Partial Derivatives Involving Generalized I–Function of Two Variables

Shanti Swaroop Dubey¹, Dr. S. S. Shrivastava²
¹ITM University, Raipur (C. G.)
²Institute for Excellence in Higher Education, Bhopal (M. P.)

Abstract: The aim of this paper is to derive partial derivatives involving generalized I–function of two variables.

I. Introduction

The generalized I–function of two variables introduced by Goyal and Agrawal [1], will be defined and represented as follows:

\[
I[x] = \int_{0}^{1} \int_{0}^{1} \phi_1(\xi, \eta) \phi_2(\epsilon, \eta) x^{\alpha} y^{\beta} d\xi d\eta,
\]

where

\[
\phi_1(\xi, \eta) = \frac{\Pi_{i=1}^{m} \Gamma(1-\beta_i-\eta_0)}{\sum_{i=1}^{m} \Gamma(1-\beta_i-\eta_0)},
\]

\[
\phi_2(\epsilon, \eta) = \frac{\Pi_{i=1}^{m} \Gamma(1-\gamma_i-\eta_0)}{\sum_{i=1}^{m} \Gamma(1-\gamma_i-\eta_0)}.
\]

x and y are not equal to zero, and an empty product is interpreted as unity if \(p_i, q_i, r_i, s_i, t_i, m, n, r_i, s, t, k\) are non-negative integers such that \(p_i \geq 0, q_i \geq 0, r_i \geq 0, s_i \geq 0, t_i \geq 0, \) \(k = 1, 2\) also all the \(A, A', B, B', \) \(i = 1, \ldots, p_i\) and \(j = 1, \ldots, q_i\) are assumed to be positive quantities for standardization purpose; the definition of I–function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour \(L_1\) is in the \(\xi\)–plane and runs from \(-\infty\) to \(+\infty\), with loops, if necessary, to ensure that the poles of \(\Gamma(d_0-\Delta_0)\) \(j = 1, \ldots, m\) lie to the right, and the poles of \(\Gamma(1-c_1+\gamma_0)\) \(j = 1, \ldots, n\) \(\Gamma(1-c_1+\alpha_0+\beta_0)\) \(j = 1, \ldots, n\) to the left of the contour.

The contour \(L_2\) is in the \(\eta\)–plane and runs from \(-\infty\) to \(+\infty\), with loops, if necessary, to ensure that the poles of \(\Gamma(d_0-\Delta_0)\) \(j = 1, \ldots, m, n\) lie to the right, and the poles of \(\Gamma(1-c_1+\gamma_0)\) \(j = 1, \ldots, n\) \(\Gamma(1-c_1+\alpha_0+\beta_0)\) \(j = 1, \ldots, n\) to the left of the contour.

Also

\[
R' = \sum_{i=1}^{p_1} a_{i1} + \sum_{i=1}^{p_2} a_{i1} \beta_i - \sum_{i=1}^{q_1} \delta_i < 0,
\]

\[
S' = \sum_{i=1}^{p_2} A_{i1} + \sum_{i=1}^{q_2} E_i - \sum_{i=1}^{q_1} B_i - \sum_{i=1}^{q_1} F_{\delta_i} < 0,
\]

\[
U = \sum_{i=1}^{q_1} \beta_i - \sum_{i=1}^{q_1} \delta_i - \sum_{i=1}^{q_1} \gamma_i - \sum_{i=1}^{q_1} \gamma_i - \sum_{i=1}^{q_1} \gamma_i > 0,
\]

\[
V = -\sum_{i=1}^{p_1} A_{i1} - \sum_{i=1}^{q_1} B_{i1} - \sum_{i=1}^{q_1} F_i - \sum_{i=1}^{q_1} \gamma_i - \sum_{i=1}^{q_1} \gamma_i - \sum_{i=1}^{q_1} \gamma_i > 0,
\]

and \(|\arg x| < \frac{1}{2} \pi, |\arg y| < \frac{1}{2} \pi\).

