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Emission of carbon dioxide from different attributes in India: A mathematical study

Sumit Nandi[1], Pijush Basak[2]
[1]*Department of Chemistry, Narula Institute of Technology, Agarpara, Kolkata-700109, West Bengal, India*
[2]*Department of Mathematics, Narula Institute of Technology, Agarpara, Kolkata-700109, West Bengal, India*

Abstract: *Increased emission of carbon dioxide from different sources is quite appalling for the last few decades which ultimately results global warming. Emission of carbon dioxide mainly comes from four attributes namely solid fuels, liquid fuels, gaseous fuels and cement industry. In this alarming situation, a mathematical model has been developed based on these attributes with a system of differential equations in Indian perspective. A statistical approach, namely search method is applied to study the behavior of the said attributes in the developed model utilizing the data set of twenty one years in India. The solutions of the environmental model are compared with the real data from where future prediction for the emission of carbon dioxide can be made.*

Keywords: *Carbon dioxide emission, Differential equation, Global warming.*

I. INTRODUCTION

Environmental degradation for the last few decades increases so rapidly that existence of human life comes to a question in future. Global warming seems to be considered as one of the most important environmental tribulations for this issue. Among the green house gases responsible for global warming, carbon dioxide (CO_2) is the most important that is being affected by human activities. Presently, the rate of CO_2 rising in the environment is about 1.5 parts per million by volume (ppmv) per year and if emission continues in this rate, the concentration will reach 500 ppmv (at present 367 ppmv) by the end of twenty first century which is very much alarming for the existence of living system. Due to rise in concentration of CO_2, it is predicted that the average global surface temperature could rise 0.6-2.5^0 C in the next 50 years and 1.4^0 to 5.8^0 C by the year 2100 (1).

CO_2 is emitted from different sources like solid fuels, liquid fuels, gaseous fuels and from cement industry. The major global sources of CO_2 emission are liquid fuels (mainly petroleum products). Solid duels (mainly coal) come second in importance. But in India, solid fuel is much more important than liquid fuels regarding CO_2 emission. For the world, the solid liquid ratio is 33:44. In India, it is 68:24. Cement manufacturing industry also releases CO_2 as it uses essentially 100% calcium oxide which is obtained by burning calcium carbonate during calcinations. Several studies [2, 3, 4, 5 and 6] have been done by different researchers for the emissions of green house gases in India. Ghoshal and Bhattacharyya [7] made a detailed survey regarding state level CO_2 emissions of India during the year 1980-2000.

Estimated emission of CO_2 by mathematical modeling has been attempted by many researchers. Bert W. Rust [8] demonstrated the connections between fossil fuel emissions, atmospheric CO_2 concentrations and global temperatures by coupled mathematical models for their measured time series. Chris P. Tokos et al [9] developed differential equations for the emission of CO_2 based on six attributes. Jin et al [10] made a dynamic evolutionary model of carbon emissions in Yangtze Delta, China and they showed that due to excessive dependency of fossil fuels, carbon emission has risen dramatically after year 2000. In 1990, Thomas J. Goreau [11] briefly mentioned that the rate of change of CO_2 emissions in the atmosphere could be studied using differential equations.

In the present study, we have developed mathematical models for the emissions of CO_2 with a system of differential equations based on the four attributes namely, solid fuels, liquid fuels, gaseous fuels and cement industry in Indian perspective. Using real historical data on the subject phenomenon, analytical form of the equations are developed. From the analytical solution, the CO_2 emissions by various sources is to be estimated for short and long range of time so that remedial measures can be taken to reduce the emissions as far as practicable without compromising economic growth.

II. MATHEMATICAL FORMULATIONS:

To generate a mathematical model of CO_2 emissions in India, we need to consider the different sources of CO_2 emissions mainly from solid fuels (S), liquid fuels (L), gaseous fuels (G) and cement industry(C) as these are the main sources. In our model, the functional form of the differential equations is represented by the following equation:

$$\frac{d(E)}{dT} = f\left(\frac{d(S)}{dT}, \frac{d(L)}{dT}, \frac{d(G)}{dT}, \frac{d(C)}{dT}\right)$$

where E is the total emission of CO_2 by all the sources. Now, we can represent the total rate of change of carbon dioxide emissions as a function of time by the following equation:

$$\frac{d(E)}{dT} = \alpha \frac{d(S)}{dT} + \beta \frac{d(L)}{dT} + \gamma \frac{d(G)}{dT} + \delta \frac{d(C)}{dT} + \mu$$

where α, β, γ and δ are the coefficients of each differential term and μ is a constant. Here, we consider the differential equations of each attribute. We use three statistical criteria, coefficient of determination (R^2) [12], PRESS statistics [13] and residual analysis [14]. The coefficient of determination R^2 is defined as the proportion of the total response variation that is explained by the model. It provides an overall measure of how well the model fits. Model with the lowest value of PRESS indicates the best structure. Models that are over-parameterized would tend to give small residuals and are included in the model-fitting but large residuals for observations are excluded. PRESS will evaluate how good the estimation will be if each time we remove one data. For the formulation and analysis of our model, we consider twenty one year's data from 1980 to 2000 in India [7].

