electricity Authority (CEA) has associated itself with the UNDP geothermal project in India and the Puga and Parvati projects for the utilization of available geothermal resources for power generation (Jonathan Craig, 2013). The Geological Survey of India (GSI) has published a special publication titled "Geothermal Atlas of India" based on data compiled from all sources of information (Ravi Shankar et al. 1991). However, the lack of uniformity in data acquisition practices and manual handling of large amounts of data has made data storage, search, retrieval, and analysis laborious and cumbersome (A.Roy, 1994)

Basic Concept of Exploratory Factor Analysis and its significance in interpretation

Exploratory factor analysis is a procedure that aims to uncover structures in large sets of variables. If there is a data set with many variables, it is possible to separate a significant few reference factors, some of them correlatable with each other, from the insignificant many multi-variate data collection, which will account for much of the sample set. The prime aim, therefore, is to reduce a large number of correlating variables to a few independent latent variables, the so-called factors. In other words, the aim of the latent variables is to clarify as much of the variance of the original variables as possible. Different portions of the total variance or the loadings of a variable can then be assigned to different factors, as shown below:



$$X_i = \sum_{r=1}^p C_{ir} f_r$$

where f_r ($r=1,2,3,\dots,p$) represents common underlying factors and C_{ir} indicates the factor loadings of variable X_i on factor f_r . The theoretical unknown factors can thus be expressed in terms of distinct groups of elemental variables, in the present case fluid geochemical elements, which when correlated with the observed features, geothermal geochemistry of the area of investigation, provide significant insight into the causal factors (Roy. A., 1984, Amitabha Roy, 2023).

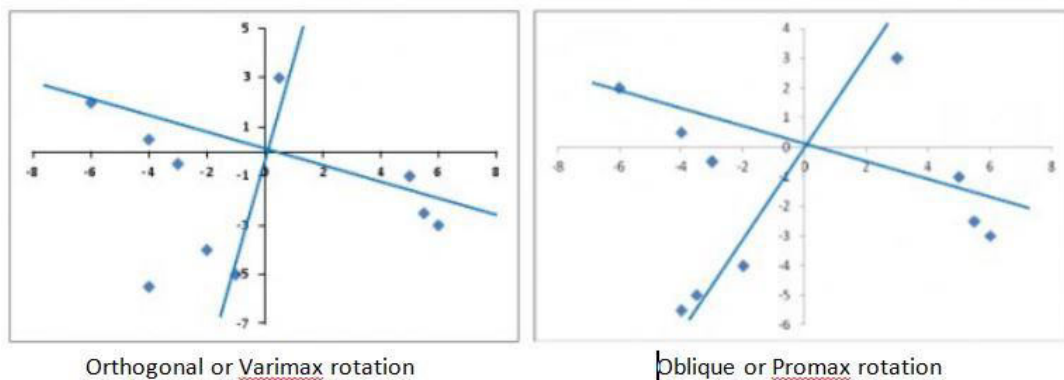


Fig. 1

To carry out this dimensional reduction with the data, the various computational steps involved and their significance in the analysis and interpretation of fluid geochemical data of hot springs in India's Peninsular and Extra-Penisular regions (Amitabha Roy, 2023) are discussed below.

- **Correlation Coefficient Matrix**, the basis for factor analysis (Table-1A- B)
- **Unrotated factor solution:** Displays unrotated factor loadings (factor pattern matrix), communalities, and eigenvalues for the factor solution (Amitabha Roy, 2023). Unrotated Principal component Analysis (PCA): Principal component transformation PCT brings about a linear orthogonal transformation of m original variables, geochemical elements in the m -dimensional measurement space to m new statistically independent variables or principal components where each new variable is a linear combination of the old. The PC analysis extracts m -eigenvectors (principal component axes) and corresponding m -eigenvalues (the variance measured along the eigenvector) from $m \times m$ symmetrical matrix of correlation. The results of PC analysis is given in Table-2A-B. The columns of this PCT matrix are all orthogonal and hence inter-column correlations are near Zero. These columns represent eigenvectors. The eigenvalues account for all of the original data variances in the decreasing order such that each has variance or eigenvalue less than previous ones. The principal components are then converted into factors by multiplying each element of the principal components or eigenvectors (V) by the square root of the corresponding eigenvalues (that is, $H=\lambda^{1/2}V$). These, besides the direction, also represent the variances.
- **Rotated Factor Analysis (Varimax)** - Rotation methods fall into two broad categories (Fig. 1): orthogonal and oblique (referring to the angle maintained between the X and Y axes) to remove the background noise imposed by $(m-p)$ unnecessary axes. To accomplish this, in varimax rotation (i.e. maintain a 90° angle between X - Y axes), p orthogonal reference axes or factors are rotated about the origin to positions such that