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II. Result Required

The following result are required in our present investigation:

From Rainville [2]:

\[ z \Gamma(z) = \Gamma(z + 1). \]  

(4)

III. Main Result

In this paper we will establish the following partial derivatives:

\[
\frac{\partial}{\partial x} \left\{ \frac{m!n!}{p_1!q_1!} \binom{m_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\} = x^{-1} \frac{m!n!}{p_1!q_1!} \binom{m_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \left[ \frac{\partial^2}{\partial x^2} \left\{ \frac{m!n!}{p_1!q_1!} \binom{m_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\} \right]
\]

\[
= \frac{\partial^2}{\partial x^2} \left\{ \frac{m!n!}{p_1!q_1!} \binom{m_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\}
\]

where \( \arg x < \frac{1}{2} U \), \( |y| < \frac{1}{2} V \), where \( U \) and \( V \) are given in (2) and (3) respectively.

IV. Special Cases

On choosing \( m = 0 \) main results, we get following partial derivatives in terms of \( I \)-function of two variables:

\[
\frac{\partial}{\partial x} \left\{ \frac{0!n!}{p_1!q_1!} \binom{0_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\} = x^{-1} \frac{0!n!}{p_1!q_1!} \binom{0_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \left[ \frac{\partial}{\partial x} \left\{ \frac{0!n!}{p_1!q_1!} \binom{0_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\} \right]
\]

\[
= \frac{\partial}{\partial x} \left\{ \frac{0!n!}{p_1!q_1!} \binom{0_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\}
\]

Now using \( \frac{\partial}{\partial x} \left\{ \frac{1!n!}{p_1!q_1!} \binom{1_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\} = \frac{1!n!}{p_1!q_1!} \binom{1_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \) and \( \frac{\partial}{\partial x} \left\{ \frac{1!n!}{p_1!q_1!} \binom{1_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \right\} = \frac{1!n!}{p_1!q_1!} \binom{1_1}{n_1} \binom{m_2}{n_2} \binom{x}{y} \),

IV. Special Cases

On choosing \( m = 0 \) main results, we get following partial derivatives in terms of I-function of two variables:
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\[
\frac{\partial}{\partial y} \left\{ \frac{1}{y} \sum_{p_1 \cdots p_{n_2}} \prod_{q_1 \cdots q_{m_2}} \left[ \sum_{i_1 \cdots i_{n_2}} \left( A_{i_1} \cdots A_{i_{n_2}} \right) \right] \right\} = y^{-1} \sum_{p_1 \cdots p_{n_2}} \prod_{q_1 \cdots q_{m_2}} \left[ \sum_{i_1 \cdots i_{n_2}} \left( A_{i_1} \cdots A_{i_{n_2}} \right) \right]
\]

(9)

\[
\frac{\partial^2}{\partial x \partial y} \left\{ \frac{1}{y} \sum_{p_1 \cdots p_{n_2}} \prod_{q_1 \cdots q_{m_2}} \left[ \sum_{i_1 \cdots i_{n_2}} \left( A_{i_1} \cdots A_{i_{n_2}} \right) \right] \right\} = (xy)^{-1} \sum_{p_1 \cdots p_{n_2}} \prod_{q_1 \cdots q_{m_2}} \left[ \sum_{i_1 \cdots i_{n_2}} \left( A_{i_1} \cdots A_{i_{n_2}} \right) \right]
\]

(10)

where \(|\arg x| < \frac{1}{2} U'\pi, |\arg y| < \frac{1}{2} V'\pi\), where \(U'\) and \(V'\) are given as follows respectively:

\(U' = \sum_{j=n+1}^{p_i} q_j - \sum_{j=1}^{q_i} p_j + \sum_{j=m+1}^{q_i} p_j - \sum_{j=n+1}^{p_i} q_j + \sum_{j=1}^{n} q_j - \sum_{j=m+1}^{p_i} q_j > 0,\)

\(V' = -\sum_{j=n+1}^{p_i} A_j - \sum_{j=1}^{q_i} B_j - \sum_{j=m+1}^{q_i} F_j - \sum_{j=m+1}^{q_i} F_j + \sum_{j=1}^{n} E_j - \sum_{j=m+1}^{p_i} E_j > 0,\)

References