III. SOLID FUELS AND LIQUID FUELS:

The differential equation for the emission of CO_2 from solid fuel and liquid fuel can be represented by

$S(T) + \frac{dS(T)}{dT} = 109.9T^2 - 4.2817 \times 10^5 T + 4.17 \times 10^8$ --------------(1) and

$L(T) + \frac{dL(T)}{dT} = 0.00011 \times T^3 - 0.24596 \times T^2 + 1.536 \times 10^3 \times T - 2.9211 \times 10^6$ -----------(2) respectively,

where T represents year in the equation (1) and (2). The solution of the equation (1) and (2) are given by

$S(T) = 109.9T^2 - 4.2839 \times 10^5 T + 4.1743 \times 10^8$ ----------------(3)

$L(T) = 0.000112 \times T^3 - 0.2463 \times T^2 + 1537.0848 \times T - 2922678$ ------------- (4) respectively.

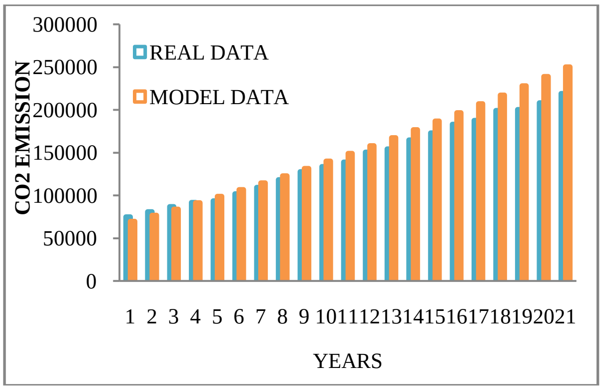


Figure 1 Emission of CO_2 due to solid fuels.

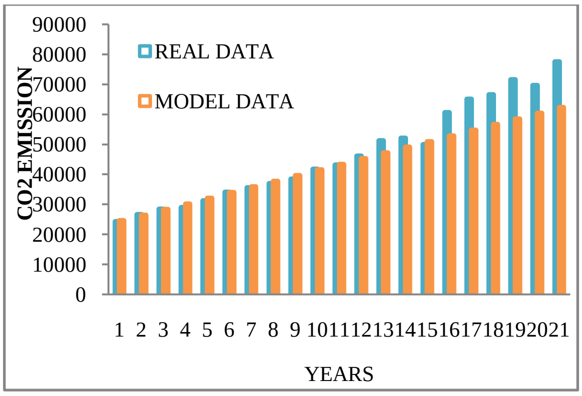


Figure 2 Emission of CO_2 due to liquid fuels

The graphical representation of the real data and the solution of the differential equation for the emission of carbon dioxide from solid fuels and liquid fuel are given by Fig. 1 and Fig. 2 respectively. It is evident from figures that our CO_2 emission model matches well with the actual status of CO_2 emission from solid fuels and liquid fuels.

The calculated values for the solid and liquid fuel model of R^2, R^2 adjusted and PRESS statistic are given by Table 1 below.

Table 1

Solid fuel			Liquid fuel		
R^2	R^2 adjusted	PRESS statistic	R^2	R^2 adjusted	PRESS statistic
0.8944	0.8758	1.30057638E+009	0.8527	0.8267	224305568

From the value of R^2 and adjusted R^2, it can be concluded that we have developed a good model for solid and liquid fuel along with a PRESS statistic value that is the smallest of several models that we have tested. Furthermore, the residual analysis on the proposed differential equation of these fuels is given in Table 2 and Table 3 below.