the variance of the loading from each variable onto each factor axis is either extreme (\pm) or near Zero. This maximization of the range of loadings can be performed by using Kaiser's (1958) varimax criterion. Scanning of each factor column for large absolute values in the varimax matrix (Table- 3) will identify a few variables with significantly high loadings and many others with insignificant loadings. The column showing communality (Σh_j^2) is the total amount of variance of each variable retained in the factors and is computed by summing the squares of the elements of the factors in each row of the varimax matrix. Fairly high commonality of each variable (Table- 3) implies the appropriateness of the four-factor model adopted in this study.

- **Promax rotation of oblique axes** : Even though factor axes X-Y of the varimax scheme no longer coincide with PC axes, they are at right angles to one another and hence are uncorrelated. But at the same time the varimax scheme assumes that correlation does exist between the variables and each of the p-mutually uncorrelated factors. Now since all geological/geochemical processes, like most natural phenomena, are seldom independent of one another, a certain amount of correlation is introduced into the factor model by rotating the orthogonal reference axes of varimax scheme to obliquity (promax rotation). This operation introduces intercorrelation between pairs of variable and pairs of factors which, in fact, brings about further maximization of the loadings on the factors (Table- 4)
- In the Exploratory Factor Analysis the most baffling issue is to determine the number of factors to retain significant few from many insignificant ones. There are different criteria suggested by different researchers namely,
 - Factors which have high eigenvalues
 - The Eigenvalue > 1
 - From the inflection point of scree or Elbow curve plot i.e. a graph of the eigenvalues (Y-axis) against the factors (X-axis) listed in the descending order.
 - Communalities for each of the variables (somewhat like R^2 from Regression analysis) at least 0.5
 - The factor loadings for each variable should be ≥ 0.6 (Awang, 2014).

Factor analysis is a way to condense the data in many variables into just a few variables. For this reason, it is also sometimes called “dimension reduction”. In the present study trial runs of PCA analysis taking into consideration of all the above criteria have been tried for determination of number of factors and came to a compromised conclusion that in the present study four factors which accounts for $75\pm 2\%$ of the total variance of the original variables “load” on a factor of Principal component is retained for rotation to avoid both overextraction and underextraction of factors that may have deleterious effects on the results. This also corroborates the criteria for communalities for each of the variable closer to one (0.9) and eigenvalues. >1 (Amitabha Roy, 2023)

Presenting the results and Visualizing the results in Tables

Table 1A. Correlation Matrix – Extra-Peninsula

Correlation matrix - Extra-Peninsula															
	TEMPC	pH	SPCMHO/cm	HCO3 mg/L	Cl mg/L	SO4 mg/L	TotHard	Ca mg/L	Mg mg/L	Na mg/L	K mg/L	F mg/L	B mg/L	SiO2 mg/L	TDS mg/L
TEMPC	1	0.05	-0.06	0.22	0.09	-0.05	-0.1	-0.06	0.03	0.25	0.17	0.26	0.18	0.55	0.14
pH	0.05	1	-0.11	0.05	-0.25	-0.2	-0.3	-0.26	0.04	-0.16	-0	0.04	-0.1	-0.12	-0.25
SPCMHO/cm	-0.06	-0.11	1	0.19	0.17	0.31	0.34	0.3	0.11	0.25	0.19	-0.13	0.13	0.13	0.34
HCO3 mg/L	0.22	0.05	0.19	1	0	0.03	0.05	0.01	0.35	0.57	0.6	0.23	0.09	0.51	0.21
Cl mg/L	0.09	-0.25	0.17	0	1	0.22	0.31	0.23	0.21	0.7	0.51	0.01	0.54	0.24	0.6
SO4 mg/L	-0.05	-0.2	0.31	0.03	0.22	1	0.96	0.97	0.34	0.37	0.35	-0	0.55	-0.09	0.83
TotHard	-0.1	-0.3	0.34	0.05	0.31	0.96	1	0.97	0.37	0.41	0.37	-0.1	0.56	-0.08	0.85
Ca mg/L	-0.06	-0.26	0.3	0.01	0.23	0.97	0.97	1	0.34	0.32	0.28	-0.12	0.46	-0.07	0.81
Mg mg/L	0.03	0.04	0.11	0.35	0.21	0.34	0.37	0.34	1	0.3	0.6	-0.26	0.48	-0.02	0.49
Na mg/L	0.25	-0.16	0.25	0.57	0.7	0.37	0.41	0.32	0.3	1	0.71	0.38	0.58	0.53	0.75
K mg/L	0.17	-0	0.19	0.6	0.51	0.35	0.37	0.28	0.6	0.71	1	0.03	0.7	0.25	0.62
F mg/L	0.26	0.04	-0.13	0.23	0.01	-0	-0.1	-0.12	-0.26	0.38	0.03	1	0.03	0.44	0.11
B mg/L	0.18	-0.1	0.13	0.09	0.54	0.55	0.56	0.46	0.48	0.58	0.7	0.03	1	-0	0.76
SiO2 mg/L	0.55	-0.12	0.13	0.51	0.24	-0.09	-0.08	-0.07	-0.02	0.53	0.25	0.44	-0	1	0.21
TDS mg/L	0.14	-0.25	0.34	0.21	0.6	0.83	0.85	0.81	0.49	0.75	0.62	0.11	0.76	0.21	1