Table 2 Residual analysis for emission of CO_2 from solid fuels in India ('000 MT)

Year	Real data	Model data	Residual	Year	Real data	Model data	Residual
1980	74800.09	69781.98	5018.10	1991	150505.18	158033.95	-7528.76
1981	80958.90	76705.89	4253.01	1992	154222.09	167375.65	-13153.56
1982	86733.96	83849.59	2884.37	1993	164893.28	176937.15	-12043.87
1983	91875.82	91213.10	662.72	1994	173186.10	186718.46	-13532.35
1984	93810.89	98796.40	-4985.51	1995	183086.90	196719.57	-13632.67
1985	101748.03	106599.51	-4851.47	1996	187459.35	206940.48	-19481.12
1986	109278.15	114622.42	-5344.26	1997	199328.32	217381.18	-18052.85
1987	118181.96	122865.12	-4683.15	1998	200189.01	228041.68	-27852.67
1988	127641.40	131327.62	-3686.21	1999	208194.59	238922.00	-30727.40
1989	133625.10	140009.93	-6384.82	2000	218232.85	250022.10	-31097.98
1990	138817.12	148912.04	-10094.92				
			Mean of residuals				-10205.49
			Standard deviation of residuals (SD)				10621.10
			Standard error of residuals (SE)				2317.71

Table 3 Residual analysis for emission of CO_2 from liquid fuels in India ('000 MT)

Year	Real data	Model data	Residual	Year	Real data	Model data	Residual
1980	24326.73	24552.58	-225.84	1991	46150.38	45272.35	878.02
1981	26667.35	26432.01	235.34	1992	51185.87	47161.05	4024.82
1982	28459.20	28312.21	146.99	1993	52056.78	49050.57	3006.21
1983	28943.68	30193.35	-1249.66	1994	49939.97	50940.91	-1000.94
1984	31234.90	32075.26	-840.36	1995	60633.19	52832.14	7801.04
1985	34041.28	33958.06	83.22	1996	65118.39	54724.20	10394.18
1986	35626.44	35841.68	-215.24	1997	66586.84	56617.14	9969.69
1987	36932.90	37726.13	-793.23	1998	71606.75	58510.91	13095.84
1988	38433.11	39611.40	-1178.28	1999	69702.04	60405.50	9262.54
1989	41738.91	41497.56	241.34	2000	72496.78	62300.98	15210.34
1990	43112.50	43384.54	-272.04				
			Mean of residuals				3267.04
			Standard deviation of residuals (SD)				5315.23
			Standard error of residuals (SE)				1159.87

Small residuals and standard error of residuals support the good quality of the proposed model for solid fuels and liquid fuels. Here our predicted value of emission of CO_2 form solid fuels in India in 2015 and 2020 are 442,877.5 and 518,160 ('000 MT) respectively and for liquid fuels in India in 2015 and 2020 are 90826.0325 and 100, 380.472 ('000 MT) respectively.

IV. GASEOUS FUELS AND CEMENT INDUSTRY

The differential equation for the emission of CO_2 from gaseous fuel and cement industry are represented by

$G(T) + \frac{dG(T)}{dT} = 6 \times 10^{-7} \times T^3 – 39.54 \times 10^{-3} \times T^2 + 759.2089 \times T – 1.3516 \times 10^6$ ------------------- (5) and

$C(T) + \frac{dC(T)}{dT} = 0.000199 \times T^3 – 12.679 \times T^2 + 5.3419 \times 10^4 \times T – 5.4503 \times 10^7$ -------------------- (6)

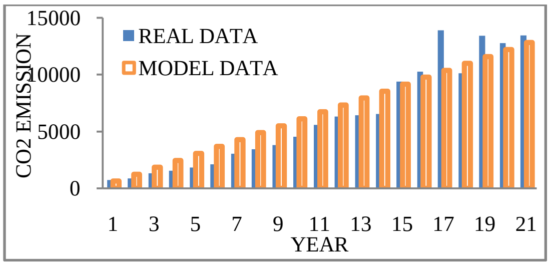
respectively.

The solution of the equation (5) and (6) are given by

$G(T) = 6 \times 10^{-7} \times T^3 – 39.542 \times T^2 + 759.288 \times T – 1.3523 \times 10^6$ -------------------------- (7) and

$C(T) = 0.0002 \times T^3 – 12.68 \times T^2 + 53445 \times T – 54557441$---------------------------------- (8) respectively.

The graphical representation of the actual data and the solution of the differential equation for the emission of carbon dioxide from gaseous fuels and cement industry are given by Fig. 3 and Fig. 4 respectively.



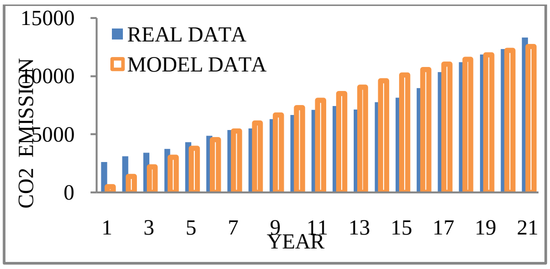


Figure 3 Emission of CO_2 due to gaseous fuels

The calculated values of R^2, R^2 adjusted and PRESS s
cement industry model are given by Table 4 below.