Table 1B. Correlation Matrix – Peninsula

Correlation matrix - Peninsula															
	Temp_C	pH	SPCMHO/cm	HCO3 mg/L	Cl mg/L	SO4 mg/L	TotHard	Ca mg/L	Mg mg/L	Na mg/L	K mg/L	F mg/L	Bmg/L	SiO2 mg/L	TDS mg/L
Temp_C	1	-0.63	0.15	0.26	0.39	0.48	0.14	0.11	-0.13	0.15	0.45	0.33	0.15	0.17	0.53
pH	-0.63	1	0.1	-0.64	-0.14	-0.52	0.12	0.31	0.08	-0.07	-0.59	0.08	0.02	0	-0.36
SPCMHO/cm	0.15	0.1	1	-0.19	0.53	-0.13	0.33	0.78	0.8	0.93	0.22	-0.13	0.08	-0.1	0.06
HCO3 mg/L	0.26	-0.64	-0.19	1	0.1	0.59	-0.21	-0.28	-0.02	0.03	0.8	-0.34	-0.07	-0.44	0.46
Cl mg/L	0.39	-0.14	0.53	0.1	1	0.36	0.73	0.39	0.12	0.36	0.43	-0.18	0.24	0.02	0.59
SO4 mg/L	0.48	-0.52	-0.13	0.59	0.36	1	0.12	-0.12	-0.19	-0.04	0.73	0.13	-0.26	-0.13	0.65
TotHard	0.14	0.12	0.33	-0.21	0.73	0.12	1	0.32	-0.07	0	-0.07	-0.06	-0.01	0.31	0.13
Ca mg/L	0.11	0.31	0.78	-0.28	0.39	-0.12	0.32	1	0.62	0.65	0.08	0.15	-0.09	-0.01	0.14
Mg mg/L	-0.13	0.08	0.8	-0.02	0.12	-0.19	-0.07	0.62	1	0.87	0.22	-0.3	-0.07	-0.32	-0.09
Na mg/L	0.15	-0.07	0.93	0.03	0.36	-0.04	0	0.65	0.87	1	0.42	-0.19	0.06	-0.24	0.1
K mg/L	0.45	-0.59	0.22	0.8	0.43	0.73	-0.07	0.08	0.22	0.42	1	-0.25	-0.08	-0.33	0.65
F mg/L	0.33	0.08	-0.13	-0.34	-0.18	0.13	-0.06	0.15	-0.3	-0.19	-0.25	1	-0.13	0.35	0.06
Bmg/L	0.15	0.02	0.08	-0.07	0.24	-0.26	-0.01	-0.09	-0.07	0.06	-0.08	-0.13	1	-0.07	0.36
SiO2 mg/L	0.17	0	-0.1	-0.44	0.02	-0.13	0.31	-0.01	-0.32	-0.24	-0.33	0.35	-0.07	1	-0.2
TDS mg/L	0.53	-0.36	0.06	0.46	0.59	0.65	0.13	0.14	-0.09	0.1	0.65	0.06	0.36	-0.2	1

Table 2A Total Variance, Eigenvalues, and, Communality
Extra-Peninsula Peninsula

Explained Total Variance				Communality		Explained Total Variance				Communality	
Component	Total	% of variance	Accumulated %	Extraction	Component	Total	% of variance	Accumulated %	Extraction	Component	Extraction
1	5.74	38.26	38.26	TEMPC	0.39	1	4.29	28.6	28.6	Temp_C	0.7
2	2.74	18.24	56.5	pH	0.38	2	3.61	24.09	52.7	pH	0.64
3	1.49	9.94	66.45	SPCMHO/cm	0.31	3	2.29	15.27	67.97	SPCMHO/cm	0.96
4	1.18	7.84	74.29	HCO3 mg/L	0.83	4	1.44	9.63	77.6	HCO3 mg/L	0.87
5	1.07	7.15	81.44	Cl mg/L	0.82	5	1.21	8.08	85.68	Cl mg/L	0.92
6	0.83	5.55	86.99	SO4 mg/L	0.94	6	0.93	6.2	91.88	SO4 mg/L	0.78
7	0.78	5.17	92.16	TotHard	0.97	7	0.39	2.61	94.49	TotHard	0.67
8	0.43	2.9	95.06	Ca mg/L	0.94	8	0.27	1.82	96.3	Ca mg/L	0.81
9	0.34	2.28	97.34	Mg mg/L	0.74	9	0.17	1.16	97.47	Mg mg/L	0.92
10	0.19	1.27	98.61	Na mg/L	0.88	10	0.17	1.14	98.61	Na mg/L	0.92
11	0.13	0.84	99.45	K mg/L	0.88	11	0.12	0.78	99.38	K mg/L	0.92
12	0.04	0.25	99.7	F mg/L	0.56	12	0.05	0.36	99.75	F mg/L	0.7
13	0.02	0.15	99.85	B mg/L	0.77	13	0.02	0.14	99.89	Bmg/L	0.49
14	0.02	0.13	99.98	SiO2 mg/L	0.79	14	0.02	0.1	99.99	SiO2 mg/L	0.6
15	0	0.02	100	TDS mg/L	0.97	15	0	0.01	100	TDS mg/L	0.74

Table 2B. Unrotated Principal Component(PCA)Matrix
Extra-Peninsula Peninsula

	Component					Component			
	1	2	3	4		1	2	3	4
TEMPC	-0.14	0.58	0.16	0.02	Temp_C	0.59	-0.29	0.45	-0.26
pH	0.26	0.14	-0.49	0.23	pH	-0.54	0.55	0.07	0.2
SPCMHO/cm	-0.38	-0.09	0.08	0.38	SPCMHO/cm	0.52	0.83*	0.04	-0.07
HCO3 mg/L	-0.35	0.62*	-0.28	0.49	HCO3 mg/L	0.6*	-0.56	-0.44	0.04
Cl mg/L	-0.6*	0.19	0.05	-0.65*	Cl mg/L	0.69*	0.21	0.49	0.38
SO4 mg/L	-0.8*	-0.45	0.19	0.23	SO4 mg/L	0.64*	-0.55	0.14	-0.21
TotHard	-0.83*	-0.46	0.18	0.16	TotHard	0.21	0.26	0.68*	0.31
Ca mg/L	-0.77*	-0.49	0.22	0.23	Ca mg/L	0.35	0.77*	0.21	-0.24
Mg mg/L	-0.56	-0.04	-0.65*	0.08	Mg mg/L	0.38	0.75*	-0.43	-0.17
Na mg/L	-0.78*	0.51	0.07	-0.12	Na mg/L	0.6*	0.69*	-0.24	-0.17
K mg/L	-0.74*	0.34	-0.45	-0.07	K mg/L	0.89*	-0.27	-0.24	-0.09
F mg/L	-0.06	0.56	0.48	0.08	F mg/L	-0.2	-0.11	0.53	-0.6*
B mg/L	-0.78*	-0.01	-0.19	-0.35	Bmg/L	0.09	0.04	0.13	0.68*
SiO2 mg/L	-0.24	0.78*	0.32	0.17	SiO2 mg/L	-0.31	-0.01	0.67*	-0.22
TDS mg/L	-0.97*	-0.06	0.11	-0.05	TDS mg/L	0.73*	-0.32	0.25	0.22

Table 3. Rotated Component Matrix (VARIMAX)