Table 4

Gaseous fuel			Cement industry		
R^2	R^2 adjusted	PRESS statistic	R^2	R^2 adjusted	PRESS statistic
0.9022	0.8850	35873032	0.8656	0.8418	1934429.25

It is shown from the above values that an excellent correlation of R^2 and adjusted R^2 are obtained for gaseous fuel and cement industry. Reasonably here, PRESS statistic values of both the cases are considered that is the smallest of several models that we tested. Furthermore, the residual analysis we performed on the proposed differential equation of gaseous fuels and cement industry are given in Table 5 and Table 6 respectively below.

Table 5 Residual analysis for emission of CO_2 from gaseous fuels in India ('000 MT)

Year	Real data	Model data	Residual	Year	Real data	Model data	Residual
1980	740.28	635.26	105.02	1991	6303.85	7338.25	-1034.40
1981	866.77	1244.98	-378.21	1992	6423.15	7947.18	-1524.03
1982	1332.13	1854.63	-522.50	1993	6548.02	8556.04	-2008.02
1983	1554.19	2464.21	-910.02	1994	9376.08	9164.83	211.24
1984	1845.15	3073.72	-1228.56	1995	10263.84	9773.54	490.29
1985	2124.87	3683.15	-1558.28	1996	13906.35	10382.19	3524.16
1986	3048.83	4292.52	-1243.68	1997	10113.36	10990.76	-877.40
1987	3451.53	4901.81	-1450.28	1998	13409.50	11599.26	1810.24
1988	3811.25	5511.02	-1699.77	1999	12761.49	12207.69	553.79
1989	4526.13	6120.17	-1594.04	2000	13678.09	12816.04	631.77
1990	5569.27	6729.25	-1159.97				
			Mean of residuals				-469.65
			Standard deviation of residuals (SD)				1339.19
			Standard error of residuals (SE)				292.23

Table 6 Residual analysis for emission of CO_2 from cement industry in India ('000)

Year	Real data	Model data	Residual	Year	Real data	Model data	Residual
1980	2611.57	500.69	2110.88	1991	7432.61	8501.94	-1069.33
1981	3118.13	1366.75	1751.38	1992	7131.91	9062.83	-1930.91
1982	3415.10	2205.19	1209.91	1993	7764.85	9595.99	-1831.14
1983	3732.61	3015.71	716.89	1994	8142.38	10101.44	-1959.06
1984	4314.21	3798.62	515.58	1995	8970.96	10579.08	-1608.11
1985	4871.30	4553.71	317.58	1996	10353.43	11028.99	-675.56
1986	5354.79	5281.09	73.70	1997	11208.25	11451.09	-242.84
1987	5508.41	5980.75	-472.34	1998	11859.96	11845.48	14.47
1988	6307.45	6652.70	-345.25	1999	12336.10	12212.15	123.95
1989	6666.10	7296.83	-630.72	2000	13876.80	12551.00	769.98
1990	7109.70	7913.25	-803.54				
			Mean of residuals				-188.78
			Standard deviation of residuals (SD)				1146.36
			Standard error of residuals (SE)				250.15

As seen from the tables, the residuals are small compared to the data and so are the standard errors. Here our predicted value of emission of CO_2 form gaseous fuels in India in 2015 and 2020 are 21932.715 and 24968.028 ('000 MT) respectively and from cement industry in India in 2015 and 2020 are 14310.325 and 13505.4 ('000 MT) respectively.

V. CONCLUSION

We have developed differential equations based on the emission of carbon dioxide for each of the four main attributes using actual data from 1980 to 2000 for India. We have used three different statistical procedures, namely R^2 (R^2 adjusted), PRESS statistic and residual analysis to evaluate the quality of the proposed differential methods. The models are analyzed by using regression analysis method and it illustrates that the model matches well with the actual status of India's carbon dioxide emission from four main attributable variables. All these statistical procedures advocate to the quality of the proposed differential systems. Finally, we predict the short and medium term total carbon emissions trend in India by utilizing our model. Proper framing of emission strategies and policies are immediately required to restrain the rapid increasing of CO_2 emission. The information from our models provides a theoretical basis for the further study on the undesirable situation of carbon dioxide emissions in India and should be useful for intended planning and formulating policies to prevent the distressed situation of global warming.

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