	<u>Extra-Peninsula</u>				<u>Peninsula</u>				
	Component				Component				
	1	2	3	4	1	2	3	4	
TEMPC	0.15	0.57	-0.07	-0.18	Temp_C	0.61	0.09	0.36	-0.43
pH	0.33	-0.11	-0.42	0.29	pH	-0.78*	0.15	0.01	0.08
SPCMHO/cm	-0.46	0.2	-0.16	0.17	SPCMHO/cm	-0.02	0.97*	0.12	0.03
HCO3 mg/L	0.01	0.61*	-0.67*	0.12	HCO3 mg/L	0.84*	-0.16	-0.14	0.34
Cl mg/L	-0.19	0.11	-0.05	-0.87*	Cl mg/L	0.3	0.45	0.79*	0.07
SO4 mg/L	-0.96*	0.03	-0.09	-0.08	SO4 mg/L	0.83*	-0.11	0.16	-0.21
TotHard	-0.97*	0	-0.09	-0.15	TotHard	-0.14	0.25	0.75*	-0.15
Ca mg/L	-0.97*	0	-0.04	-0.05	Ca mg/L	-0.12	0.86*	0.11	-0.24
Mg mg/L	-0.29	-0.16	-0.77*	-0.19	Mg mg/L	0	0.87*	-0.34	0.22
Na mg/L	-0.3	0.59	-0.33	-0.58	Na mg/L	0.18	0.93*	-0.13	0.12
K mg/L	-0.24	0.22	-0.73*	-0.48	K mg/L	0.91*	0.25	0.01	0.16
F mg/L	0.1	0.7*	0.22	-0.08	F mg/L	-0.06	-0.12	0	-0.83*
B mg/L	-0.45	-0.01	-0.36	-0.66*	Bmg/L	-0.1	-0.04	0.52	0.46
SiO2 mg/L	0.11	0.87*	-0.07	-0.14	SiO2 mg/L	-0.3	-0.16	0.3	-0.63*
TDS mg/L	-0.78*	0.24	-0.24	-0.48	TDS mg/L	0.68*	0.06	0.52	0.08

Table 4. Rotated Component Matrix (PROMAX)

<u>Extra-Peninsula</u>					<u>Peninsula</u>				
Factanal (x = data, factors = 4, rotation = "promax")					Factanal (x = data, factors = 4, rotation = "promax")				
	Factor1	Factor2	Factor3	Factor4	Temp_C				
TEMPC	-0.134		0.234	0.105	Temp_C	0.413		0.211	0.120
pH	-0.300	-0.106	0.206		pH	-0.649*		0.129	
SPCMHO.cm	0.323	0.181			SPCMHO.cm		0.925*	0.225	
HCO3.mg.L		0.909*	-0.484	0.462	HCO3.mg.L	0.898*	-0.107	-0.215	
Cl.mg.L			0.869*		Cl.mg.L	0.271	0.194	0.767*	0.148
SO4.mg.L	0.998*				SO4.mg.L	0.847*	-0.197	0.219	.318
TotHard	0.976*				TotHard	-0.156		0.996*	-0.122
Ca.mg.L	1.077*		-0.117		Ca.mg.L	-0.124	0.681*	0.302	-0.140
Mg.mg.L			0.725*		Mg.mg.L		0.929*	-0.222	
Na.mg.L	0.100	0.650*	0.458	0.165	Na.mg.L	0.165	0.99*	-0.116	
K.mg.L		0.315	0.228	0.776*	K.mg.L	0.990*	0.256		-0.109
F.mg.L		0.454	0.186	-0.251	F.mg.L	-0.197	-0.149		-0.128
B.mg.L		-0.157	0.620*	0.559*	Bmg.L	-0.246			1.048*
SiO2.mg.L		0.685*			SiO2.mg.L	-0.309	-0.222	0.324	
TDS.mg.L	0.623*	0.162	0.374	0.166	TDS.mg.L	0.618*		0.233	0.335
SS loadings	3.752	2.171	1.853	1.839	SS loadings	3.816	3.414	2.112	1.431
Proportion Var	0.250	0.145	0.124	0.123	Proportion Var	0.254	0.228	0.141	0.095
Cumulative Var	0.250	0.395	0.518	0.641	Cumulative Var	0.254	0.482	0.623	0.718
Factor Correlations:					Factor Correlations:				
	Factor1	Factor2	Factor3	Factor4		Factor1	Factor2	Factor3	Factor4
Factor1	1.0000	0.0698	0.332*	0.136	Factor1	1.000	-0.076	0.038	.213
Factor2	-0.0698	1.0000	-0.448*	-0.443*	Factor2	-0.076	1.000	-0.273	-.210
Factor3	0.3322*	-0.4479*	1.000	0.226	Factor3	0.038	-0.273	1.000	0.152
Factor4	0.1360	-0.4426*	0.226	1.000	Factor4	0.213	-0.210	0.152	1.000
# above ± 0.32 = criterion for Promax rotation					Test of the hypothesis that 4 factors are sufficient.				
Test of the hypothesis that 4 factors are sufficient.					The chi square statistic is 106.33 on 51 degrees of freedom.				
The chi square statistic is 106.33 on 51 degrees of freedom.					The p-value is 9.03e-06				
The p-value is 9.03e-06									

II. Interpretation of the results

Table 5A. Results of Factor Analysis (PCA vs VARIMAX)

EXTRA-PENINSULA		PENINSULA	
Unrotated PCA	Rotated VARIMAX	Unrotated PCA	Rotated VARIMAX
F1 Cl, SO4, Ca, Na, K, B, TDS	F1 SO4, Ca, TDS	F1 HCO3, Cl, SO4, Na, K, TDS	F1 pH, HCO3, SO4, K, TDS
F2 HCO3, SiO2	F2 HCO3, F, SiO2	F2 SPCMHO*, Ca, Mg, Na	F2 SPCMHO*, Ca, Mg, Na
F3 Mg-	F3 HCO3-, Mg-, K-	F3 SiO2	F3 Cl
F4 Cl -	F4 Cl-, F-	F4 B, F-	F4 F, SiO2

SPCMHO/Cm* - is correctly defined as the electrical conductance of 1 cubic centimeter of a solution at 25 °C used to estimate the salinity, ionic strength and concentrations of major TDS solutes in natural waters.

Table 5B. Results of Factor Analysis (VARIMAX vs PROMAX)

Ex-Peninsula	Peninsula	Ex-Peninsula	Peninsula
Varimax Rotation		Promax Rotation	Varimax Rotation
F1 SO4, Ca, TDS	F1 pH-, HCO3, SO4, K, TDS	F1 SO4, TotHard, Ca, TDS	F1 pH, HCO3, SO4, K, TDS
F2 HCO3, F, SiO2	F2 SPCMHO*, Ca, Mg, Na	F2 HCO3, Na, SiO2	F2 SPCMHO, Ca, Mg, Na
F3 HCO3, Mg, K	F3 Cl	F3 Cl, Mg, B	F3 Cl, TotHard
F4 Cl, F	F4 F-, SiO2-	F4 K, B	F4 B

III. Discussion

Given the sophistication of computational tools available today, orthogonal or varimax rotation is unlikely to be the best rotation practice, as oblique rotation can accurately represent both orthogonal and oblique rotations (Kimani Chege Gabriel, 2019). This broad generalization about rotation techniques is undesirable, because there is no conclusive answer to the question of which factor rotation approach is ideal. According to Fabrigar et al. (1999), both approaches have limits but give comparable results, and it appears to be safe to utilize the software package's default settings.

The following points should be underlined in order to achieve progressive dimension reduction from Varimax rotation versus Promax rotation:

1. Consistency in the number of factors, for example, 4
2. Promax rotation necessitates a big data set, often more than 150 rows.
3. Unless there are compelling grounds for orthogonal rotation, the factor correlation matrix for correlations around ± 0.32 and higher necessitates oblique rotation.
4. The number of variables with high loadings on each component is reduced by an orthogonal rotation. This strategy makes the components easier to comprehend.
5. By far the most used rotation method is Varimax, which was created by Kaiser (1958).
6. At the very least, it appears good to test one oblique rotation approach, such as promax, while analyzing the factor correlation matrix for values greater than ± 0.32 using the criterion described above, and one orthogonal rotation method (such as the ever-popular varimax rotation).
7. Both rotation techniques have been investigated in EFA. Some of the factors are correlated with one another, while others are uncorrelated with oblique rotation (Table 4), suggesting that orthogonal rotation is more beneficial.
8. With the number of factors kept constant, e.g., 4, four factors account for $75 \pm 2\%$ of the total variance of the original variable "load on a factor, whether it is in the case of the extra-peninsula or the peninsula.
9. In contrast, four factors account for 64% of the total variation of the original variables "load" on a factor in extra-Peninsula, where some factors are correlated, but 72% of the total variation of the original variables "load" on a factor in Peninsula, where factors are uncorrelated.
10. Overmaximization appears to be less successful for spatial analysis in the current study, resulting in some redundancy and complexity in the components and making it more difficult to compare the results across different locations and samples (Table- 5A-B)

11. According to A.Fog (2014). Why Is Factor Rotation Always Recommended, Though It Obscures General Factors.

Given all of the aforementioned considerations, there are persuasive as well as compromising arguments in favor of orthogonal rotation (varimax rotation).

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